Nuclear Fusion and Plasma Physics

The basics of thermonuclear fusion

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Contents

1 Fusion and the world's energy needs

(P.P. and F.E. by J.Freidberg, Ch. 1 and 2 - Fusion Physics, IAEA, Ch. 1)

The need for ways to supply energy that are economically and environmentally sustainable represents one of the main issues mankind faces today. Discussing the whole energy issue in general would take an entire course. Nevertheless, it is useful to reflect upon it, even if in a necessarily superficial way, as an introduction to the course, and as a motivation for our efforts. The attached slides remind us of some of the issues, including an overview of the energy options available to meet the demand, including nuclear fusion, and have been used as a basis for our in-class discussion.

2 Nuclear fusion

2.1 Fusion reactions

The net fusion reaction in the sun consists of 4 hydrogen fusing to produce one helium nucleus; the 'basic process' is H + H + H + H $\rightarrow$ He^4. Fusion consists in converting light nuclei into heavier ones, freeing large amounts of energy. The main fusion reactions of interest for the production of energy are:

1. $D_1^2 + T_1^3 \rightarrow He_2^4\ (3.5\ \text{MeV}) + n_0^1\ (14.1\ \text{MeV});\quad E_{\min} \simeq 4\ \text{keV},\ \Delta E_f = 17.6\ \text{MeV}$

> **Note 1.2.1**: The energy share between α particles (2 protons + 2 neutrons - that is, the nucleus of He_2^4 atoms) and neutrons n is dictated by the masses:
>
> $$\begin{cases} \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \Delta E \\ m_1\mathbf{v}_1 + m_2\mathbf{v}_1 = 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{2}m_1 v_1^2 = \frac{m_2}{m_1+m_2}\Delta E \\ \frac{1}{2}m_2 v_2^2 = \frac{m_1}{m_1+m_2}\Delta E \end{cases} \Rightarrow \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{m_2}{m_1}$$

2. $$D_1^2 + D_1^2 \begin{array}{lllll} \nearrow (50\%) & T_1^3\ (1\ \text{MeV}) & + & p_1^1\ (3\ \text{MeV}), & \Delta E_f = 4\ \text{MeV} \\ \searrow (50\%) & He_2^3\ (0.8\ \text{MeV}) & + & n_0^1\ (2.45\ \text{MeV}), & \Delta E_f = 3.2\ \text{MeV} \end{array}$$

   $E_{\min} \simeq 10$ keV

3. $D_1^2 + He_2^3 \rightarrow He_2^4\ (3.7\ \text{MeV}) + p_1^1 (14.5\ \text{MeV});\ E_{\min} \simeq 30\ \text{keV};\ \Delta E_f \cong 18.2\ \text{MeV}$

Et cetera, with increasing $E_{\min}$; He^3 is a T-decay product, but could also be mined from the surface of the moon.

> **Note 1.2.2**: Reaction 3 is the 'cleanest', as it does not produce (fast) neutrons, has no radioactive elements. No neutron irradiation $\Rightarrow$ no activation of structure materials.

> **Definition** (reminder): **Deuterium** D is an isotope of hydrogen with 1 proton and 1 neutron in its neucleus. **Tritium** T is a hydrogen atom that has 1 proton and 2 neutrons in its nucleus.

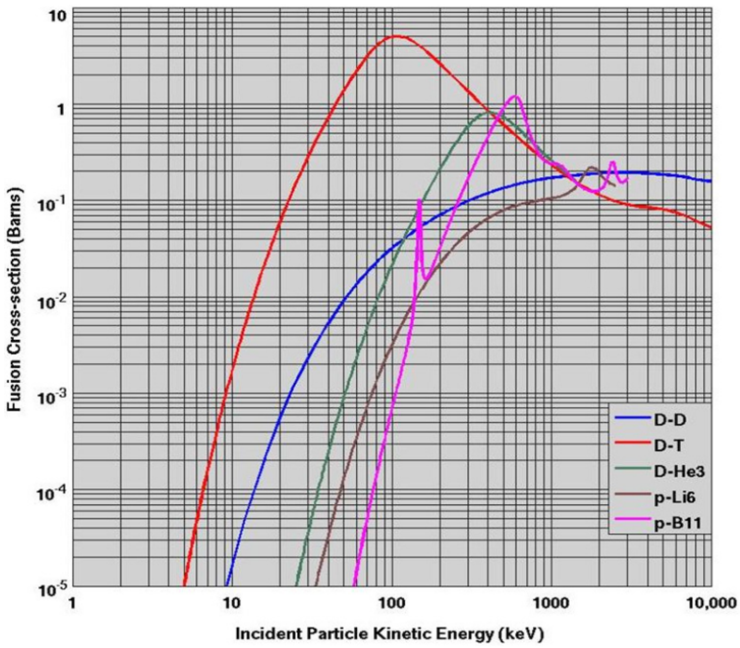


Figure 1: Cross-section dependence upon energy for the most important fusion reactions for use as energy source.

Note 1.2.3: There is no energy at which a mono-energetic beam can produce more fusion reactions than Coulomb interactions ($\sigma_{\text{Coul}} \geq \sigma_{\text{fus}} \; \forall \; E$). This implies:

- No chance to make fusion energy with beams (thay are scattered before they can fuse).
- We need to deal with a Maxwellian equilibrium generated by Coulomb collisions, like in a gas.
- That is why we speak of *thermonuclear fusion*.

2.2 Sources of fuel for fusion (D-T)

D is $\frac{1}{6700}$ of hydrogenic atoms in all the oceans i.e. $\sim$ 1.6 g/l $\Rightarrow \sim$ 5 x 10^{16} kg total in the oceans. But 1 kg of D-T gives 2.7 x 10^{14} J (or 7.5 x 10^7 kWh, the world's need for 1 min), so the supply is practically unlimited ($\sim 10^{16} \times 10^{14} J$, or $\sim 10^{16}$ x 1 min $= \frac{10^{16}}{60\text{x}24\text{x}365} \sim 2 \times 10^{10}$ years).

By comparison, 1 kg of coal gives (700 K) 3 x 10^7 J.

Tritium does not exist in nature, as it is radioactive and relatively short-lived (half-life of 12.5 years). It can be produced from nuclear reactions with Lithium:

$$n_0^1 + Li_3^6 \rightarrow T_1^3 + He_2^4 + 4.8 \text{ MeV} \;\; (Li^6 \text{ is} \sim 7\% \text{ abundant})$$

$$n_0^1 + Li_3^7 \rightarrow T_1^3 + He_2^4 + n_0^1 - 2.5 \text{ MeV } (Li^7 \text{ is} \sim 93\% \text{ abundant})$$

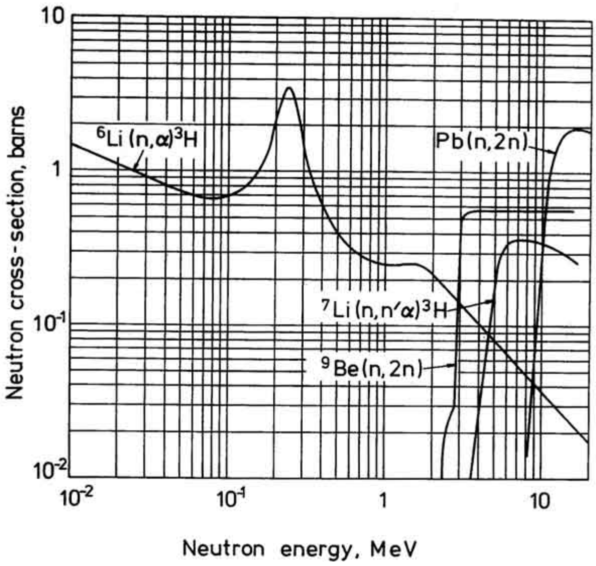


Figure 2: Cross-section for neutron reactions for n-multiplication and Tritium breeding.

As we can see in Fig. 2, the cross section for Li^6 is much larger than for Li^7, much so that we may not even need to use artificially Li^6 - enriched Lithium. But we need to have low-energy neutrons, i.e. to moderate them (from 14 MeV down to thermal) before they interact with Lithium.
The second thing to note is that, if we need to multiply neutrons before we breed T (and we do need to do that, as we must guarantee one new T atom for each fusion reaction), we must do it while the neutrons are still at high energy, for example using reactions with Lead or Beryllium.
So, the logical sequence is to first use neutron-multipliers, then moderate the neutrons, and finally, breed Tritium.

Li is in the Earth's crust, and also in the ocean waters ($\sim$ 0.17 g/m^3 ; for comparison, Uranium is 0.003 g/m^3).

As we will see, in practice T is generated 'in situ', in a blanket containing Lithium that surrounds the plasma reactor. To compare the 'feasibility' of the different reactions, one should consider not just the minimum energy, but also the reaction rate as a function of energy, which stems from the concept of **cross-section**. This will also allow us to calculate the conditions for fusion energy production.

2.3 Fusion as a collisional process
(P.P. and F.E. by J.Freidberg, Ch. 3)

Note 1.2.4: We focus only on DT reactions.

Fusion reaction rate (# of fusion reactions per unit volume per unit time)
Let n_D and n_T denote the densities of deuterium and tritium atoms respectively. With σ_{DT}

the cross-section of the reaction, the fusion reaction rate between the two is thus:

$$R_{DT}(v) = n_D(n_T\sigma_{DT}(v)v) \quad \left(\frac{1}{\text{m}^3\text{s}}\right) \quad \text{for velocity v}$$

Fusion power density for velocity v (or corresponding energy) = $R_{DT}(v)\Delta E_f$

$$= \overbrace{(n_D n_T \sigma_{DT}(v)v)}^{\#\text{ of reactions/s/volume}} \times \underbrace{(\Delta E_f)}_{\text{energy of one reaction}} \quad \left(\frac{\text{W}}{\text{m}^3}\right)$$

ΔE_f is the energy released in one reaction (for DT, ΔE_f = 17.6 MeV).

> **Note 1.2.5**: The fusion power density, similarly to $R_{DT}(v)$, is velocity (or energy) dependent, so it does not represent a global property of a plasma. We must perform an average over the distribution function.

Fusion power density:

$$n_D n_T \langle\sigma v\rangle_{DT}\Delta E_f = \frac{1}{4}n_e^2\langle\sigma v\rangle_{DT}\Delta E_f \tag{2.1}$$

with n_e denoting the electron density and $\langle\sigma v\rangle_{DT}$ the average over the distribution function.

> **Note 1.2.6**: We can assume that $n_D \simeq n_T \simeq \frac{1}{2}n_e$, where n_α is assumed to be negligible. Otherwise, $\sum Z_j n_j = n_D + n_T + 2n_\alpha = n_e$ to satisfy quasi-neutrality, where we sum over the type of atoms (D,T,α) and Z_j represents their atomic number.

> **Note 1.2.7**: The symbol $\langle\ \rangle_{DT}$ indicates an average over the D and T distributions (σ depends on v, so it cannot be taken out of the integral defining the average). Formally:
>
> $$R_{DT} = n_D n_T\langle\sigma v\rangle_{DT} = \iint d\mathbf{v}_D d\mathbf{v}_T f_D(\mathbf{v}_D) f_T(\mathbf{v}_T)\sigma_{DT}(|\mathbf{v}_D - \mathbf{v}_T|)(|\mathbf{v}_D - \mathbf{v}_T|)$$
>
> v is in fact the relative velocity: $(|\mathbf{v}_D - \mathbf{v}_T|)$

For two Maxwellian distributions with the same temperature ($T_D = T_T = T$), we have:

$$\langle\sigma v\rangle_{DT} = \left(\frac{m_r}{2\pi T}\right)^{3/2}\int_0^\infty 4\pi v^3 \exp\left(-\frac{m_r v^2}{2T}\right)\sigma_{DT}(v)dv$$

where m_r is the reduced mass $m_r = \frac{m_D m_T}{m_D + m_T}$. This is represented as a function of temperature in Fig. 3.

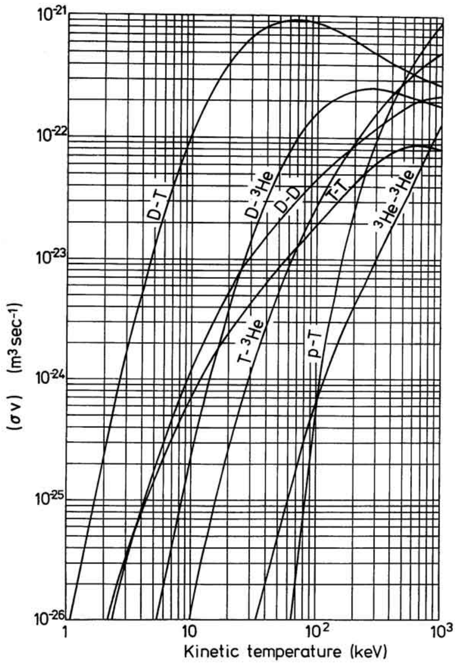


Figure 3: Fusion $\langle \sigma v \rangle_{DT}$ as a function of the plasma temperature.

Note 1.2.8: The maximum of $\langle \sigma v \rangle_{DT}$ *vs.* T is around 60 keV (it was around 100 keV for the mono-energetic case, as illustrated in Fig. 1).

Note 1.2.9: At T = 9 keV the curve is at 10% of its maximum, while to be at 10% of the max. for the mono-energetic case we need to be at 35 keV.

2.3.1 Numerical example to evaluate if fusion can be obtained in a reasonable size reactor

We want a D-T fusion reactor with $P_{\text{fusion}} \sim 1$ GW (note: $P_{\text{fusion}} \neq P_{\text{electric}}$): what volume do we need?
In magnetic fusion we can take $n_e \sim 5 \times 10^{20} \text{m}^{-3}$; $\langle \sigma v \rangle_{DT} \sim 4 \times 10^{-22} \frac{\text{m}^3}{\text{s}}$ (for T $\sim$ 20 keV)

$$\text{volume} = \frac{P_{\text{fusion}}}{n_D n_T \langle \sigma v \rangle_{DT} \Delta E_f} =$$

$$\frac{10^9 \text{J/s}}{\frac{1}{4}(5 \times 10^{20})^2 \text{m}^{-6} \times 4 \times 10^{-22} \text{m}^3/\text{s} \times 17.6 \times 10^6 \text{eV} \times 1.6 \times 10^{-19} \text{J/eV}} \cong 14 \text{ m}^3$$

which is a reasonable value.

Note 1.2.10: The characteristic (thermal) velocity of ions is $v_{th_i} = \sqrt{\frac{T_i}{m_i}}$. Assuming T_i = 20 keV for a Deuterium ion,

$$\sqrt{\frac{T_i}{m_i}} = \sqrt{\frac{20 \times 10^3 \text{eV} \times 1.6 \times 10^{-19} \text{J/eV}}{3.3 \times 10^{-27} \text{kg}}} \simeq 10^6 \text{m/s}$$

For 1 m (reactor dimension) we have $\Delta t = \frac{1\text{m}}{10^6 \text{m/s}} = 1\mu\text{s} \rightarrow$ we need a confinement scheme!

2.4 Power balance in fusion
(P.P. and F.E. by J.Freidberg, Ch. 4)

We need (at least) a power produced by fusion reactions that exceeds the losses from the plasma. This 'starting point' is called "break-even", at which the fusion power coincides with the input power.

Break-even: $\frac{\text{fusion power}}{\text{input power}} = \frac{P_f}{P_{in}} = 1$

We know that $\frac{P_f}{\text{volume}} = \frac{1}{4} n_e^2 \langle \sigma v \rangle_{DT} \Delta E_f$ $\qquad (\Delta E_f = 17.6 \text{ MeV})$

and that in steady-state, $P_{in} = P_l - P_\alpha$, where P_l = lost power (as plasma heating = losses).

We have two kinds of losses:

1. **direct losses** from plasma, described by a characteristic time over which energy is 'transported' outside the plasma by conduction and convection. The 'energy confinement time' τ_E is defined by:

$$\frac{P_{dl}}{\text{volume}} = \frac{\text{energy/volume}}{\tau_E} = \frac{3 n_e T}{\tau_E}$$

Note 1.2.11: The average energy is $\frac{3}{2}T$ for both ions and electrons (assumed to be at the same temperature T), so the total energy per unit volume is 3nT.

2. **radiation** due to acceleration (deceleration) of electrons in the field of ions ('bremsstrahlung'), releasing a power P_b:

$$\frac{P_b}{\text{volume}} = A \sum_{\text{species } j} Z_j^2 n_j n_e (T_e|_{\text{keV}})^{1/2} = A n_e (T_e|_{\text{keV}})^{1/2} \sum_j n_j Z_j^2, \qquad A \simeq 5 \times 10^{-37} \text{Wm}^3$$

where $T_e|_{\text{keV}}$ is the numerical value of the temperature expressed in keV (the physical units of T_e are already included in A).

We can write $\sum_j n_j Z_j^2 = n_e Z_{\text{eff}}$ with $Z_{\text{eff}} = \frac{\sum_j n_j Z_j^2}{n_e}$ where Z_j = charge number

Therefore, we obtain that $\frac{P_b}{\text{volume}} \simeq A n_e^2 Z_{\text{eff}} T_e^{1/2}$

Note 1.2.12: As shown in Fig. 4, P_b is in the form of X-rays, which are not re-absorbed by the plasma since the plasma is transparent at these wavelengths. Also, the X-rays go through the metal walls and are therefore lost from the plasma region.

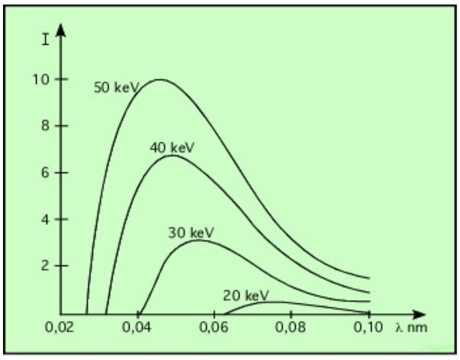


Figure 4: Emission spectrum for bremsstrahlung radiation for different plasma temperatures.

Note 1.2.13: As the emission spectrum depends on T_e, a measurement of it provides information on the electron temperature.

Note 1.2.14: Z_{eff} is the parameter that describes such emission. In pure D-T (only H-species) plasma $Z_{\text{eff}} = 1$.

The total lost power can therefore be written as:

$$\boxed{\frac{P_l}{\text{volume}} \simeq \frac{P_{dl}}{\text{volume}} + \frac{P_b}{\text{volume}} = \frac{3n_eT_e}{\tau_E} + An_e^2 Z_{\text{eff}} T_e^{1/2}}$$

Note 1.2.15: Radiation due to acceleration around magnetic field lines, known as cyclotron emission, can be significant ($\text{P}_{\text{cycl}}/\text{volume} \simeq \text{const} \times \text{n}_\text{e}\text{T}_\text{e}\text{B}^2$). $P_{cycl} \propto n_e$ as it is not a binary collisional effect, and $f_{cycl} \propto B$ ($\omega = n\Omega_{ce}, n = 1, 2, \ldots$). P_{cycl} is very large, but it does not constitute a loss, as it is mostly reabsorbed by the plasma or it is reflected (at least most of it) by the conducting vessel walls. In fact, the small fraction that does come out is used as a diagnostic tool, to get information on n_e and T_e for example.

2.4.1 The Lawson criterion

The power losses per unit of volume are thus given by:

$$\frac{P_l}{\text{volume}} = \frac{3n_eT}{\tau_E} + An_e^2 Z_{\text{eff}} T^{1/2} \qquad (T_e = T_i = T) \tag{2.2}$$

The ratio between the fusion power and the input power is then:

$$\frac{P_f}{P_{in}} = \frac{P_f}{P_l - P_\alpha} \cong \frac{P_f}{\frac{3n_eT}{\tau_E} + An_e^2 Z_{\text{eff}} T^{1/2} - P_\alpha} = \frac{\frac{1}{4}n_e^2\langle\sigma v\rangle_{DT}\Delta E_f}{\frac{3n_eT}{\tau_E} + An_e^2 Z_{\text{eff}} T^{1/2} - \frac{1}{4}n_e^2\langle\sigma v\rangle_{DT}\Delta E_\alpha}$$

$\Delta E_\alpha = 3.5$ MeV is the energy associated with the α-particle released by the fusion reaction. To achieve break-even or do better ($P_f/P_{in} \geq 1$):

$$n_e\tau_E(\langle\sigma v\rangle_{DT}\Delta E_f) \geq 12T + 4An_e\tau_E Z_{\text{eff}}T^{1/2} - n_e\tau_E(\langle\sigma v\rangle_{DT}\Delta E_\alpha)$$

or

$$n_e\tau_E(\langle\sigma v\rangle_{DT}(\Delta E_f + \Delta E_\alpha) - 4AZ_{\text{eff}}T^{1/2}) \geq 12T \quad \Rightarrow \quad n_e\tau_E \geq \frac{12T}{\frac{6}{5}\langle\sigma v\rangle_{DT}\Delta E_f - 4AZ_{\text{eff}}T^{1/2}}$$

considering

$$\Delta E_f + \Delta E_\alpha \cong \left(1 + \frac{1}{5}\right)\Delta E_f \cong \frac{6}{5}\Delta E_f$$

When can we neglect the bremsstrahlung term? This corresponds to when the following relation is verified:

$$\frac{6}{5}\langle\sigma v\rangle_{DT}\Delta E_f \gg 4AZ_{\text{eff}}T^{1/2} \quad \xrightarrow{(Z_{eff}=1)} \quad \frac{6}{5}\langle\sigma v\rangle_{DT}\Delta E_f \gg 4AT^{1/2}$$

where we have assumed $Z_{\text{eff}} = 1$ for simplicity. The approximate form of $\langle\sigma v\rangle_{DT}$ in the range 10-20 keV is $\langle\sigma v\rangle_{DT} \simeq 1.1 \times 10^{-24}(T|_{\text{keV}})^2\text{m}^3/\text{s}$. This gives:

$$\frac{6}{5}1.1 \times 10^{-24}T|_{\text{keV}}^2\text{m}^3/\text{s} \times 17.6 \times 10^6 \times 1.6 \times 10^{-19}\text{J} \gg 4 \times 5 \times 10^{-37}T|_{\text{keV}}^{1/2}\text{Wm}^3$$

$$T|_{\text{keV}}^{3/2} \gg 0.5 \quad \Rightarrow \quad T|_{\text{keV}} \gg 0.5^{2/3} \simeq 0.7$$

So for $T \gg 1$ keV we can neglect the bremsstrahlung contribution (for Z not much larger than 1).

Note 1.2.16: We consider here the steady-state power balance. Otherwise, we would need to include, as an effective 'loss' (or utilization of input power) the variation of the plasma energy $\frac{\text{d}}{\text{dt}}(\text{3nT})$.

Coming back to the steady-state break-even (or better) condition, assuming $\frac{P_l}{\text{volume}} \simeq \frac{3n_eT}{\tau_E}$, we have:

$$n_e\tau_E\left\{\frac{6/5\langle\sigma v\rangle_{DT}\Delta E_f}{12T}\right\} = n_e\tau_E\left\{\frac{\langle\sigma v\rangle_{DT}\Delta E_f}{10T}\right\} \geq 1$$

We can indicate $\frac{\langle\sigma v\rangle_{DT}\Delta E_f}{10T} = \text{f(T)}$ and thus:

$$\boxed{n_e\tau_e \geq \frac{1}{\text{f(T)}}} \tag{2.3}$$

This condition is the **Lawson Criterion**. It is expressed in graphical form in Fig. 5.

Typically, we need $\begin{cases} n_e\tau_E & \geq & 10^{20}\text{m}^{-3}\text{s} \\ T & \geq & 10 \text{ keV} \end{cases}$

Two approaches are possible in order to achieve this (to be discussed later):

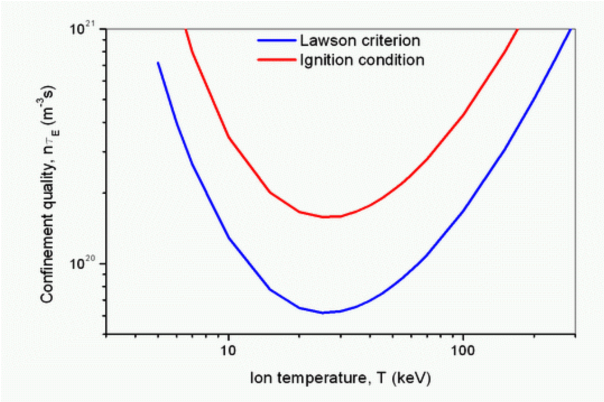


Figure 5: The conditions for break-even (Lawson Criterion) and ignition in a D-T plasma.

- large n_e ($\sim 10^{31}\text{m}^{-3}$) , small τ_E ($\sim 10^{-11}$s): 'inertial' confinement
- small n_e ($\sim 10^{20}\text{m}^{-3}$), large τ_E ($\sim$ 1s): 'magnetic' confinement

Beyond break-even: Naturally, in reality the break-even condition is not at all sufficient to make a reactor. We must have $\frac{P_f}{P_{in}} \gg 1$, for two reasons:

- In D-T reactions, the neutron energy escapes the plasma (and is used to produce heat, then electricity), while the α energy can (and should) remain in the plasma.
- The conversion efficiency of the fusion power into electricity, and of electricity into plasma heating power are significantly less than 100%

2.4.2 Ignition and burning plasma regime

If we do not want to rely on external power to maintain fusion reactivity ($P_{in} = 0$), we have to consider an equilibrium between the α-particle power and the losses.

$$\frac{P_\alpha}{P_l} \geq 1 \Rightarrow \frac{1}{4} n_e^2 \langle \sigma v \rangle_{DT} \Delta E_\alpha \geq \frac{3 n_e T}{\tau_E}$$

Analogously to the break-even condition (Eq. 2.3), this can be written as

$$n_e \tau_E \geq \frac{12T}{\langle \sigma v \rangle_{DT} \Delta E_\alpha}, \quad \text{or} \quad n_e \tau_E \geq \frac{6}{f(T)}, \quad \text{or} \quad n_e \tau_E T \geq \frac{6T}{f(T)}$$

Graphically, Fig. 6 presents the ignition condition (see also Figure 5):

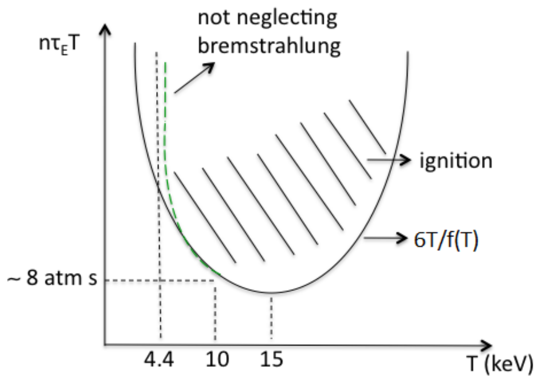


Figure 6: Ignition condition in terms of the fusion triple-product.

Note 1.2.17: If we consider that bremsstrahlung radiation as an unavoidable (the only one!) energy loss process, we must require for ignition that $P_\alpha \geq P_b$:

$$\frac{P_\alpha}{P_b} = \frac{1/4\langle\sigma v\rangle_{DT} E_\alpha n^2}{A n^2 Z_{\text{eff}} (T_e|_{\text{keV}})^{1/2}} \geq 1$$

Using the values of $\langle\sigma v\rangle_{DT}$, this gives $T \geq 4.4\ keV$. This is known as the '*ideal ignition temperature*'.

Looking at Fig. 6, we can identify the minimum (or easiest) ignition conditions:

$$\begin{cases} T_{\text{min}} & \simeq & 15 \text{ keV} \\ (n\tau_E T) & \simeq & 8 \text{ atm s} \end{cases} \qquad \text{(nT is plasma pressure)}$$

In practice, a reactor can (and, most likely, will) operate above break-even but below ignition, so that its 'burn' can be controlled and the conditions to be reached are not as tough as for ignition. This intermediate situation is described by the fusion gain factor, Q.

2.4.3 Physics fusion gain factor Q

$$\text{Q} \equiv \frac{P_{\text{fusion out}} - P_{in}}{P_{in}} \tag{2.4}$$

where $P_{\text{fusion out}}$ is the fusion power that comes out from a reactor, and P_{in} is the external heating power. The difference $P_{\text{fusion out}} - P_{in}$ thus corresponds to the net power coming out from the Tokamak reactor.

In the steady-state: $P_{\text{fusion out}} = P_{\text{neutrons}} + P_{\text{losses}} = P_{\text{neutrons}} + P_{\text{heating}}$

$$= P_{\text{neutrons}} + P_\alpha + P_{\text{in}}$$

$$= P_f + P_{\text{in}}$$

$$\Rightarrow \text{Q} = \frac{P_{\text{fusion out}} - P_{\text{in}}}{P_{\text{in}}} = \frac{P_f + P_{in} - P_{\text{in}}}{P_{\text{in}}} = \frac{P_f}{P_{\text{in}}}$$

Here $P_{heating}$ is the effective power that can be used to heat the plasma, namely the injected power P_{in} and the α particle power P_α. In addition, the fusion power P_f is the power produced by neutrons and alpha particles in fusion processes.

Note 1.2.18: Ignition: Q $\to \infty$; Break-even: Q = 1

This can be written as $\text{Q} = 5\frac{nT\tau_E}{(nT\tau_E)_{\text{ignition}} - nT\tau_E}$ (not demonstrated here) and plotted as a function of $n\tau_E T$ in Fig. 7:

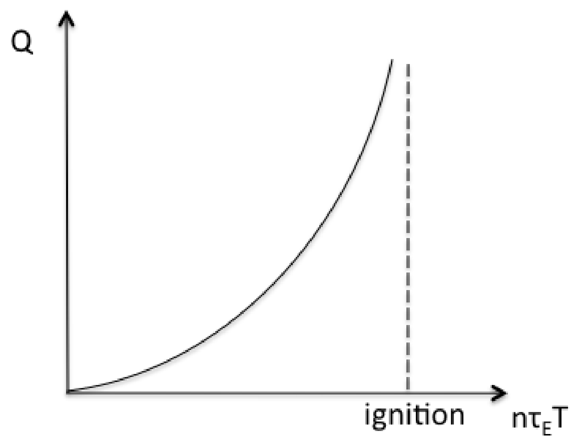


Figure 7: Qualitative behaviour of plasma fusion gain factor as a function of the fusion triple-product.

Note 1.2.19: Fraction of α-particle heating:

$$f_\alpha = \frac{P_\alpha}{P_{\text{heating}}} = \frac{P_\alpha}{P_\alpha + P_{\text{in}}} \tag{2.5}$$

$f_\alpha \cong \frac{\text{Q}}{\text{Q}+5}$; $f_\alpha \geq \frac{1}{2} \Leftrightarrow \text{Q} \geq 5$ (burning plasma)

$(\text{Q} = \frac{P_f}{P_{\text{in}}} \cong \frac{5P_\alpha}{P_{\text{heating}} - P_\alpha} = \frac{5}{\frac{1}{f\alpha} - 1} = \frac{5f_\alpha}{1 - f_\alpha} \Rightarrow (1 - f_\alpha)\text{Q} = 5f_\alpha \Rightarrow f_\alpha(5 + \text{Q}) = \text{Q})$

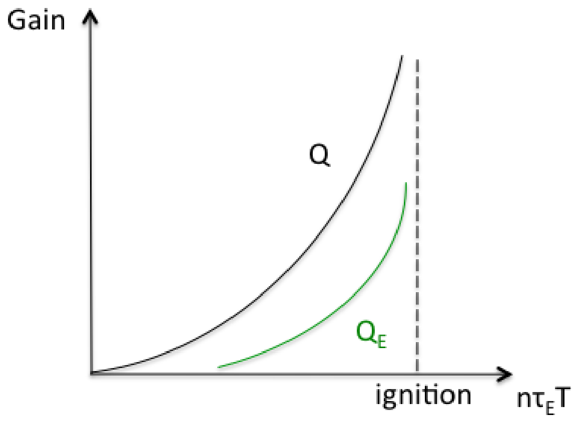


Figure 8: Qualitative behaviour of plasma fusion gain factor and engineering gain factor for increasing values of fusion triple-product.

2.4.4 Finite conversion efficiency

The engineering gain factor Q_E is defined as:

$$Q_E = \frac{\text{net electric power out}}{\text{net electric power in}} = \frac{P_{\text{out}}^{(E)} - P_{\text{in}}^{(E)}}{P_{\text{in}}^{(E)}} \tag{2.6}$$

Typically, $P_{\text{in}}^{(E)} = \frac{P_{\text{in}}}{\eta_e}$, as $P_{\text{in}} = P_{\text{in}}^{(E)} \times \eta_e$, where η_e is the efficiency of the power source (ex. generator of microwaves); a reasonable number is $\eta_e \sim 70\%$.

The power out $P_{\text{fusion out}}$ is converted into electricity at a finite efficiency, η_t (a reasonable number is $\eta_t \sim 40\%$).

Therefore, we can express the electrical power out as: $P_{\text{out}}^{(E)} = \eta_t[P_f + P_{\text{in}}]$

Note 1.2.20: We neglect the power not absorbed by the plasma (included in η_e) and rejected.

Reinserting the expression for $P_{\text{out}}^{(E)}$ in Eq. 2.6, we find:

$$Q_E = \frac{\eta_t(P_f + P_{\text{in}}) - P_{\text{in}}/\eta_e}{P_{\text{in}}/\eta_e} = \frac{\eta_e\eta_t(P_f + P_{\text{in}}) - P_{\text{in}}}{P_{\text{in}}}$$

Let's define an efficiency $\eta = \eta_e\eta_t$ in order to establish a relation between the engineering gain factor and the fusion gain factor Q:

$$Q_E = \frac{\eta[P_f + P_{in}] - P_{\text{in}}}{P_{\text{in}}} = \eta\frac{P_f}{P_{\text{in}}} - (1-\eta) = \eta Q - (1-\eta)$$

Example: $\eta_e \simeq 70\%$; $\eta_t \simeq 40\% \Rightarrow \eta \simeq 28\%$ and $Q_E \simeq 0.28Q - 0.72$

$Q_E = 10 \Leftrightarrow Q \cong 40$; $Q_E = 2 \Leftrightarrow Q \simeq 10$

Nuclear Fusion and Plasma Physics

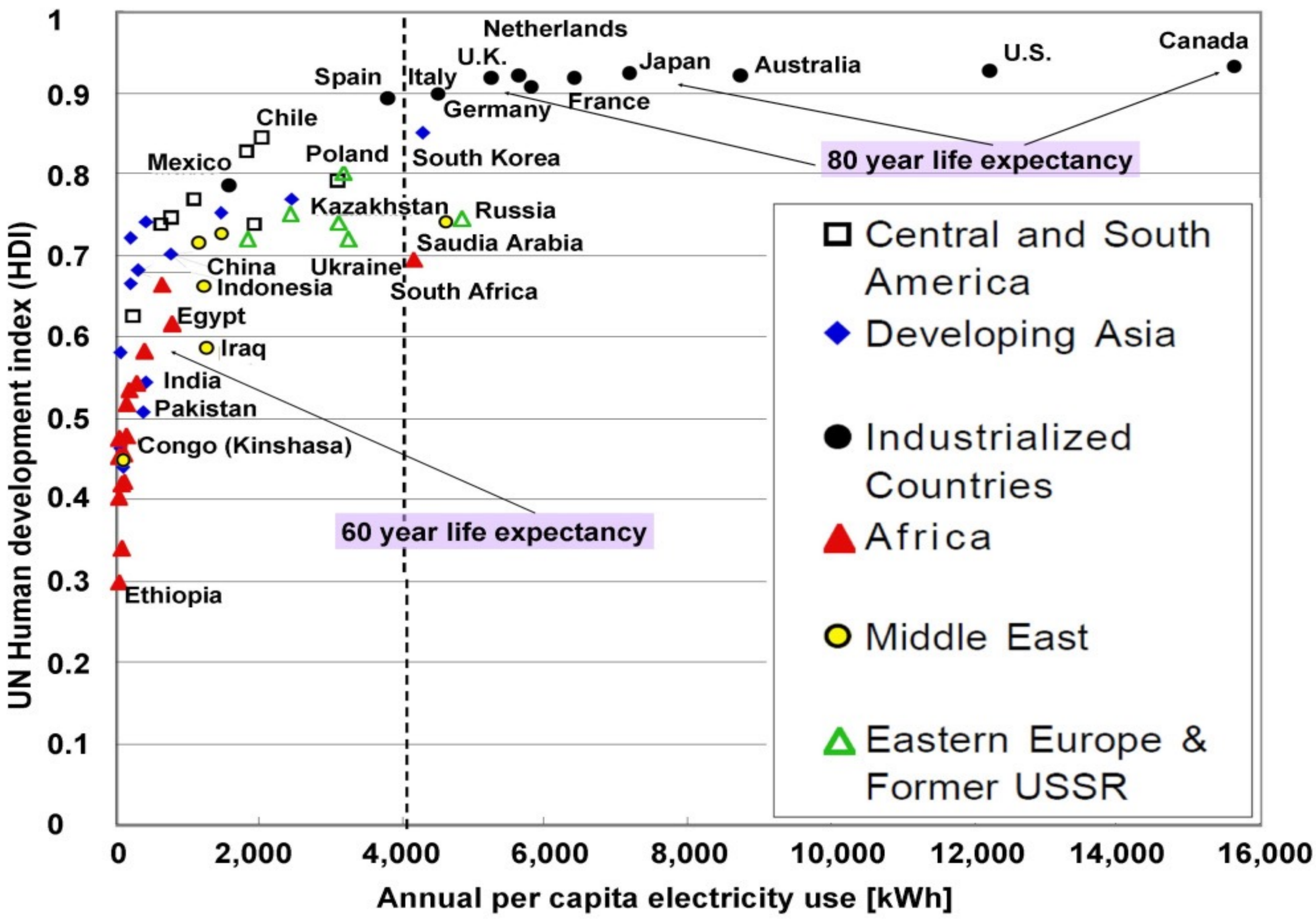
Lecture 1

Ambrogio Fasoli

Swiss Plasma Center

EPFL

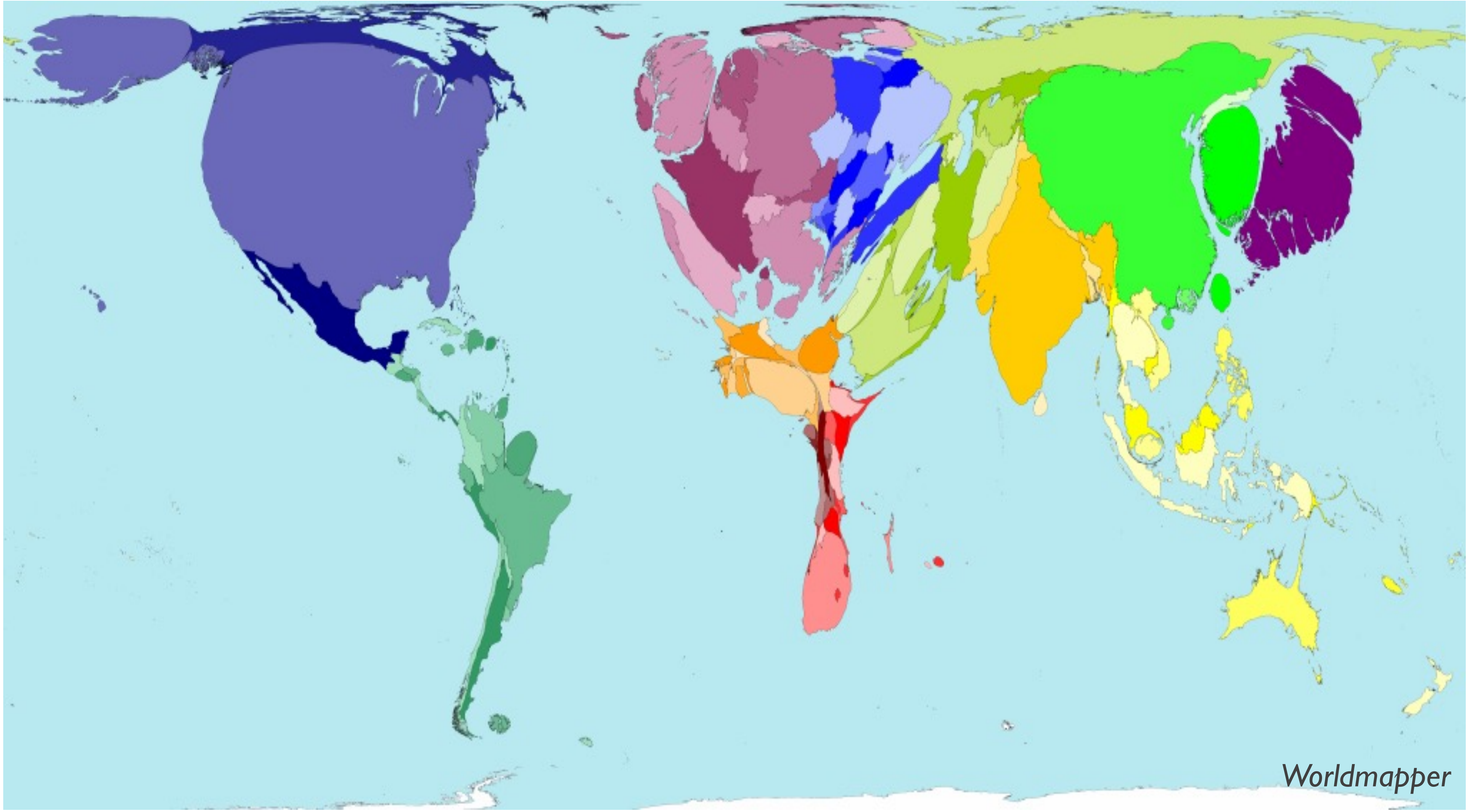
Energy and human development



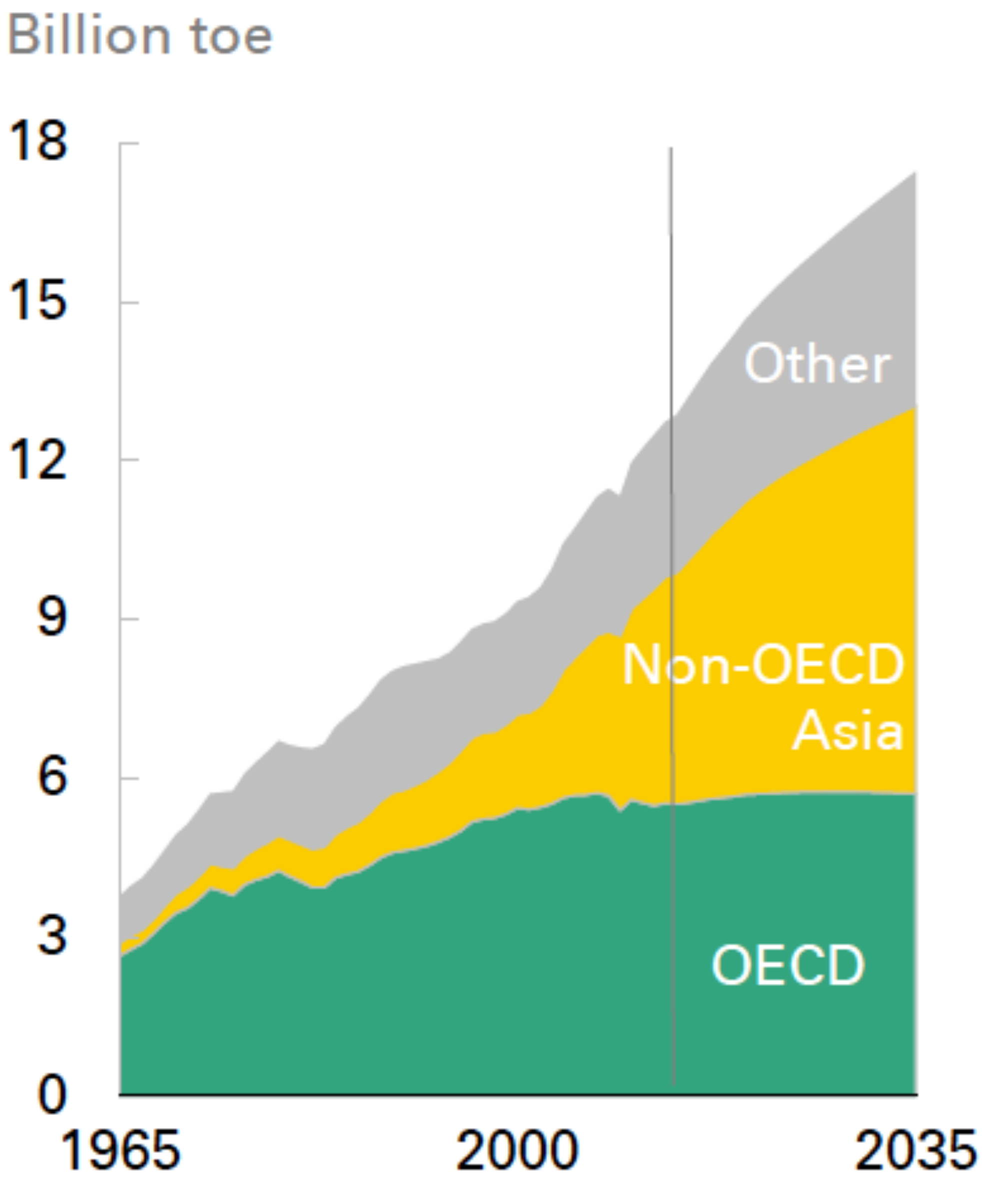
PASTERNAK, A., US Department of Energy Report UCRL-ID- 140773, LLNL , Livermore, CA (2000) 38.

EPFL

Present unequal distribution of energy consumption

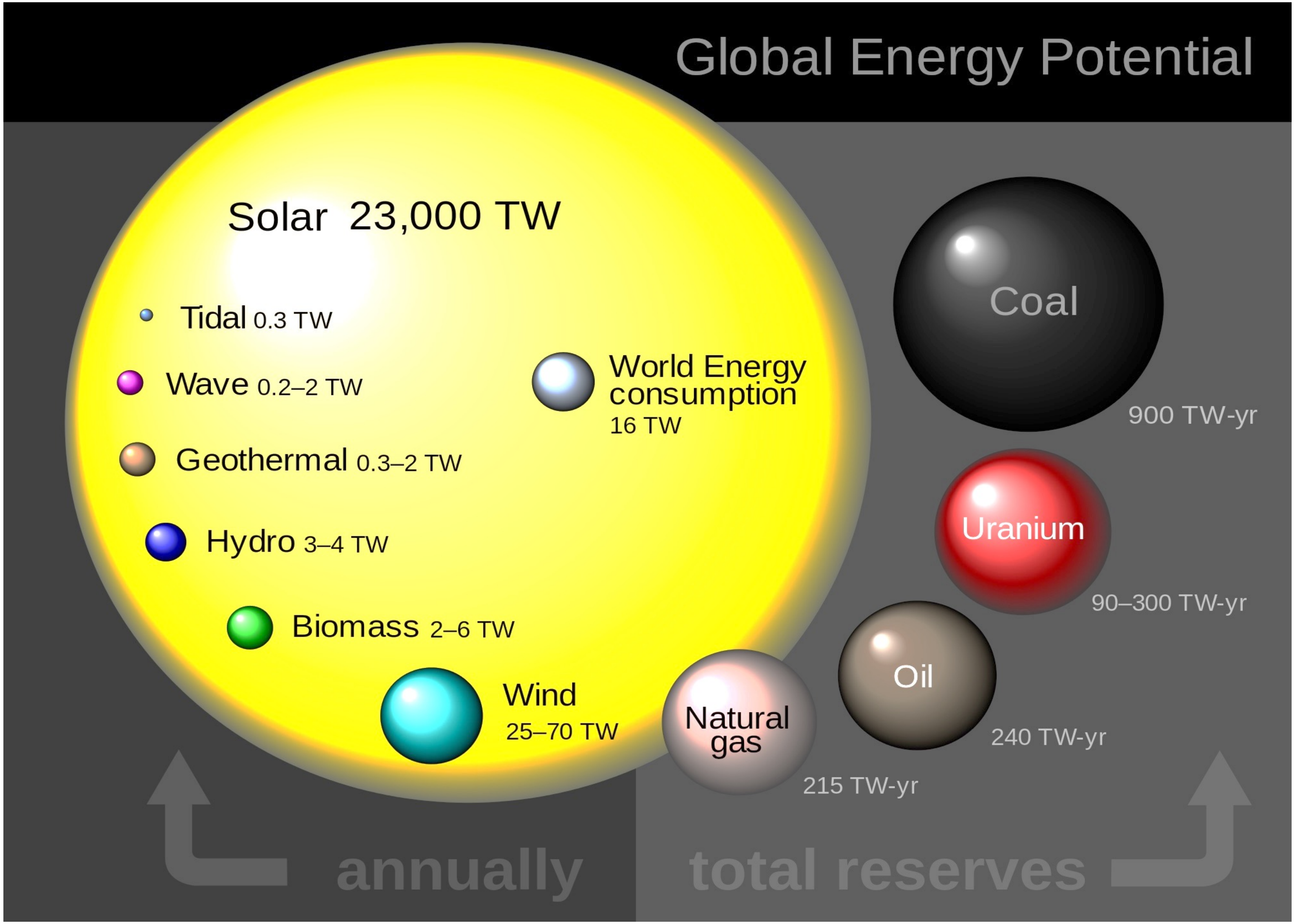


Evolution of distribution of energy consumption




SUSTAINABLE DEVELOPMENT GOALS
1 NO POVERTY
2 ZERO HUNGER
3 GOOD HEALTH AND WELL-BEING
4 QUALITY EDUCATION
5 GENDER EQUALITY
6 CLEAN WATER AND SANITATION
7 AFFORDABLE AND CLEAN ENERGY
8 DECENT WORK AND ECONOMIC GROWTH
9 INDUSTRY, INNOVATION AND INFRASTRUCTURE
10 REDUCED INEQUALITIES
11 SUSTAINABLE CITIES AND COMMUNITIES
12 RESPONSIBLE CONSUMPTION AND PRODUCTION
13 CLIMATE ACTION
14 LIFE BELOW WATER
15 LIFE ON LAND
16 PEACE, JUSTICE AND STRONG INSTITUTIONS
17 PARTNERSHIPS FOR THE GOALS

Discussion of today's energy options
Global view of energy fluxes



Perez et al., 2009, "A Fundamental Look At Energy Reserves For The Planet"

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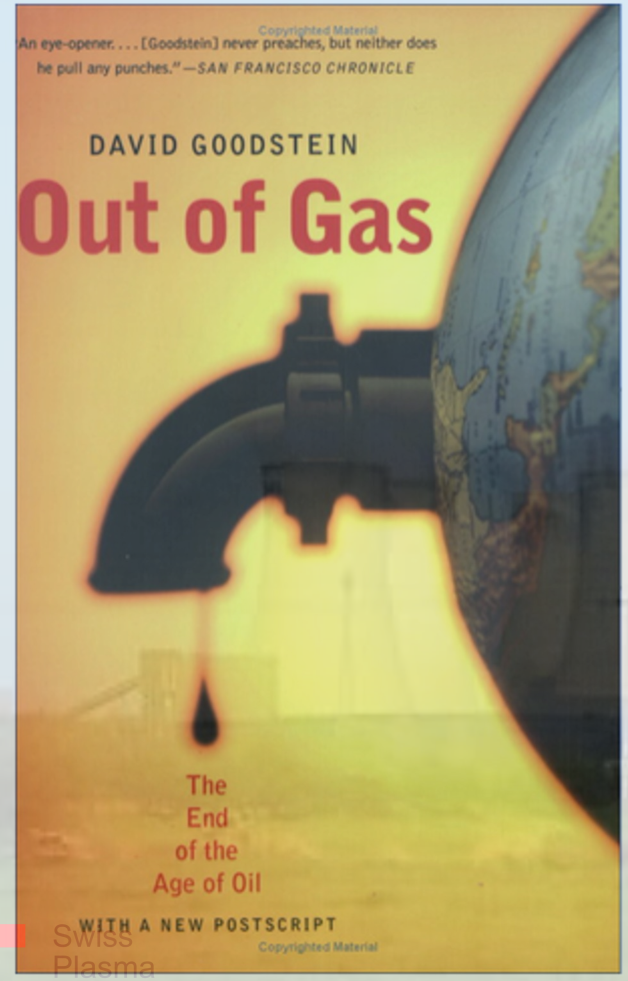
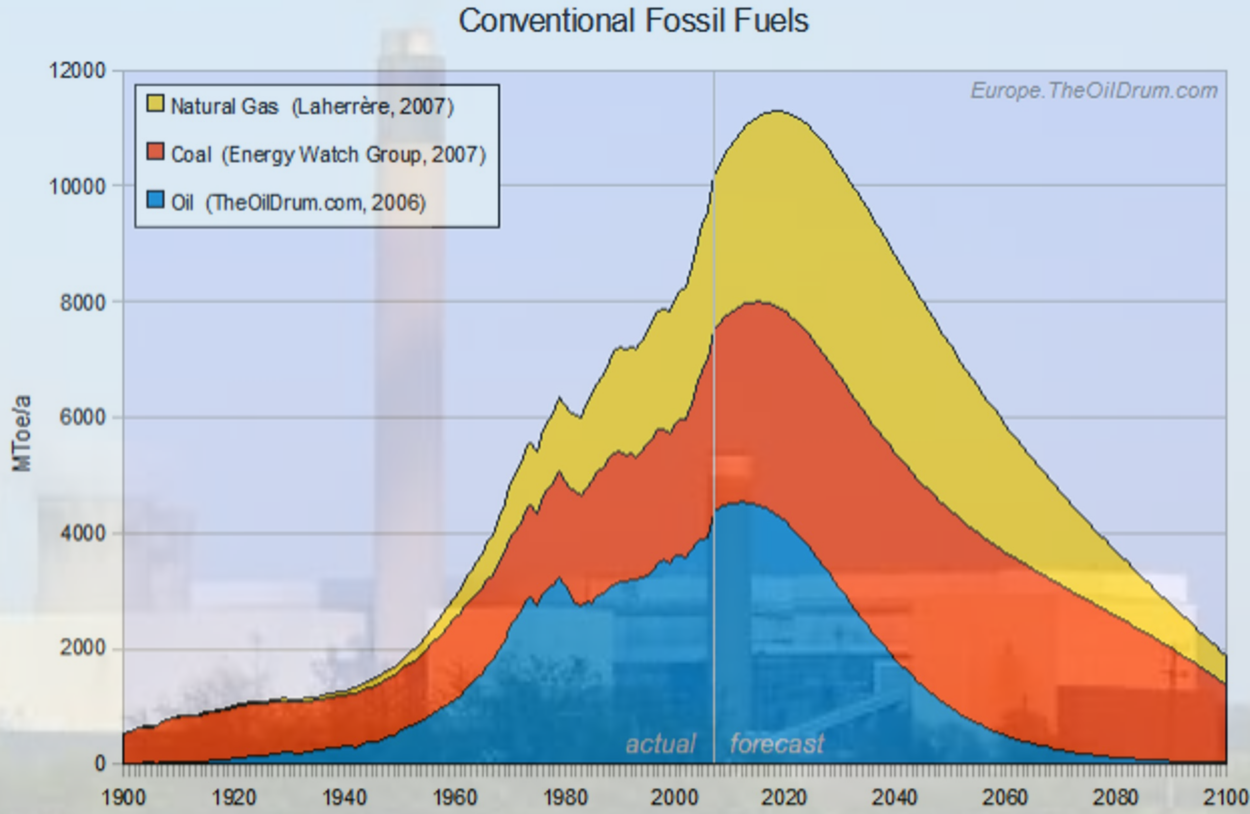
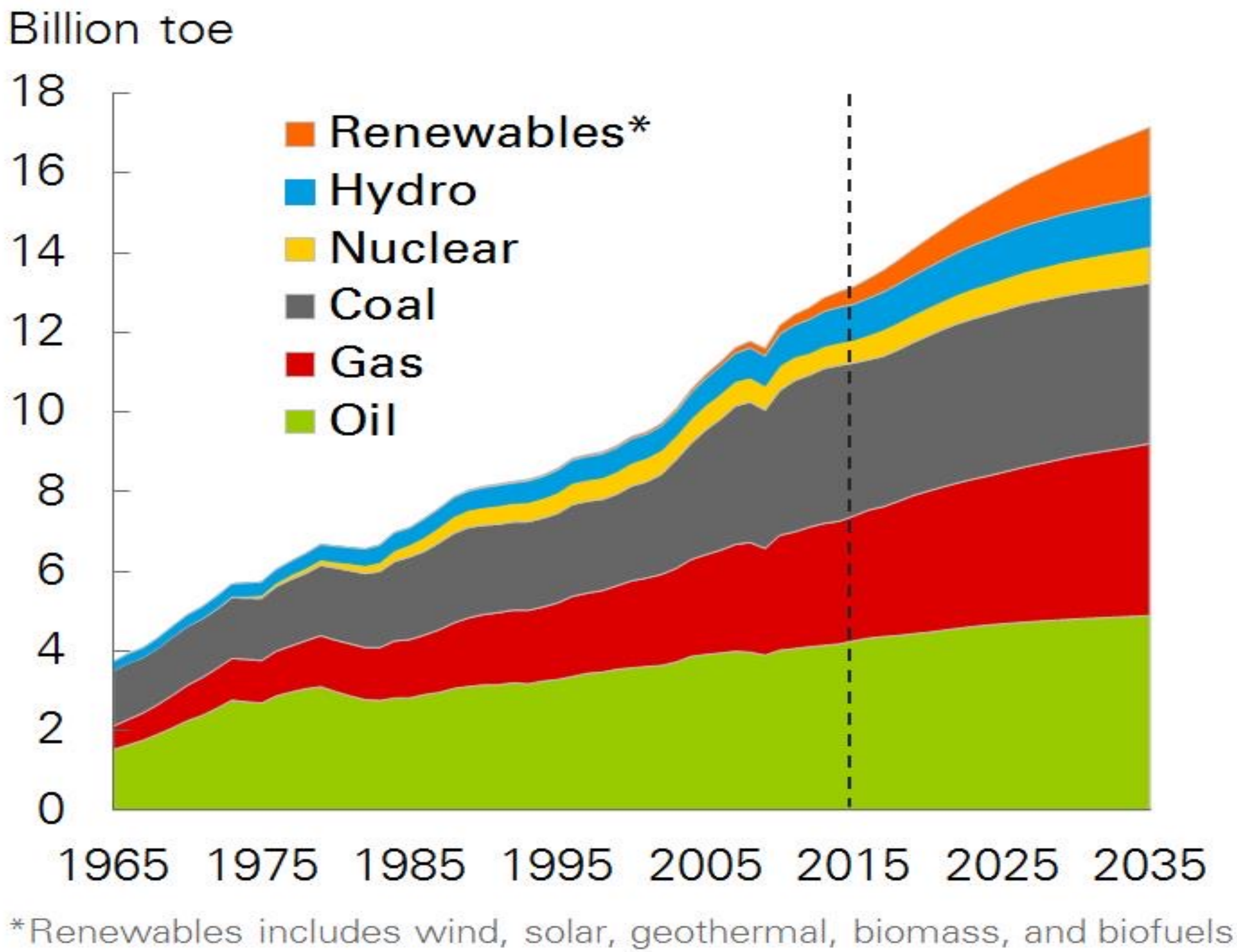
Reliance on fossil fuels

Fossil Fuels have been produced from decayed plant and animal matter over millions of years, cannot be re-formed in time



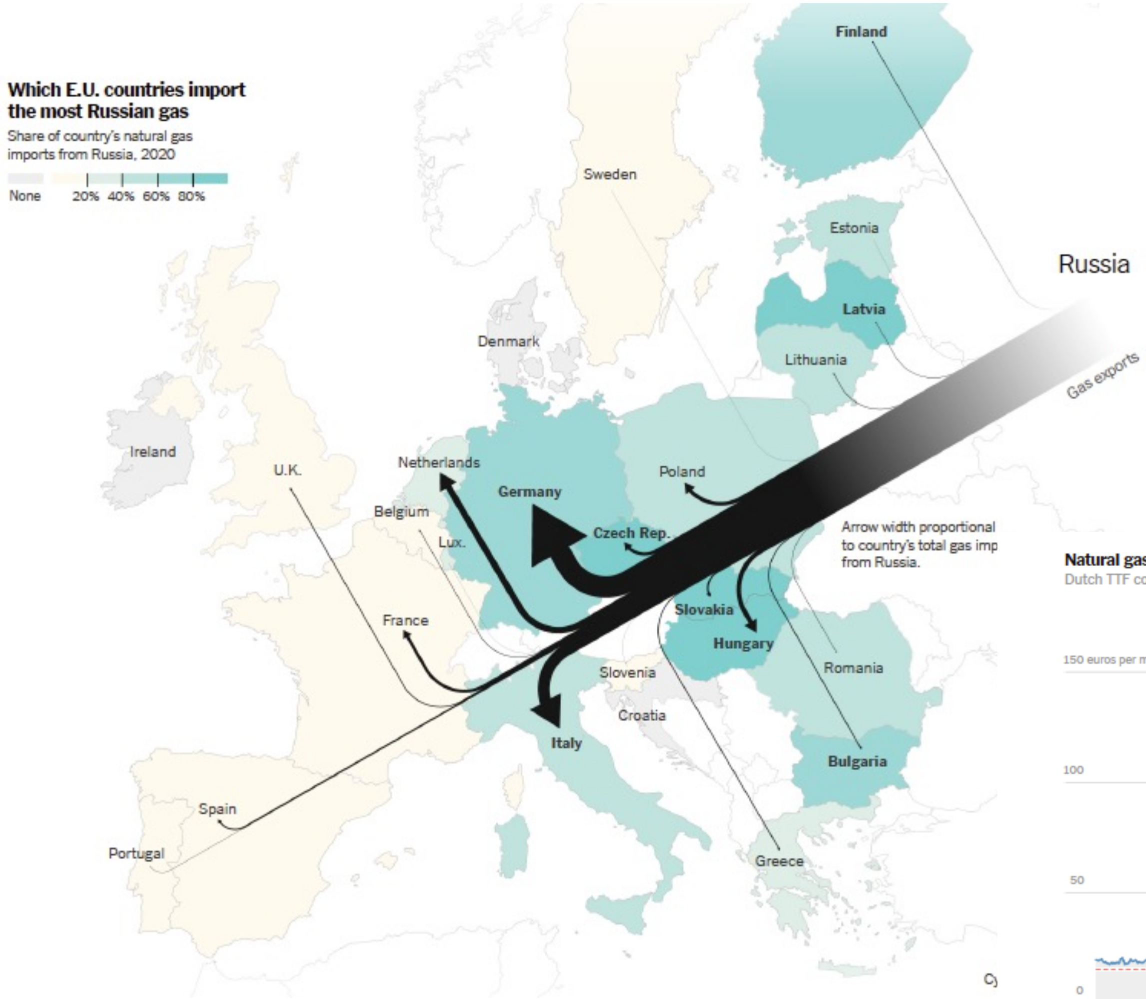


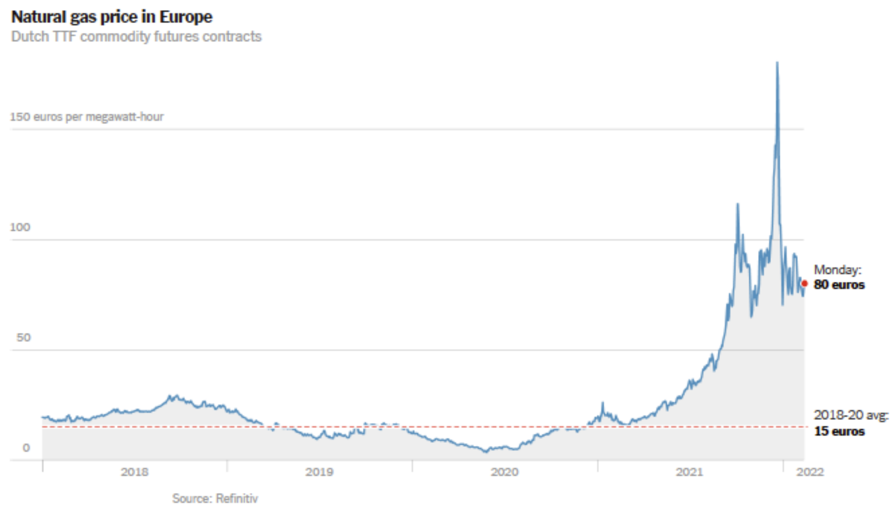
Primary energy consumption by fuel



Urgency of alternates development

Geopolitical issues

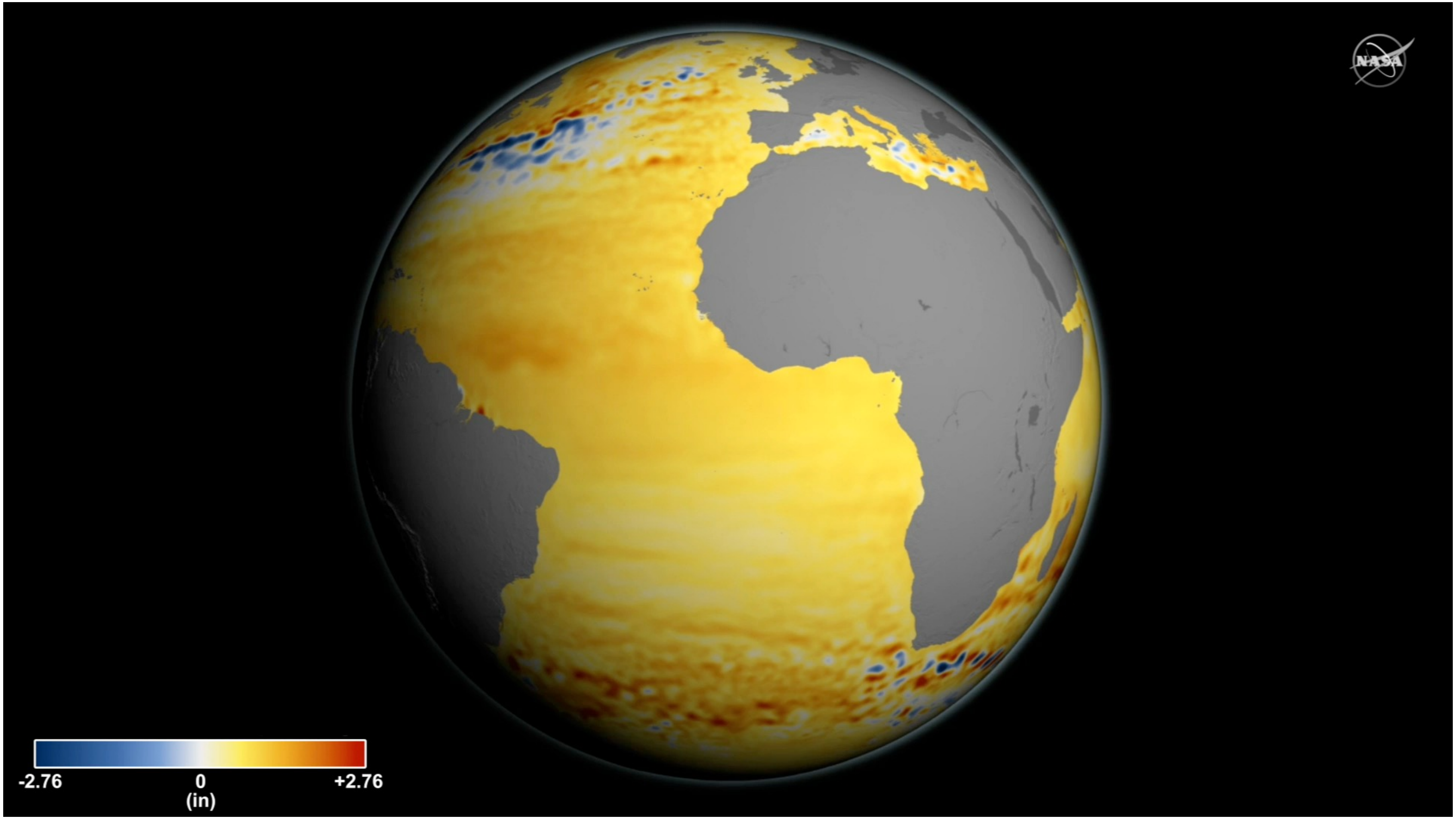




The New York Times

ENERGY & ENVIRONMENT | How a Ukraine Conflict Could Reshape Europe's Reliance on Russia

The effects of climate change

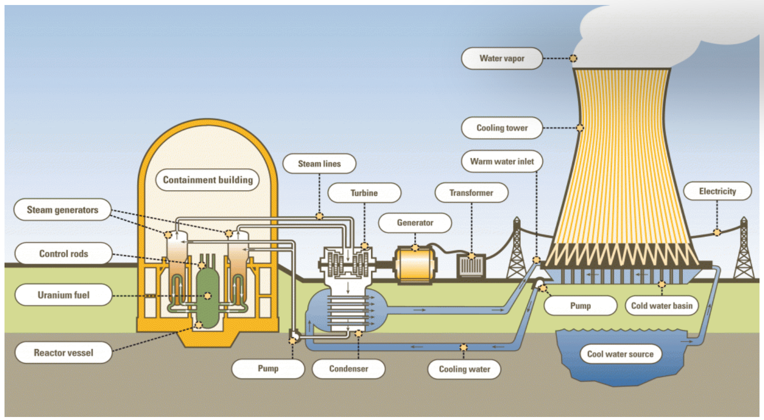


Discussion of today's energy options

Fossil
- Coal
- Natural gas
- Oil

Renewables
- Hydro-electric
- Wind
- Solar

Nuclear fission



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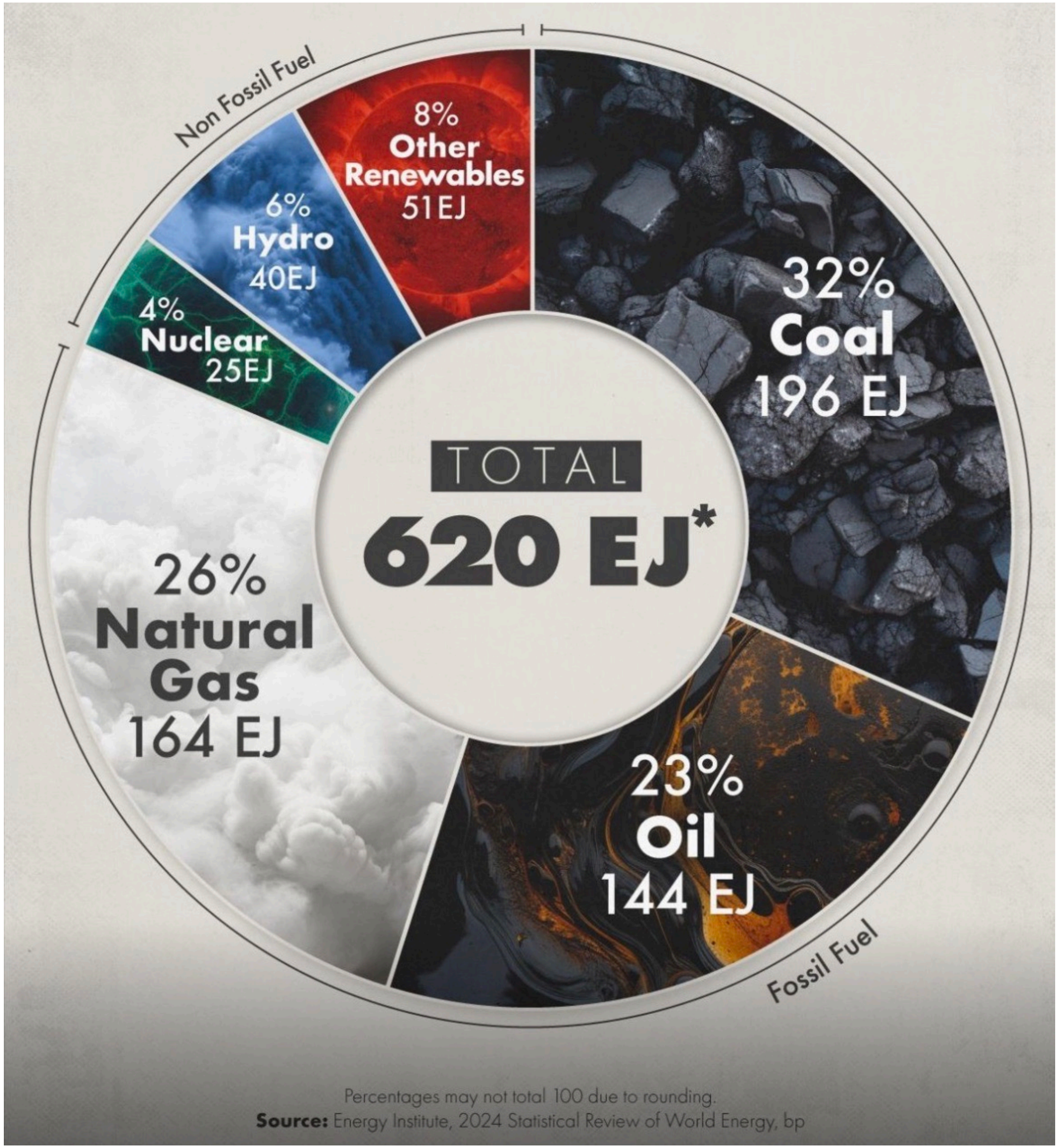
Discussion of today's energy options

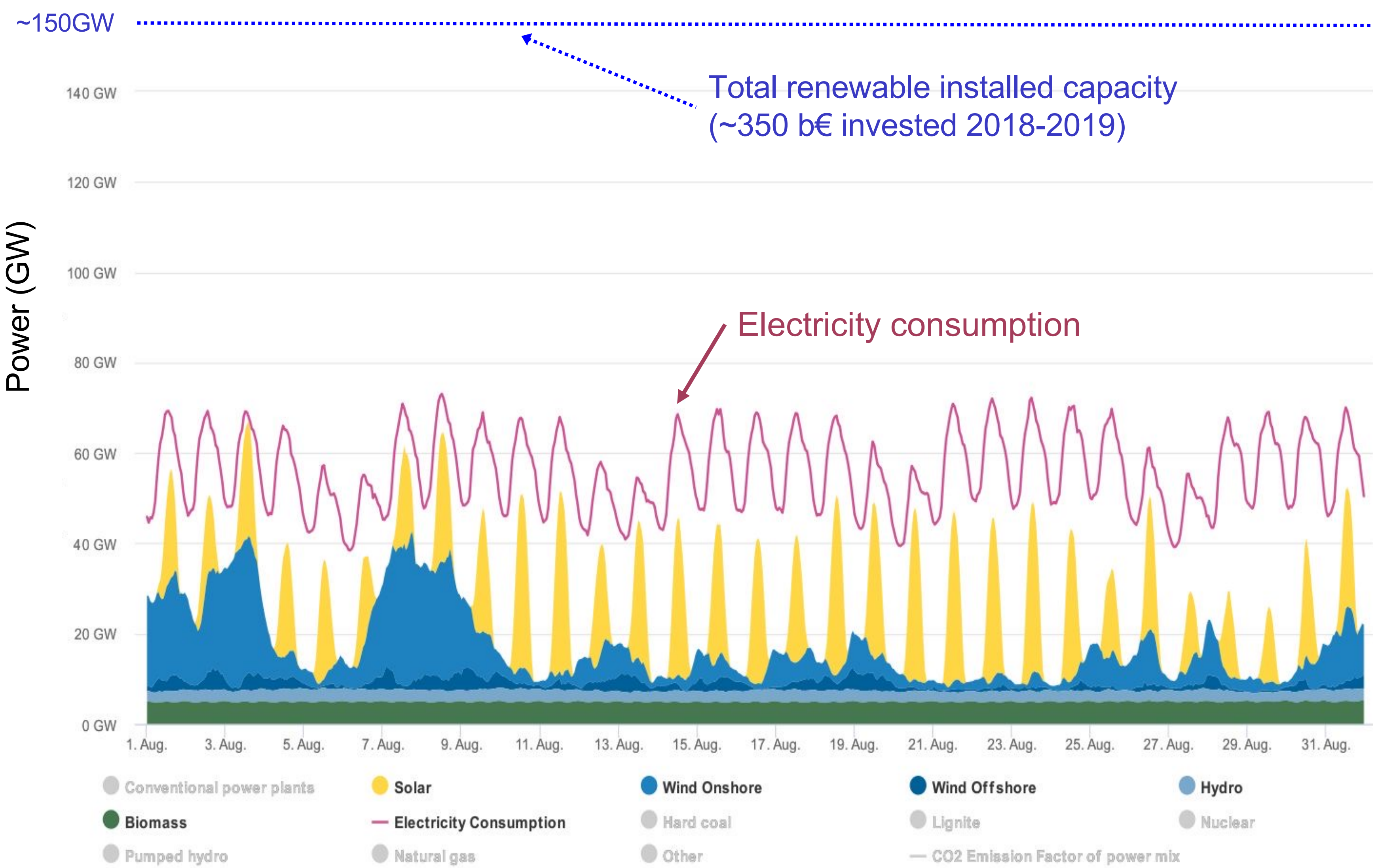
Why not only renewables ?

Renewables
- Hydro-electric
- Wind
- Solar

What powered the world in 2023 ?



The example of Germany - August ‘23



Agora Energiewende (2023): Agorameter. Model version 3.0, Berlin, 13.09.21

Germany - August ‘23 - March ‘24

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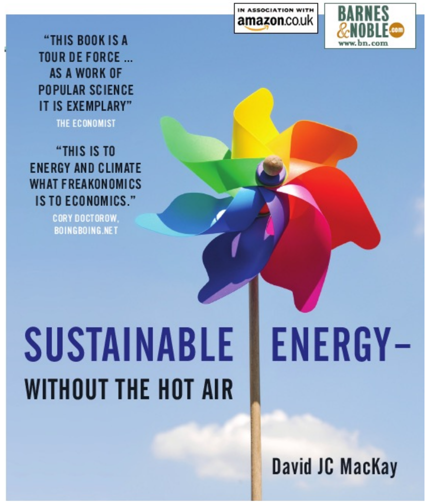
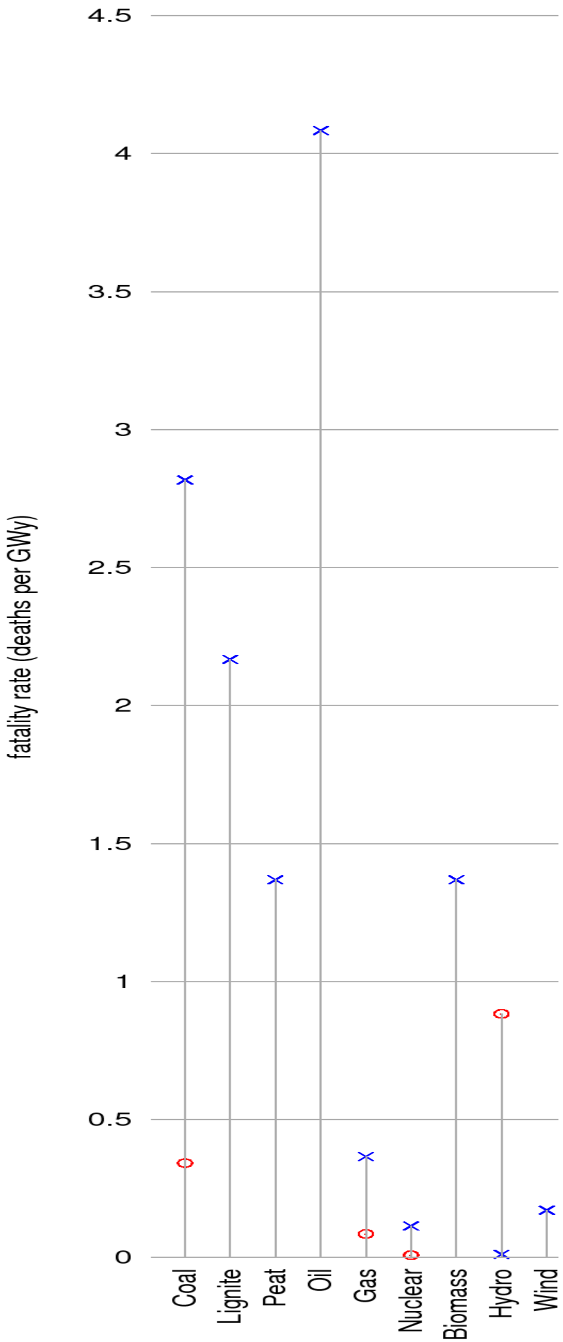
Agora Energiewende (2023): Agorameter. Model version 3.0, Berlin, 13.09.21

Discussion of today's energy options

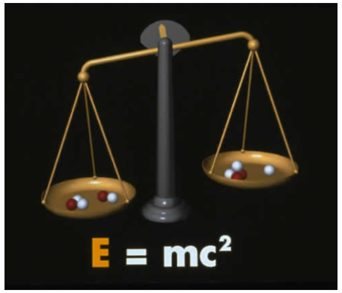
A politically incorrect view

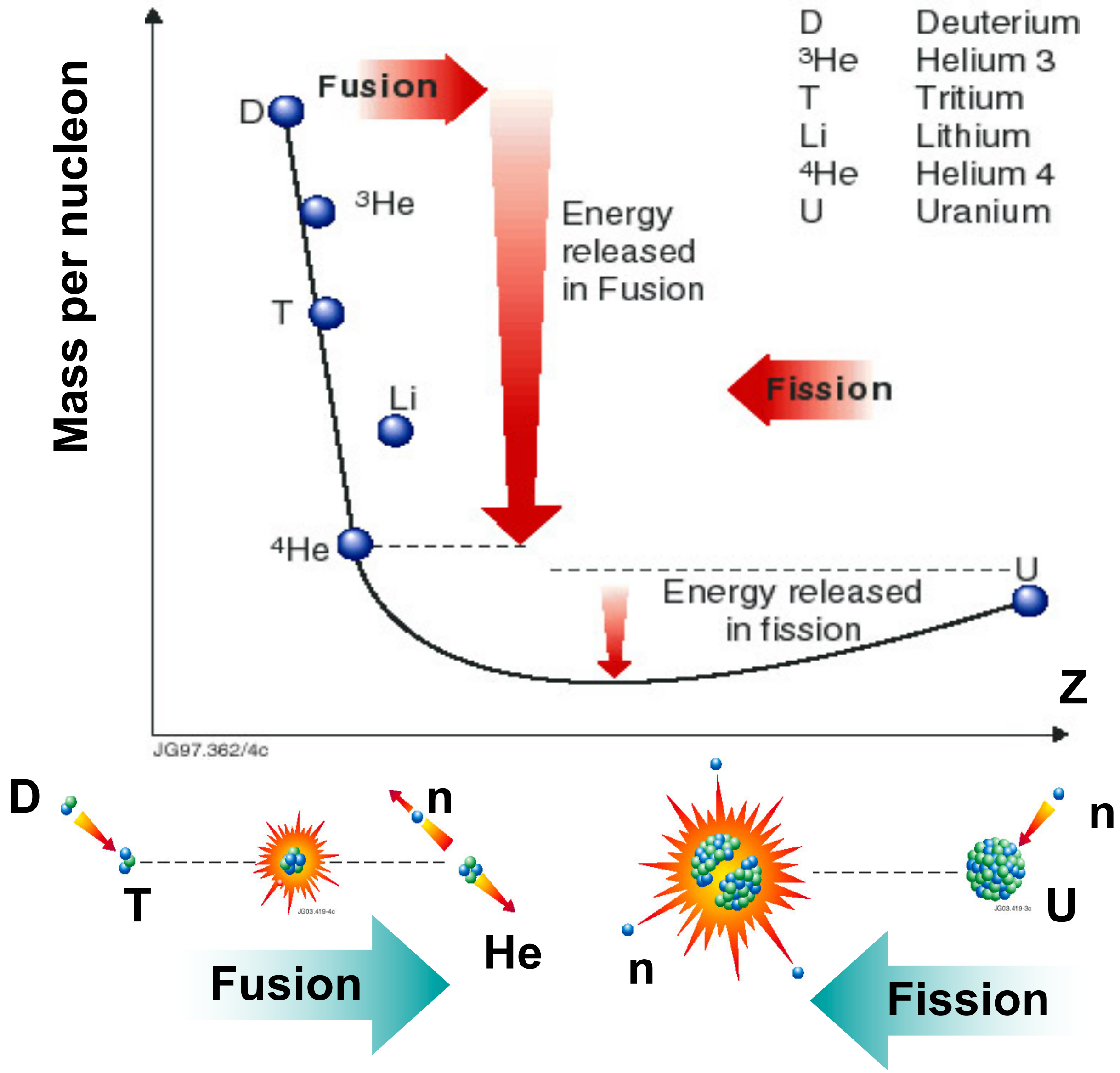
Human deaths per energy



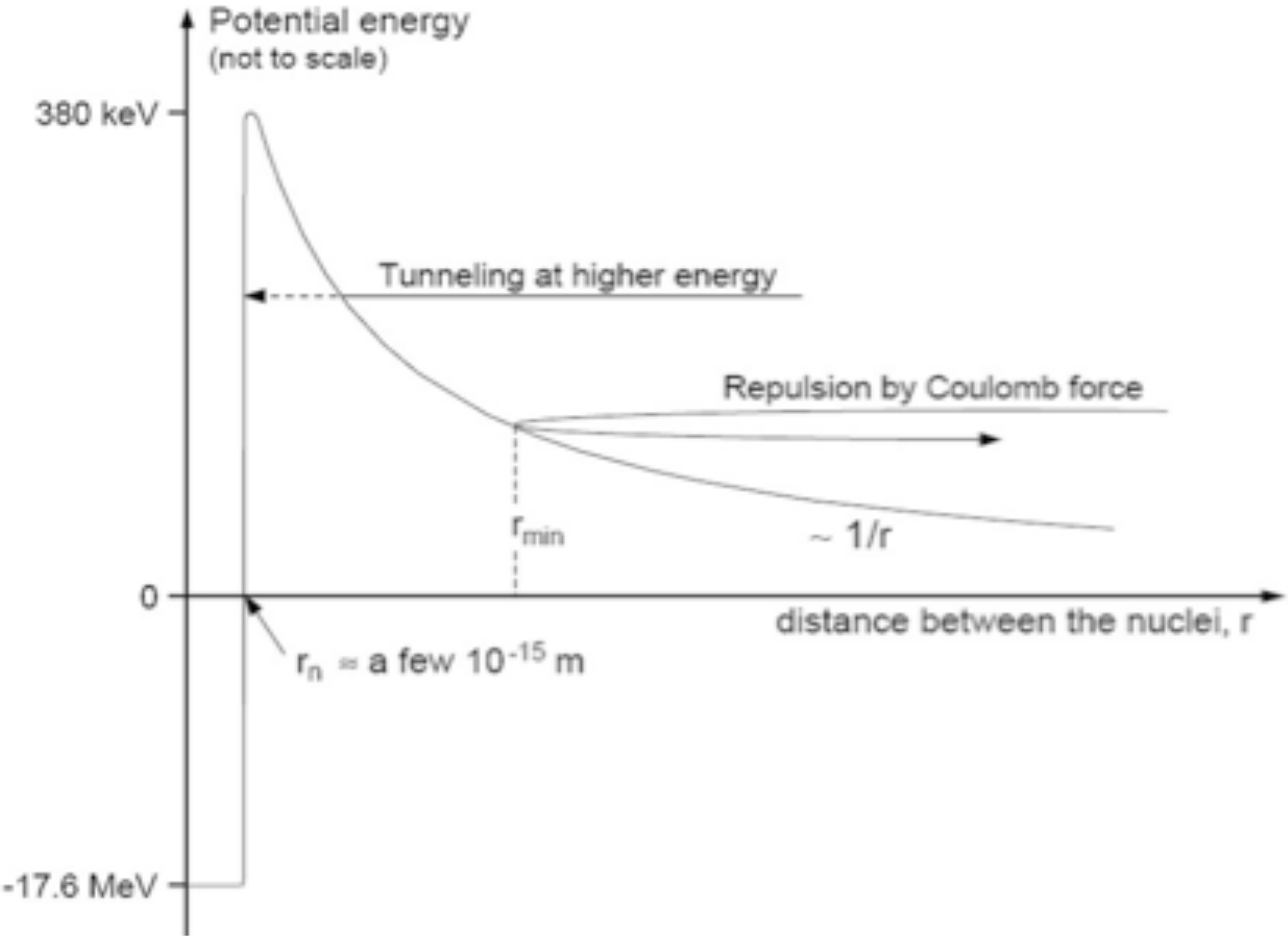


A solution in the nucleus ? Fusion vs. fission





Large energies are required for fusion



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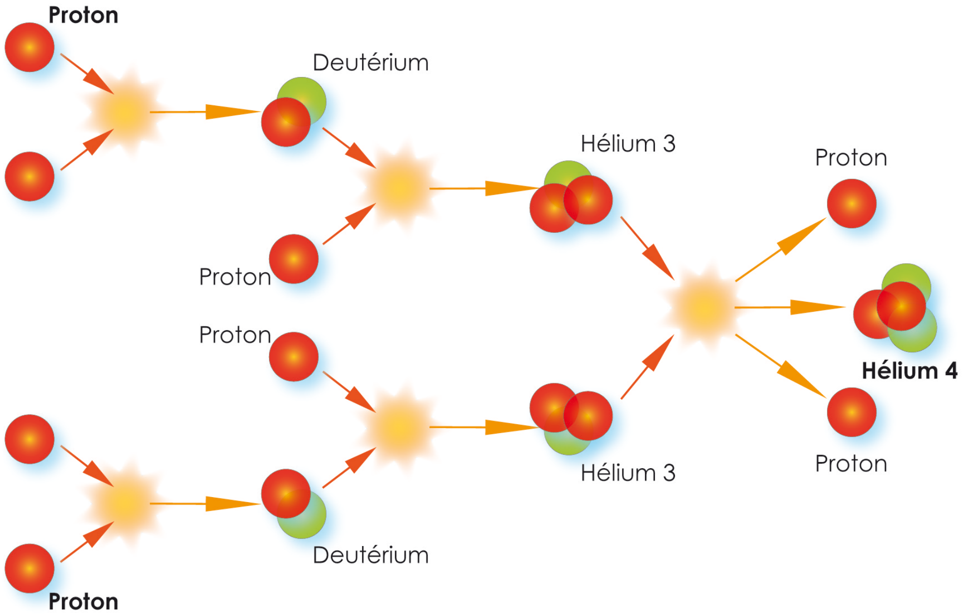
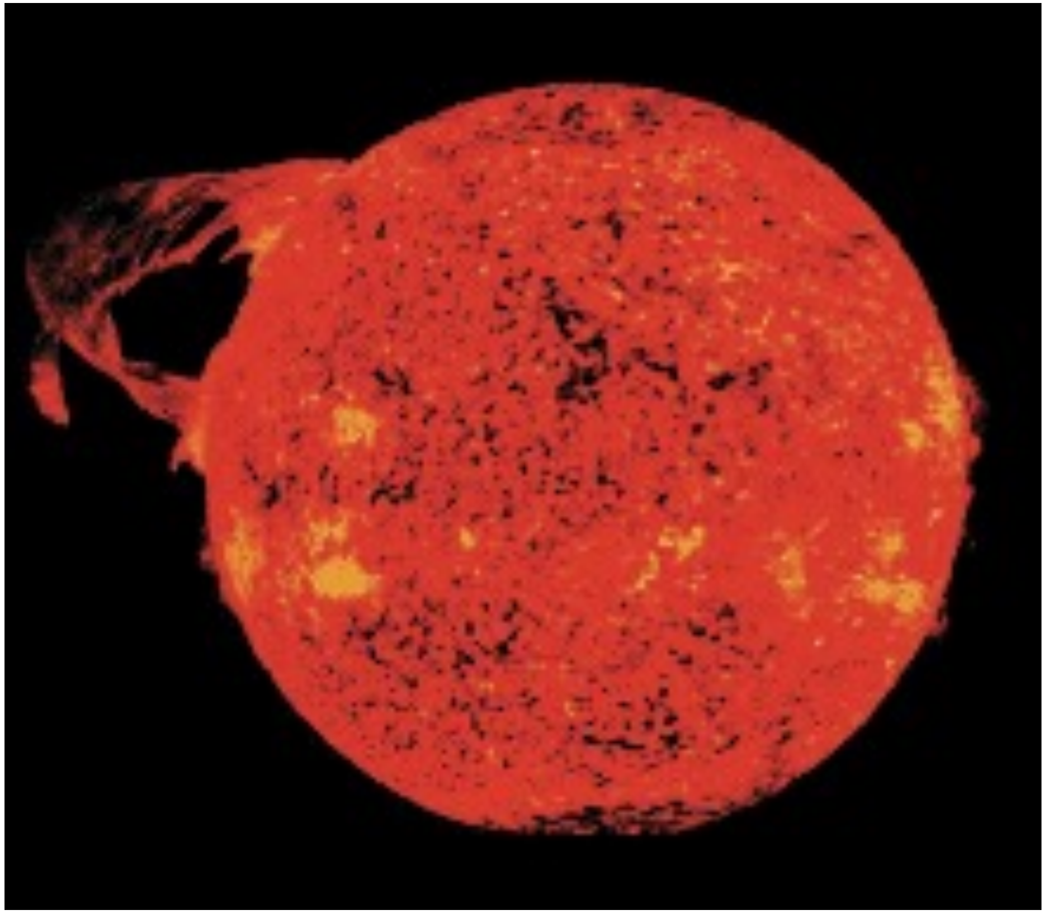
At such large energies matter is in the form of **plasma**

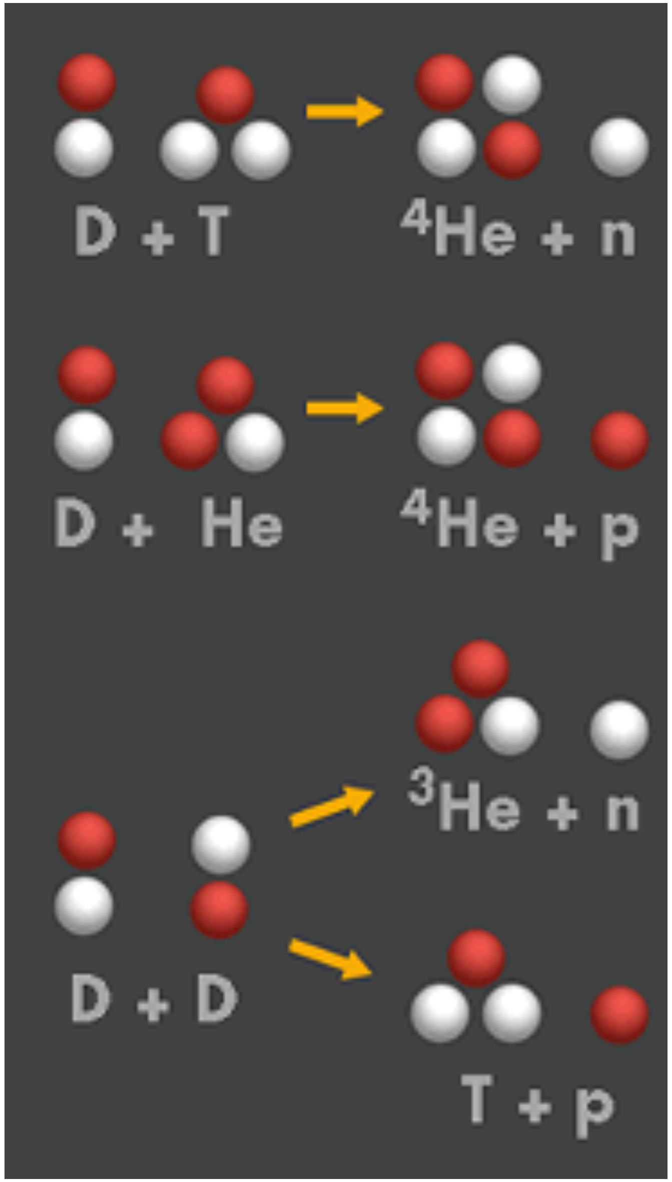
Examples of plasmas

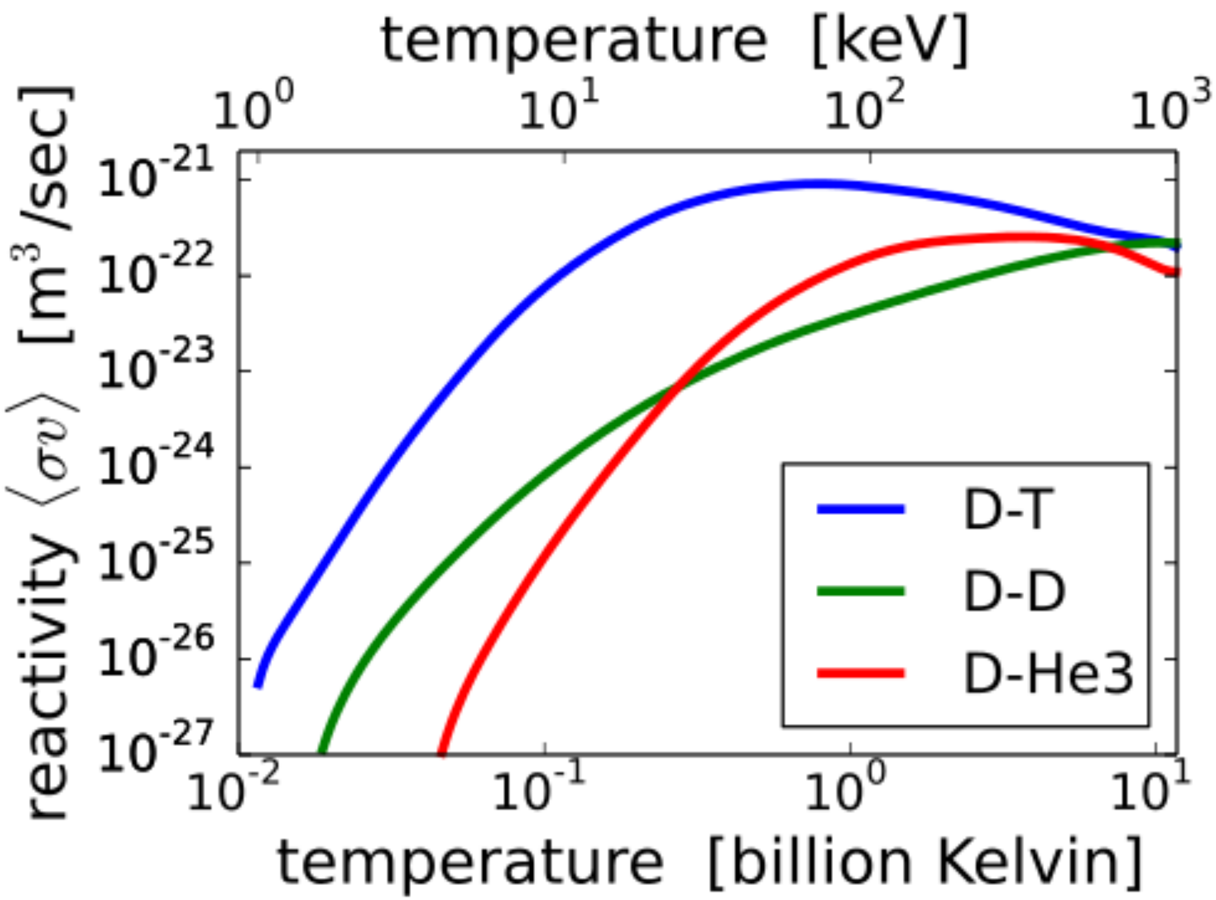
Fusion powers the sun and all the stars



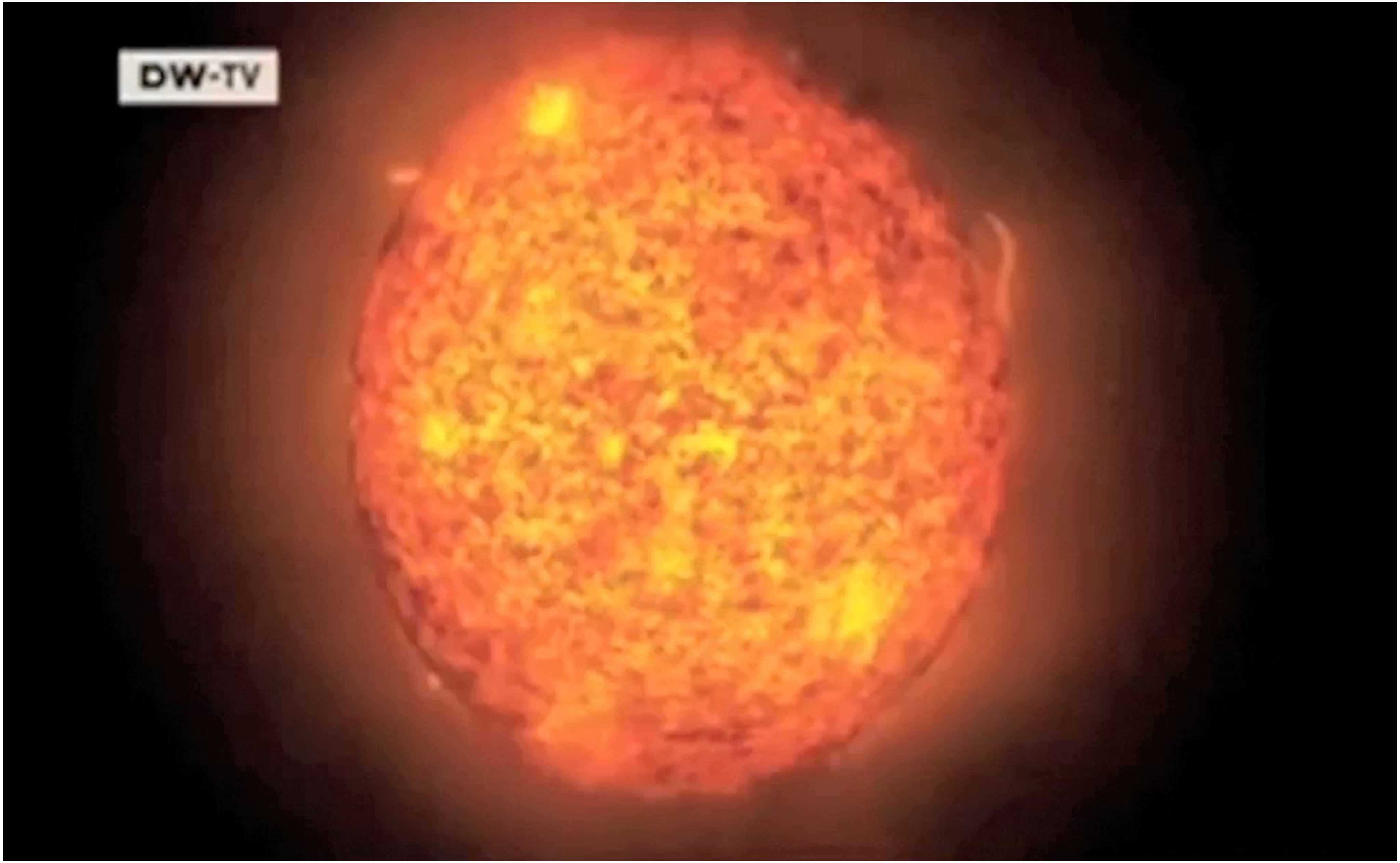


Fusion reactions and cross-sections

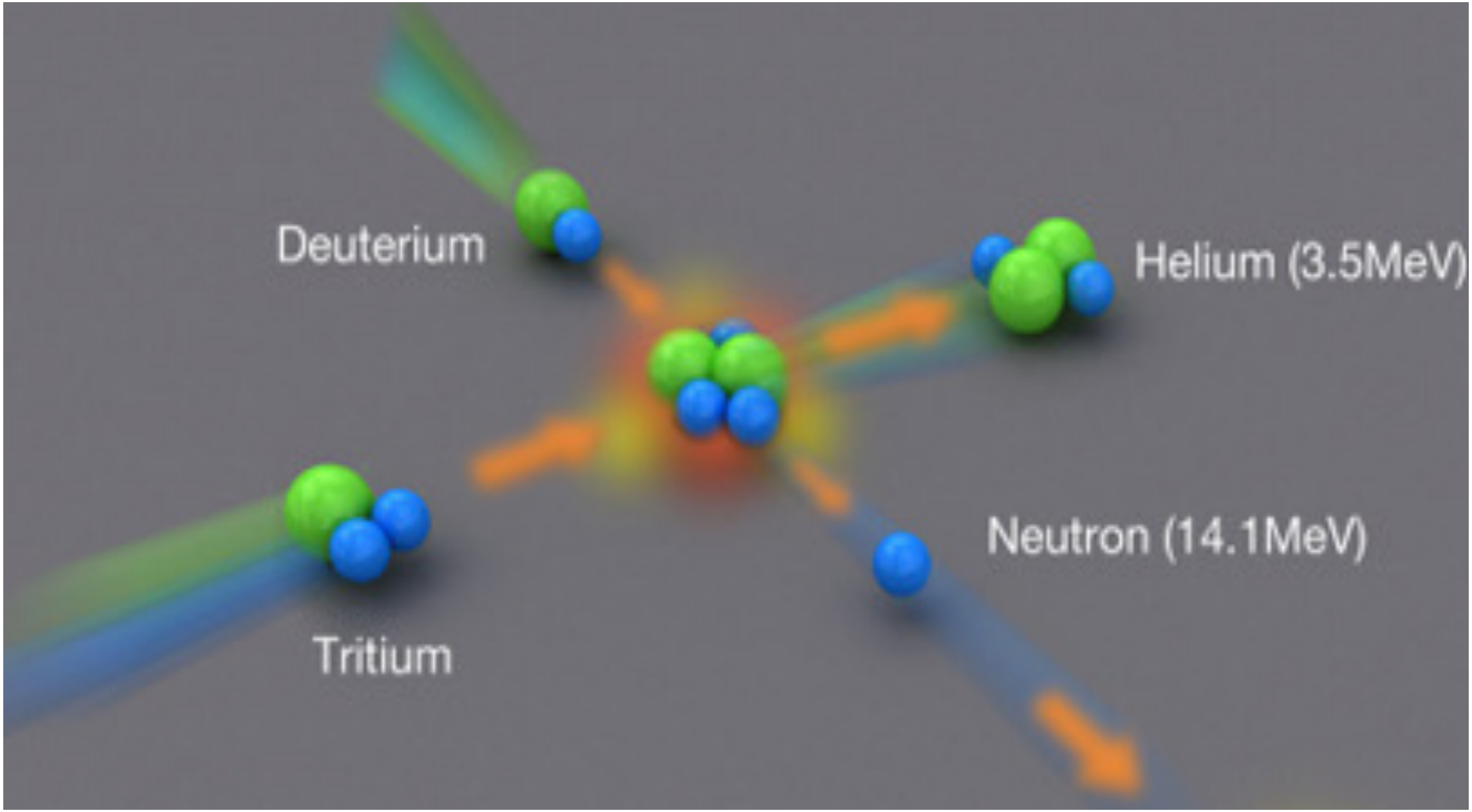




The fusion challenge



The D-T reaction



Sources of fuel for D-T reaction

D is 1/6700 of hydrogenic atoms in oceans, i.e. 1.6g/l

T does not exist in nature, as is radioactive and short-lived (12.5y)

But can be produced from Lithium

$n_0^1 + Li_3^6 \rightarrow T_1^3 + He_2^4 + 4.8MeV$

(Li_3^6 is ~7%)

$n_0^1 + Li_3^7 \rightarrow T_1^3 + He_2^4 + n_0^1 - 2.5MeV$

(Li_3^7 is ~93%)

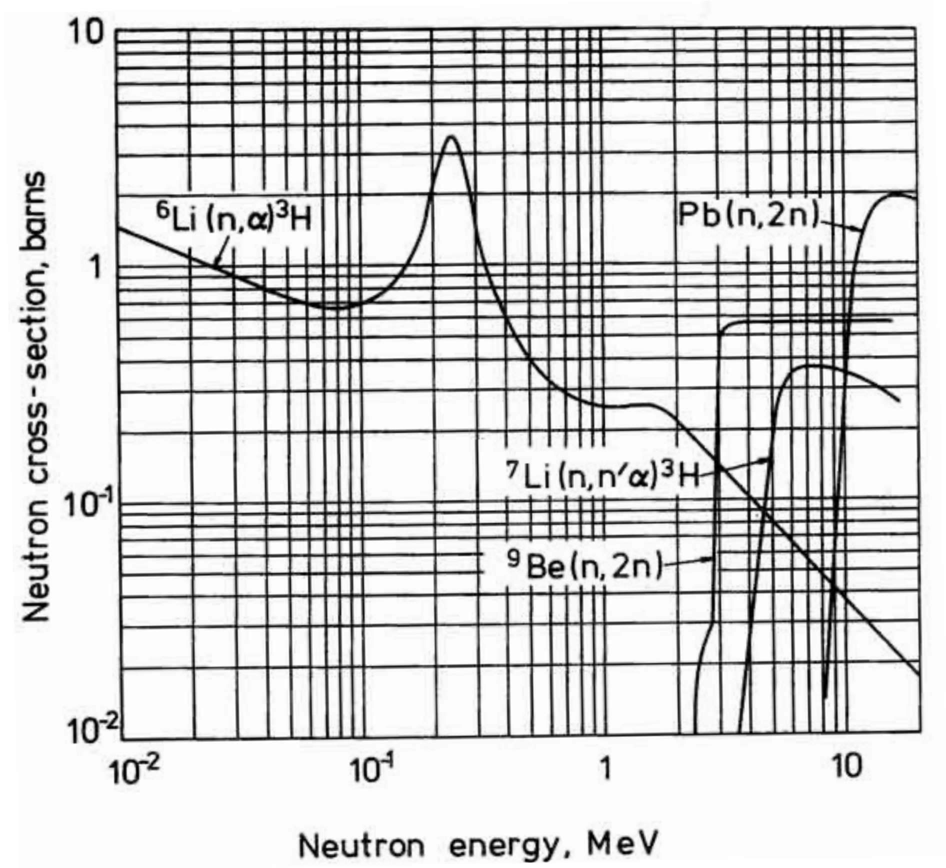
Note that the Li^6 cross section is

much larger for low energies

moderate neutrons first

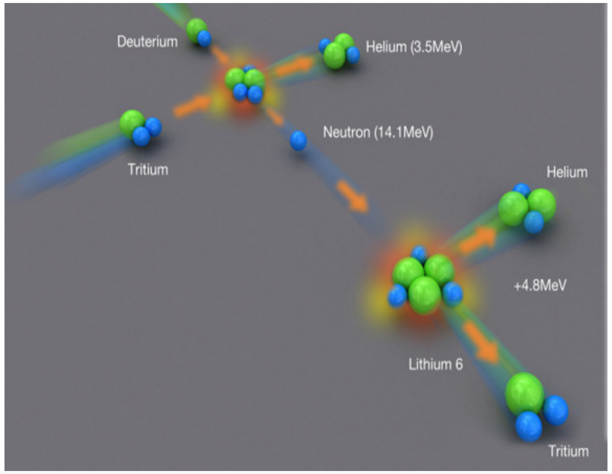
Neutron multiplication is needed, and

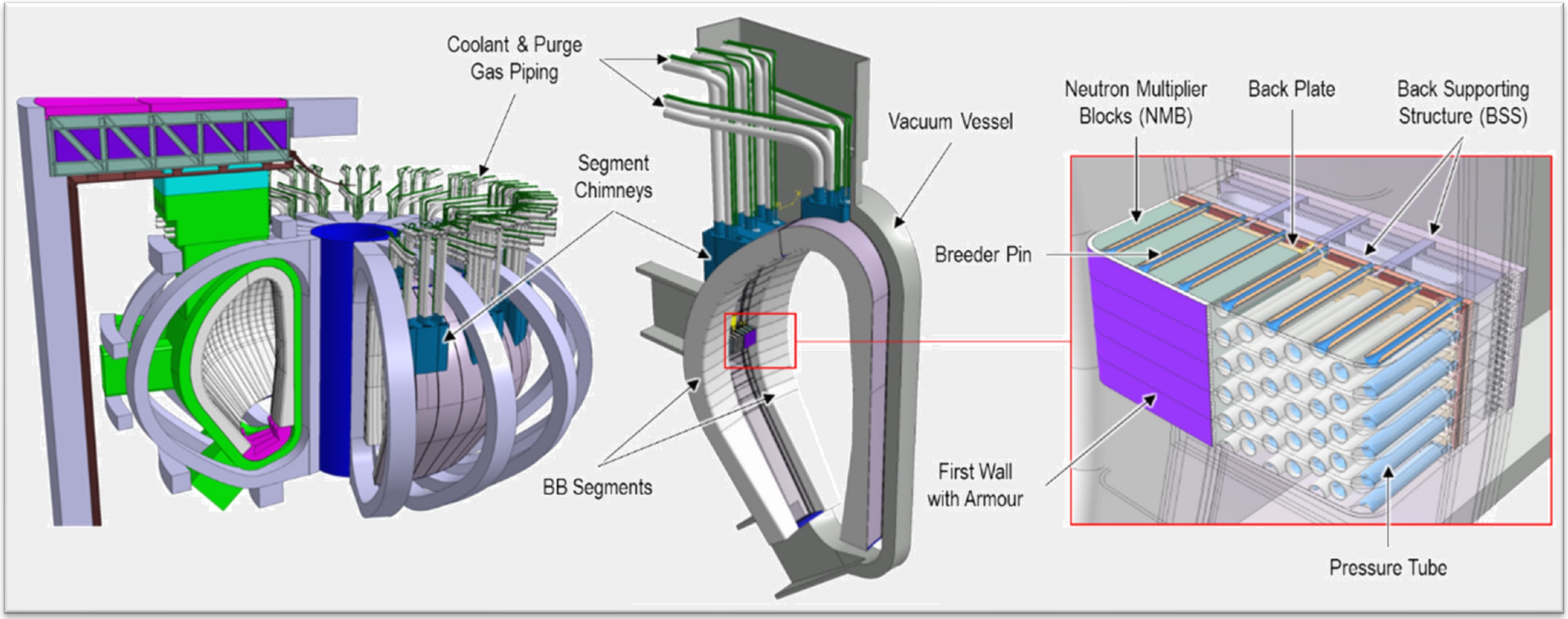
is possible above ~3MeV with Pb or Be



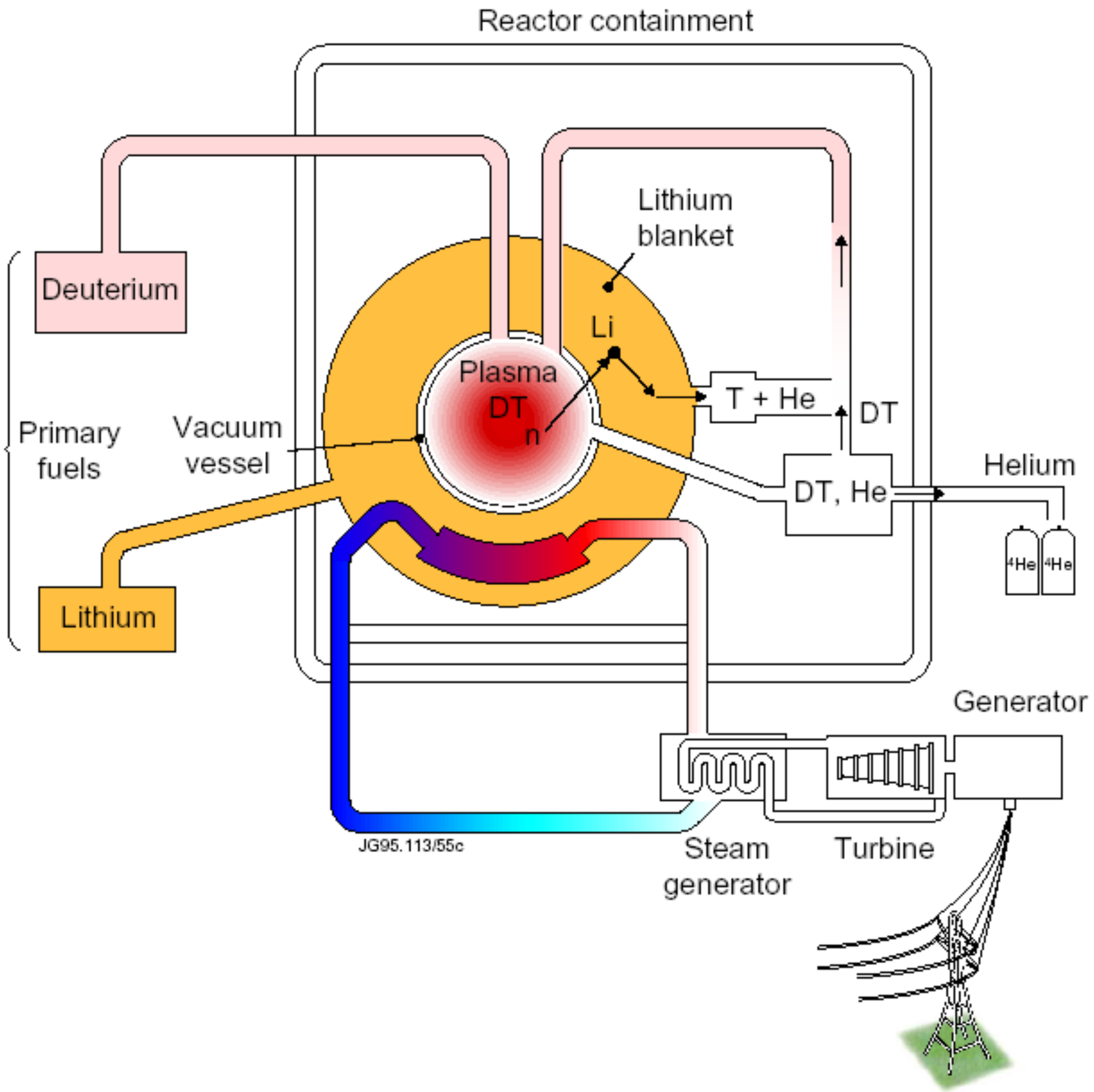
Lithium is in earth crust and in ocean water (0.15g/m^3)

Tritium breeding





Schematic of a fusion power plant



Advantages of fusion energy

High energy density fuel

Fuel	Specific Energy (MJ/kg)
Water, 100m-high dam	0.001
Coal, oil, food	30-50
Fission (U-235)	85’000’000
Fusion (D-T)	**350’000’000**

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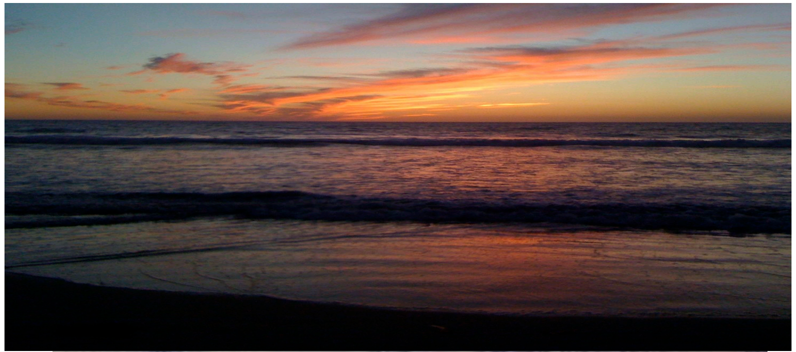
Advantages of fusion energy



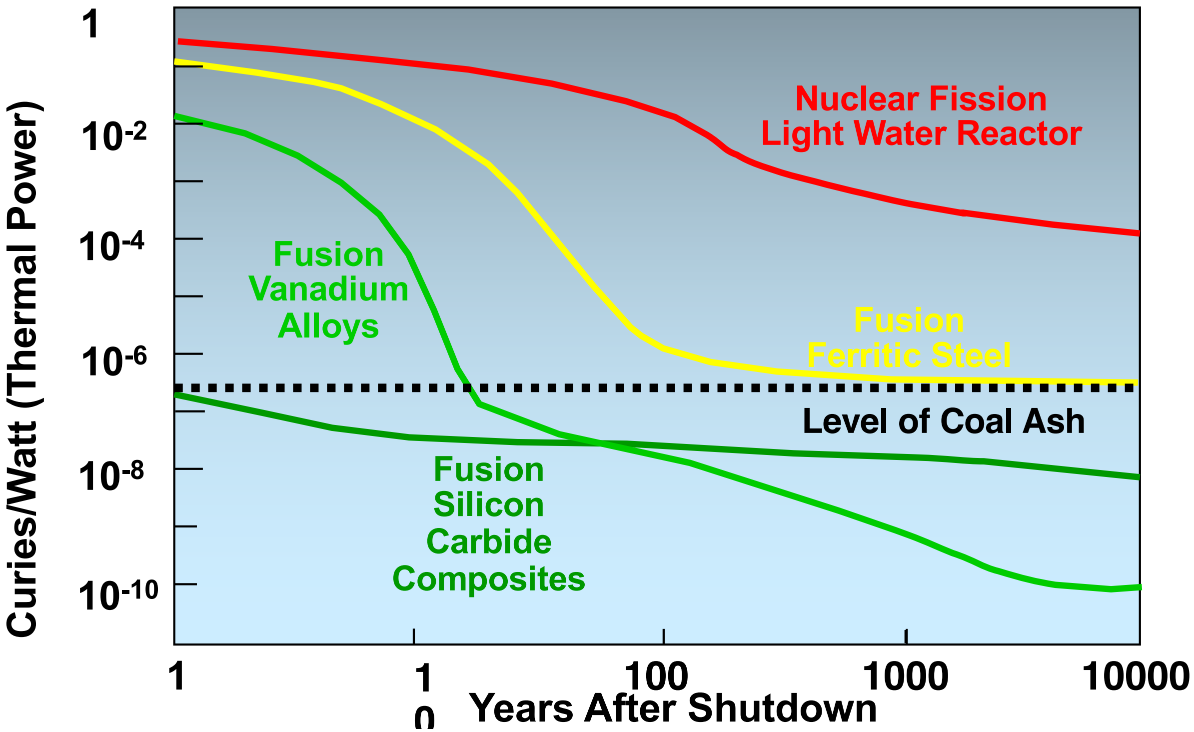
High energy density fuel

Practically inhestaustible fuel

Environmental

No CO_2 emission

No high-level radioactive wastes



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Advantages of fusion energy

High energy density fuel

Practically inhestaustible fuel

Environmental

- No CO_2 emission
- No high-level radioactive wastes

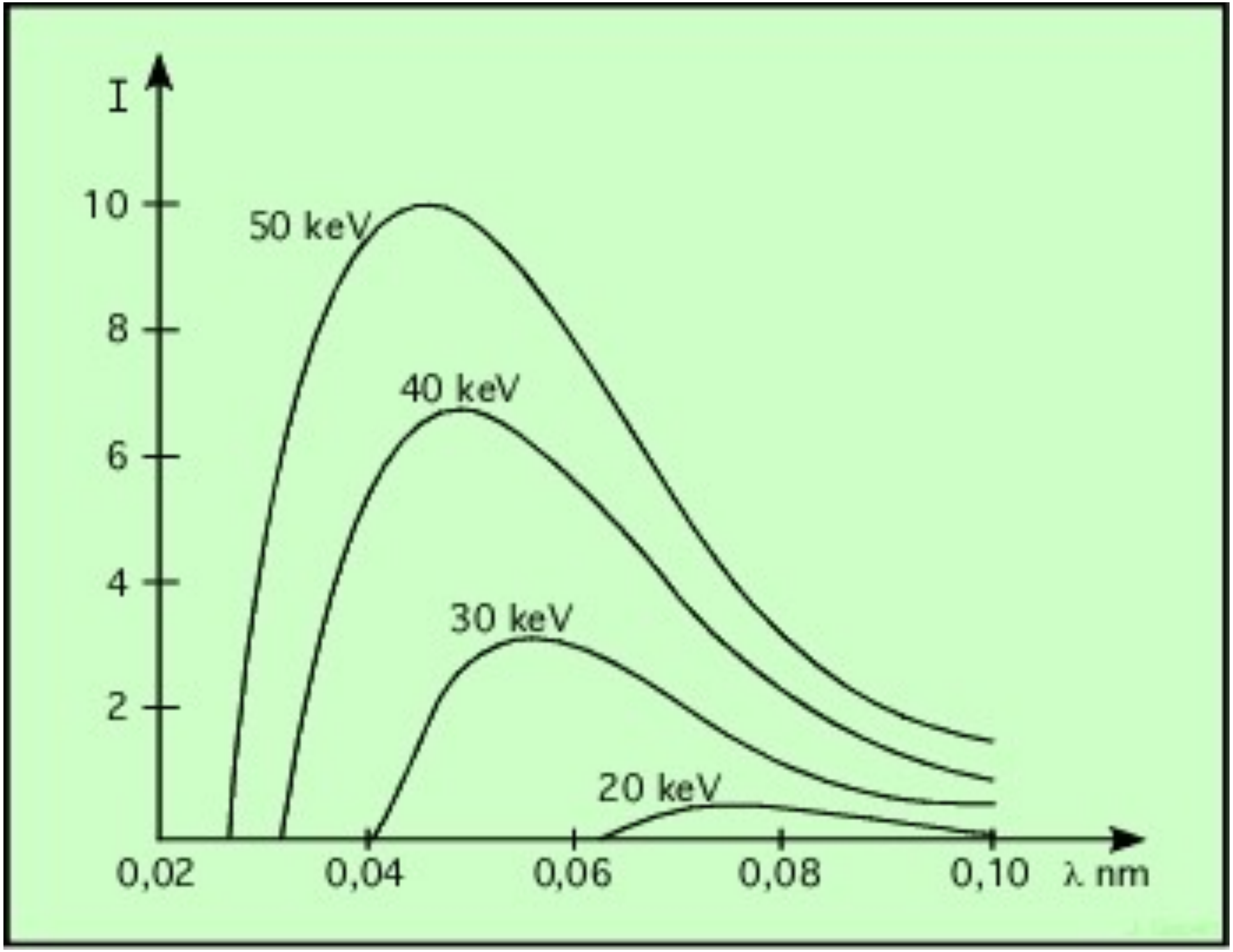
No risk of nuclear accidents

- Only ~1g of fuel in reactor
- No chain reactions

No generation of weapons material

Geographically concentrated, not subject to wheather variations

Bremsstrahlung radiation spectrum



Fusion <σv> vs. plasma temperature