

Homework 11

Exercise 1. [Sampling from a posterior]

Assume you are given some "data points" y_1, \dots, y_N which are known to be i.i.d samples of $q(y | \theta)$ where $\theta \in \mathbb{R}$ is a random parameter with (continuous) prior $p_0(\theta)$.

We want to sample θ from the posterior distribution $p(\theta | y_1, \dots, y_N)$. We decide to construct a MH Markov chain $\theta^0, \theta^1, \theta^2, \dots, \theta^t, \theta^{t+1}, \dots$ where the proposal move consists of the transition $\theta^t \rightarrow \theta^{t+1}$ with probability $p_0(\theta^{t+1})$.

- a) Formulate the MH algorithm and in particular give a *simple* formula for the acceptance probabilities that does not involve the prior and/or any potentially difficult to compute integral.
- b) Is it true that you do not need to know this prior in order to run the chain ? What is the advantage of MH here w.r.t directly sampling the posterior ?