

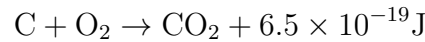
Problem Set 1

Exercise 1 - Combustion, Fission or Fusion?

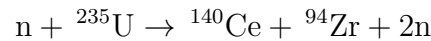
A Swiss household consumes 18780 kWh per year on average. With 1 kg of *fuel*, for how many years can you supply energy for one household?

Consider three cases:

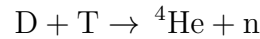
- a) The conventional **chemical reaction**, which produces 6.5×10^{-19} J of energy per reaction.



- b) The **nuclear fission** reaction



- c) The **nuclear fusion** reaction



Hint: For points b) and c) use Einstein's relation:

$$E = \Delta m_0 c^2$$

where Δm_0 is the *mass deficit*, i.e. the difference between the rest masses before and after the reaction.

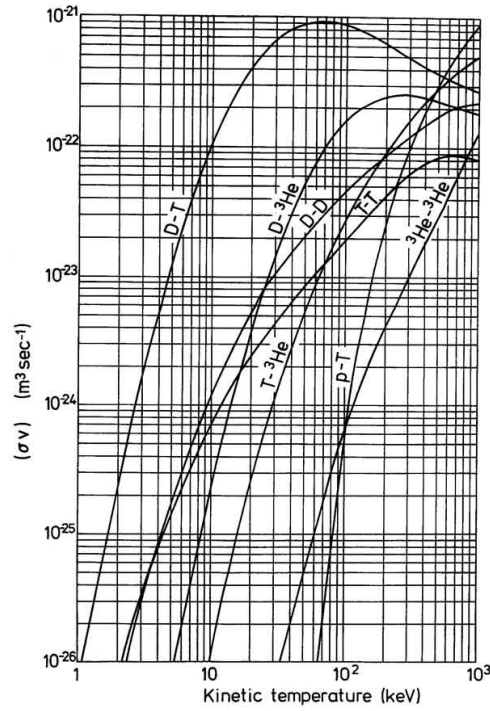
Indications: $m_{\text{C}} = 12.011 \text{ u}$, $m_{\text{O}} = 15.999 \text{ u}$, $m_{\text{n}} = 1.0087 \text{ u}$, $m_{\text{U}} = 235.04 \text{ u}$, $m_{\text{Ce}} = 139.91 \text{ u}$, $m_{\text{Zr}} = 93.91 \text{ u}$, $m_{\text{D}} = 2.014 \text{ u}$, $m_{\text{T}} = 3.0164 \text{ u}$, $m_{\text{He}} = 4.0027 \text{ u}$, with $1 \text{ u} = 1.66 \cdot 10^{-27} \text{ kg}$.

Exercise 2 - A “small” Tokamak

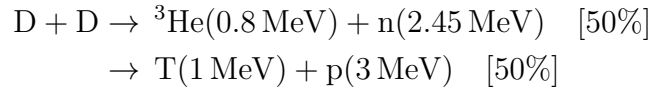
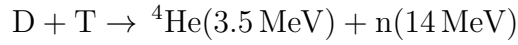
Consider a Tokamak plasma uniformly distributed in a volume $V = 10 \text{ m}^3$ with the following parameters: $B = 2 \text{ T}$; $n_e = 10^{20} \text{ m}^{-3}$; $T_i = T_e = 10 \text{ keV}$. The total heating power delivered to the plasma is $P_{\text{in}} = 28 \text{ MW}$ and the tokamak is assumed to be in “steady state”.

- a) Using the cross-sections in the figure below, calculate:

- The power in the α -particle population and the total fusion power in D-T (50:50 mixture).
- The total fusion power in D-D.



Hint: The fusion reactions and energies of the products are:



- b) Calculate the *physics fusion gain* “ Q ” and the Lawson parameter $n_e \tau_E$. How close are we to break-even?
- c) Making simple assumptions on the tokamak geometry (torus with major radius $R_0 = 1\text{m}$ and circular cross-section), consider the situation of a D-T mixture plasma with the heat flux concentrated only in 1/10 of the wall surface. Is it suitable to cover that portion of surface with a material having a heat tolerance of 4 MW/m^2 ?

Exercise 3 - The effect of impurity contamination on the plasma power balance

1. Assuming a $T = 10 \text{ keV}$ plasma with $n_e = 2 \times 10^{20} \text{ m}^{-3}$ and a volume $V = 10 \text{ m}^3$, calculate the radiated power due to impurity contamination and the fractional reduction of the fusion power due to dilution of the fusion fuel, in two cases:

a) Carbon impurities:

- Assume that the carbon atoms are fully ionized ($Z = 6$), and $n_{C^{6+}}/n_e = 4\%$.
- The radiated power function in $T = 10$ keV, assuming coronal equilibrium, is $R_C = 10^{-34} \text{ W m}^3$.

b) Tungsten impurities:

- Assume that the charge state of the tungsten atoms is 50 ($Z = 50$), and $n_{W^{50+}}/n_e = 10^{-5}$.
- The radiated power function in $T = 10$ keV, assuming coronal equilibrium, is $R_W = 10^{-31} \text{ W m}^3$.

Note:

- The impurity radiation losses are calculated as $\frac{P_{rad}^Z}{V} = n_Z n_e R_Z [W/m^3]$, where R_Z is the radiative power function for the impurity species.
 - The plasma retains quasi-neutrality: $n_e = n_{DT} + Z n_Z$, where Z is the charge state of the impurity species, and n_Z the impurity density.
 - The fractional reduction of fusion power is defined as $F = \frac{P_{norm} - P_{dilu}}{P_{norm}}$, where P_{dilu} is the fusion power density with diluted fuel and P_{norm} is the fusion power density without dilution.
2. If we define the parameter ρ^* as the ratio between the global alpha particle confinement time and the energy confinement time, can you qualitatively justify the importance of ρ^* for plasma ignition, evident in the figure below?

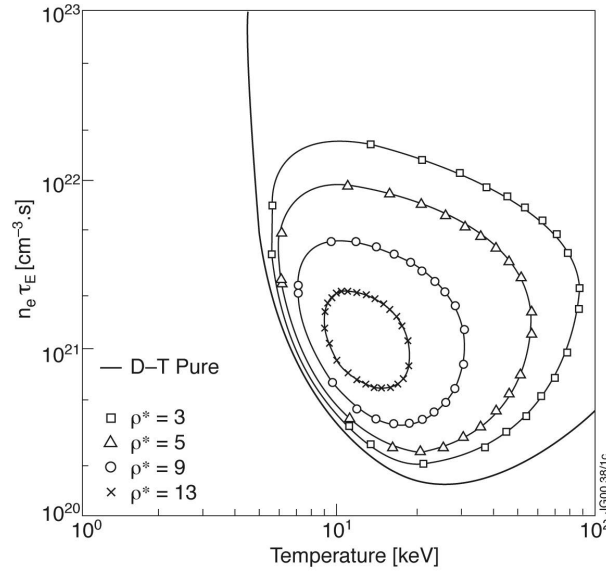


Figure 1: Ignition condition in the presence of fusion-produced α -particles, with varying confinement times [D. Reiter, G.H. Wolf, H. Kever, Nucl. Fusion, 30, (1990), 2141].