10) Existence of non-measurable sets:

Consider the set $\Omega = [0,1]$ with the “addition modulo 1” i.e. $x \oplus y$ is the decimal part of the sum $x + y$; $\Omega$ can be seen as a flat representation of a circle.
On $\Omega$, consider the following equivalence relation:

$x \sim y$ if $\exists r \in \Omega \cup \Omega$ such that $x = y + r$

This (strange) equivalence relation allows us to separate the set $\Omega$ into disjoint equivalence classes. Define now $A$ to be the set constructed with a representative of each equivalence class (the axiom of choice guarantees that such a set exists).

For $r \in \Omega \cup \Omega$, define $A_r = A + r = \{ x + r : x \in A \}$

and it holds that $A_r \cap A_s = \emptyset$ for $r \neq s$, $r, s \in \Omega \cup \Omega$

and $\Omega = \bigcup_{r \in \Omega \cup \Omega} A_r$ is therefore a countable & disjoint union.
We also want to define a probability measure \( P \) on \( \Omega = [0, 1] \) such that \( P(B) = P(B \circ z) \) for every measurable set \( B \subseteq \Omega \) and \( z \in \Omega \). This is the translation invariance of the Lebesgue measure on \( \Omega \) (or the rotation invariance if \( \Omega \) is viewed as a circle).

**Problem:** By the axioms of probability, we should have
\[
\underbrace{P(\Omega)}_{=1} = \sum_{r \in \mathbb{Q} \cap \Omega} P(A_r) = \sum_{r \in \mathbb{Q} \cap \Omega} P(A)
\]
and this is impossible (both \( P(A) = 0 \) or \( P(A) > 0 \) lead to a contradiction). **Conclusion:** \( A \) is not measurable.
2. Any cdf has at most a countable number of jumps: 

F makes a jump at position \( t \in \mathbb{R} \) if 

\[
F(t) - F(t^{-}) > 0
\]

Let us now consider 

\[ A = \left\{ t \in \mathbb{R} : F(t) - F(t^{-}) > 0 \right\} \]

and show that \( A \) is countable. Define

\[ A_n = \left\{ t \in \mathbb{R} : \frac{1}{n+1} < F(t) - F(t^{-}) \leq \frac{1}{n} \right\} \quad \text{for} \quad n \geq 1 \]

Observe that each \( A_n \) is finite (more precisely, \( |A_n| \leq n+1 \)) and 

\[ A = \bigcup_{n \geq 1} A_n \]. Therefore, \( A \) is countable.