Nuclear Fusion and Plasma Physics - Exercises

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Problem Set 1 - 25 September 2023

Exercise 1 - Combustion, Fission or Fusion?

A Swiss household consumes 18780 kWh per year on average. With 1 kg of *fuel*, for how many years can you supply energy for one household?
Consider three cases:

a) The conventional **chemical reaction**, which produces 6.5×10^{-19}J of energy per reaction.

$$\mathrm{C} + \mathrm{O_2} \rightarrow \mathrm{CO_2} + 6.5 \times 10^{-19}\mathrm{J} \tag{1}$$

b) The **nuclear fission** reaction

$$\mathrm{n} + {}^{235}\mathrm{U} \rightarrow {}^{140}\mathrm{Ce} + {}^{94}\mathrm{Zr} + 2\mathrm{n} \tag{2}$$

c) The **nuclear fusion** reaction

$$\mathrm{D} + \mathrm{T} \rightarrow {}^{4}\mathrm{He} + \mathrm{n} \tag{3}$$

Hint: For points b) and c) use Einstein's relation:

$$E = \Delta m_0 c^2 \tag{4}$$

where Δm_0 is the *mass deficit*, i.e. the difference between the rest masses before and after the reaction.

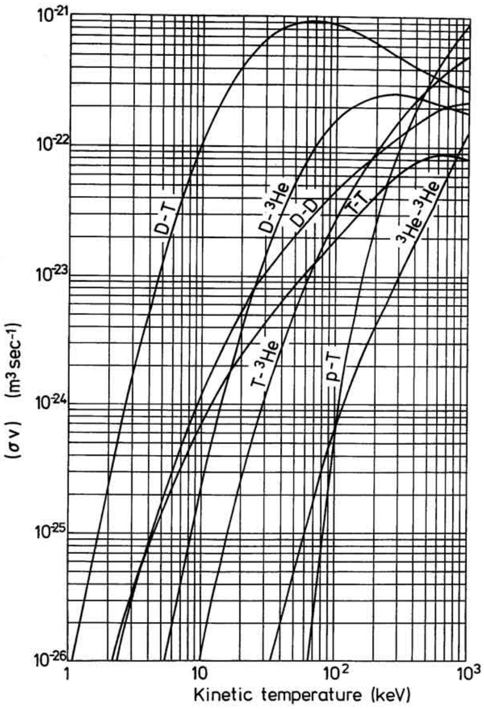
Indications: $m_n = 1.0087\,\mathrm{u}$, $m_U = 235.04\,\mathrm{u}$, $m_{Ce} = 139.91\,\mathrm{u}$, $m_{Zr} = 93.91\,\mathrm{u}$, $m_D = 2.014\,\mathrm{u}$, $m_T = 3.0164\,\mathrm{u}$, $m_{He} = 4.0027\,\mathrm{u}$, with $1\,\mathrm{u} = 1.66 \cdot 10^{-27}$ kg.

Exercise 2 - A "small" Tokamak

Consider a Tokamak plasma uniformly distributed in a volume $V = 10\,\mathrm{m}^3$ with the following parameters: $B = 2\,\mathrm{T}$; $n_e = 10^{20}\,\mathrm{m}^{-3}$; $T_i = T_e = 10\,\mathrm{keV}$. The total heating power delivered to the plasma is $P_{in} = 28\,\mathrm{MW}$ and the tokamak is assumed to be in "steady state".

a) Using the cross-sections in the figure below, calculate:

- The power in the α-particle population and the total fusion power in D-T (50:50 mixture).
- The total fusion power in D-D.



Hint: The fusion reactions and energies of the products are:

$$\mathrm{D} + \mathrm{T} \rightarrow {}^4\mathrm{He}(3.5\,\mathrm{MeV}) + \mathrm{n}(14\,\mathrm{MeV})$$

$$\begin{aligned} \mathrm{D} + \mathrm{D} &\rightarrow {}^3\mathrm{He}(0.8\,\mathrm{MeV}) + \mathrm{n}(2.45\,\mathrm{MeV}) \quad [50\%] \\ &\rightarrow \mathrm{T}(1\,\mathrm{MeV}) + \mathrm{p}(3\,\mathrm{MeV}) \quad [50\%] \end{aligned}$$

b) Calculate the *physics fusion gain* "Q" and the Lawson parameter $n_e\tau_E$. How close are we to break-even?

c) Making simple assumptions on the tokamak geometry (torus with major radius $R_0 = 1\mathrm{m}$ and circular cross-section), consider the situation of a D-T mixture plasma with the heat flux concentrated only in 1/10 of the wall surface. Is it suitable to cover that portion of surface with a material having a heat tolerance of 4 $\mathrm{MW/m}^2$?

Exercise 3 - The effect of impurity contamination on the plasma power balance

1. Assuming a T = 10 keV plasma with $n_e = 2 \times 10^{20}\,\mathrm{m}^{-3}$ and a volume $V = 10\,\mathrm{m}^3$, calculate the radiated power due to impurity contamination and the fractional reduction of the fusion power due to dilution of the fusion fuel, in two cases:

   a) Carbon impurities:

      - Assume that the carbon atoms are fully ionized (Z = 6), and $n_{C^{6+}}/n_e = 4\%$.
      - The radiated power function in T = 10 keV, assuming coronal equilibrium, is $R_C = 10^{-34}\,W\mathrm{m}^3$.

   b) Tungsten impurities:

      - Assume that the charge state of the tungsten atoms is 50 (Z = 50), and $n_{W^{50+}}/n_e = 10^{-5}$.
      - The radiated power function in T = 10 keV, assuming coronal equilibrium, is $R_W = 10^{-31}\,W\mathrm{m}^3$.

   <u>Note:</u>

   - The impurity radiation losses are calculated as $\frac{P^Z_{rad}}{V} = n_Z n_e R_Z [W/m^3]$, where R_Z is the radiative power function for the impurity species.
   - The plasma retains quasi-neutrality: $n_e = n_{DT} + Zn_Z$, where Z is the charge state of the impurity species, and n_Z the impurity density.
   - The fractional reduction of fusion power is defined as $F = \frac{P_{norm} - P_{dilu}}{P_{norm}}$, where P_{dilu} is the fusion power density with diluted fuel and P_{norm} is the fusion power density without dilution.

2. If we define the parameter ρ^* as the ratio between the global alpha particle confinement time and the energy confinement time, can you qualitatively justify the importance of ρ^* for plasma ignition, evident in the figure below?

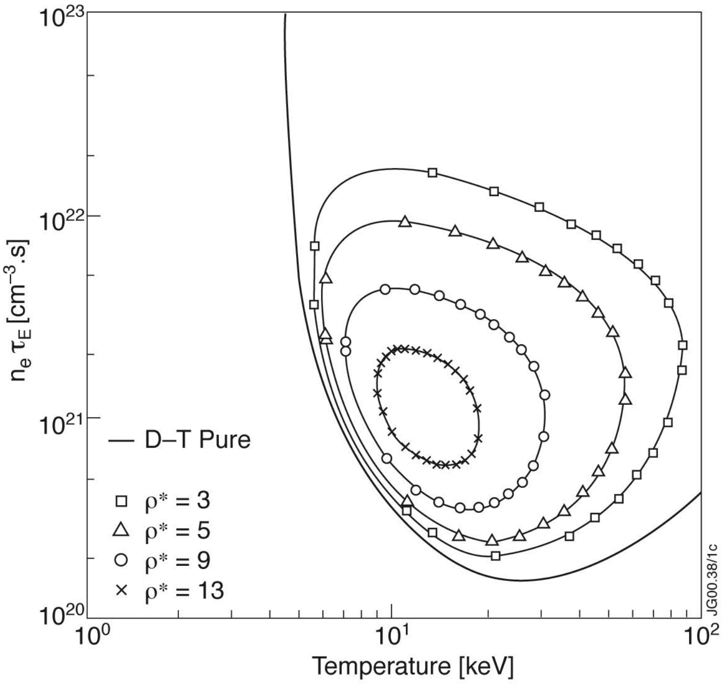


Figure 1: Ignition condition in the presence of fusion-produced α-particles, with varying confinement times [D. Reiter, G.H. Wolf, H. Kever, Nucl. Fusion, 30, (1990), 2141].