Exercise Set 4: Solution
Quantum Computation

Exercise 1 *IBM Q practice: Implementation and tests with the Toffoli gate*

Please refer to the Jupyter Notebook on Moodle.

Exercise 2 *Square-root of the NOT gate*

(a) The solutions to the eigenvalue-eigenvector equation are:

$$\lambda_0 = 1, \quad w^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad \text{and} \quad \lambda_1 = -1, \quad w^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

so $\Lambda = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ and $W = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$.

(b) We deduce from the above that a possible V is

$$V = W\sqrt{\Lambda}\,W^\dagger = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & i\end{pmatrix}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1+i & 1-i\\1-i & 1+i\end{pmatrix}$$

and indeed, one can check directly that $V^2 = U$.

(c) Yes, V is also unitary, as $V^\dagger = \frac{1}{2}\begin{pmatrix}1-i & 1+i\\1+i & 1-i\end{pmatrix}$ and $VV^\dagger = I$.

Remark: Please note that we have already encountered $W = H$, $\Lambda = Z$ and $\sqrt{\Lambda} = S$!