

Mécanique des composites 2026

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Mécanique des composites

Micro et macromécanique

Tests mécaniques

Endommagement et rupture

CADFEM

Applications



Les stratifiés

Les structures sandwich

Les composites textiles

Biblio

Les matériaux composites organiques
Traité des matériaux, vol 15, PPUR, Lausanne

Matériaux composites, Composites Materials
D. Gay, Lavoisier Hermes 2015

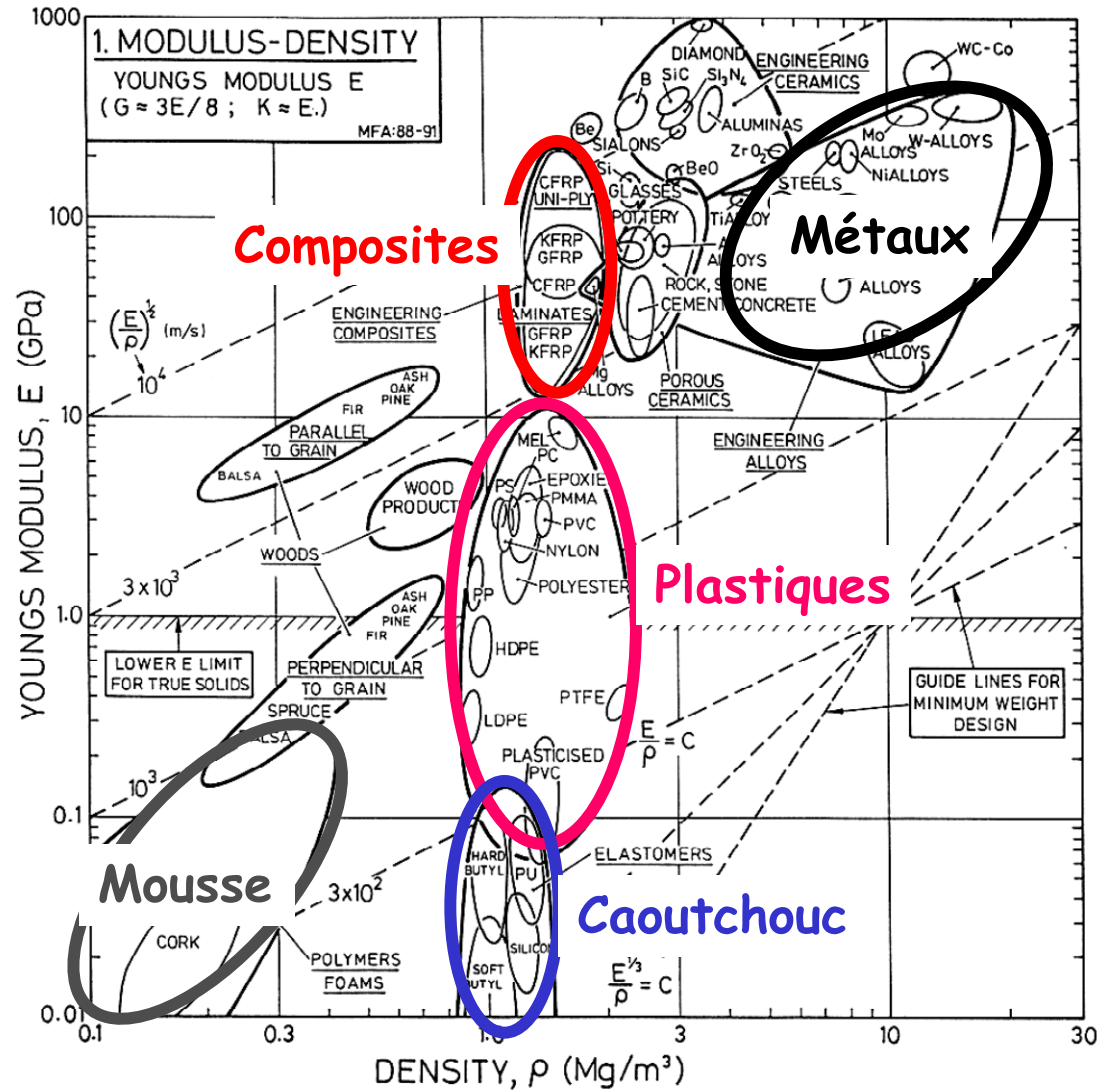
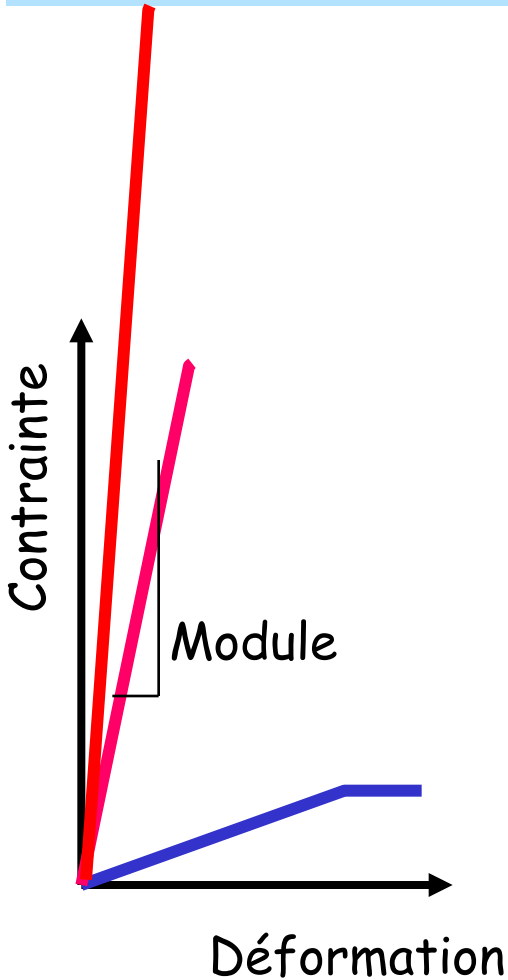
Comprehensive composite materials,
Zweben C. ed., Elsevier 2017

Engineering mechanics of composites materials,
I.M. Daniel, O. Ishai, Oxford University Press

...

...

Propriétés: module - densité



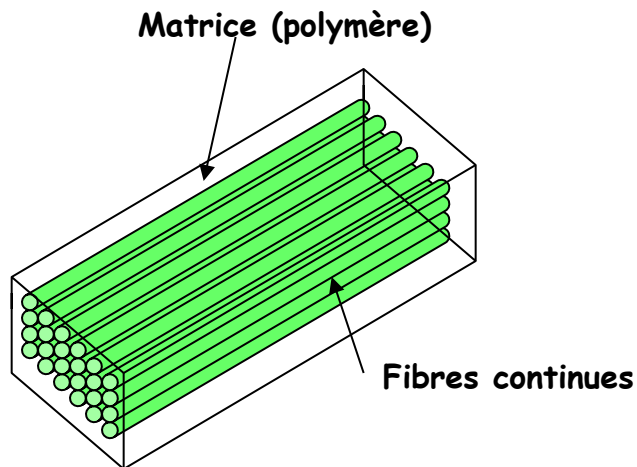
M.F. Ashby, *Materials Selection for Mechanical design*, Pergamon Press

Propriétés des matériaux composites unidirectionnels

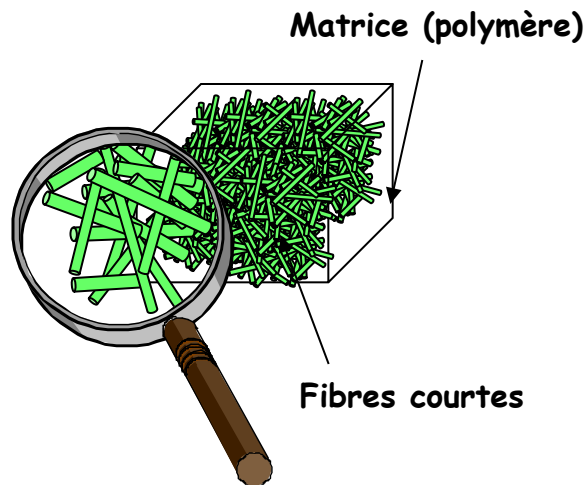
	METAUX			COMPOSITES A MATRICE ORGANIQUE			
	Ac35NDC16	Alliage Al	Alliage Ti	Bore époxy	Carbone HR époxy	Carbone HM époxy	Verre R époxy
Module de Young E (MPa)	200'000	72'000	110'000	220'000	130'000	200'000	53'000
Résistance à la rupture en traction σ_r (MPa)	1850	500	1000	2000	1000 à 1300	1000	1800 à 2000
Masse volumique ρ (g/cm ³)	7.9	2.8	4.45	2.1	1.5	1.7	2
Module spécifique E/ρ (MPam ³ /kg)	25	25	25	105	87	118	26
Résistance spécifique σ/ρ (MPam ³ /kg)	0.24	0.18	0.23	0.95	0.65 à 0.85	0.60	0.90 à 1

Propriétés comparées de composites unidirectionnels avec 60% de fibres en volume

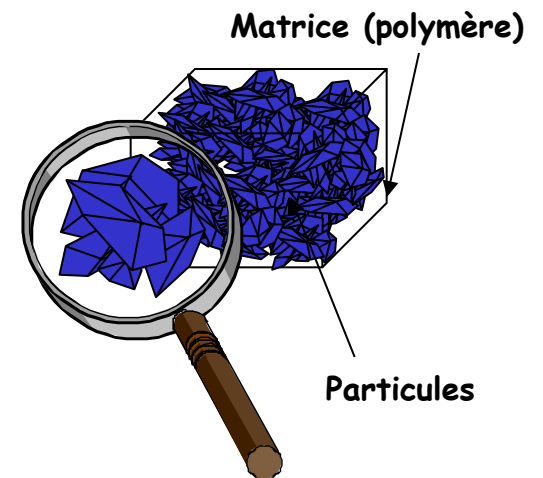
Matériaux composites



- Fibres orientées
- Composites unidirectionnels (UD)
- Thermosets, thermoplastics
- Haute performance mécanique
- Anisotropy
- Aerospace, sport, pressure vessels...



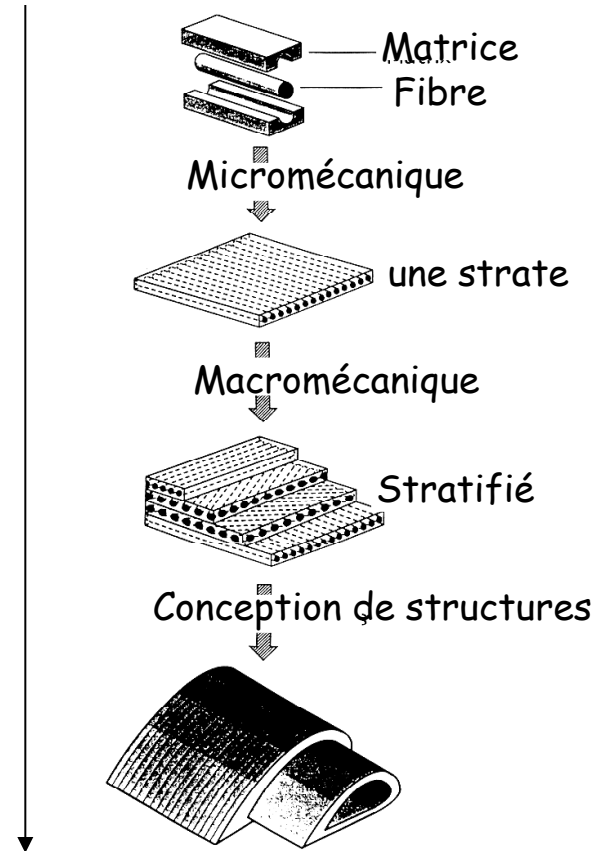
- Distribution of short fibers (< 3mm)
- Especially for thermoplastic matrices
- Isotropy
- Injection molding
- Mechanics, microtechnology, automotive....



- Particles (1-400µm), silica, clay...
- Thermosets and thermoplastics
- Improvement of properties
- Dimensional stability
- Reduction of cost
- Injection molding, casting
- Mechanics, microtechnology, dentistry....

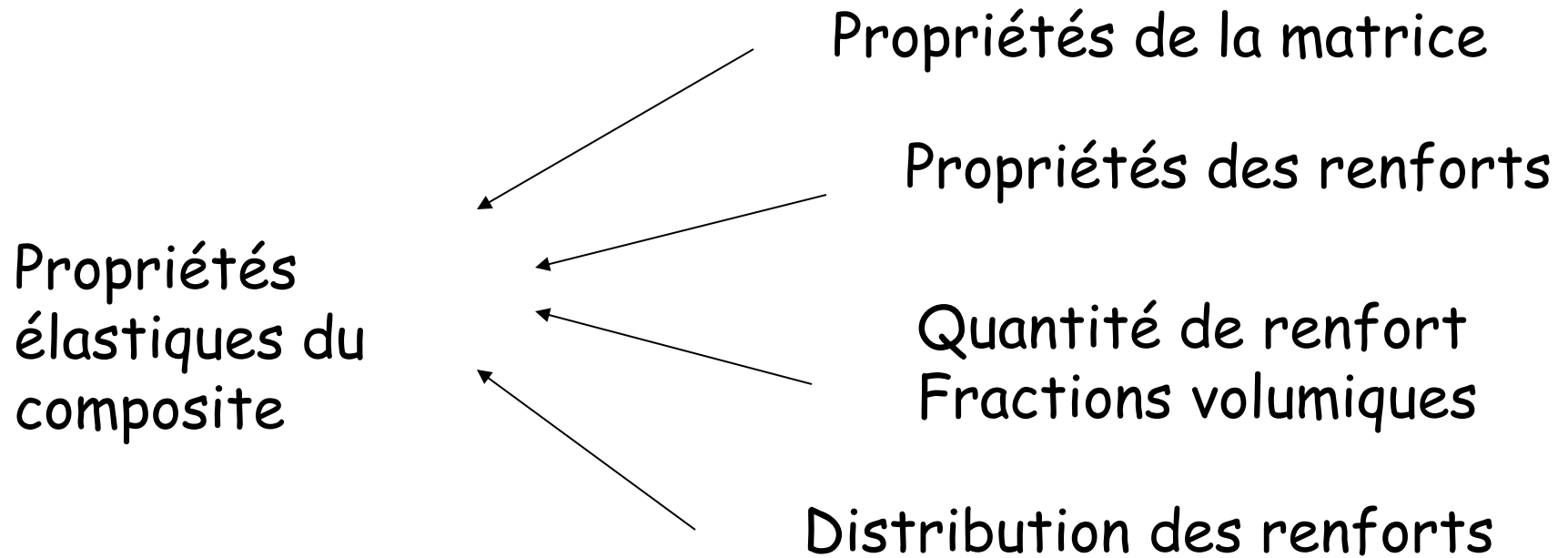
Micromécanique

- Introduction
- Lois des mélanges
- Halpin-Tsai
- Renforts discontinus



Traité des Matériaux, TMX Vol 15, chap 4 et 8

Introduction



Introduction

Polymère/ Interface/ renfort

Fractions volumiques et massiques

L'importance des fibres

Éléments de volume représentatif

Fractions volumiques et massiques

$$V_f + V_m = 1$$

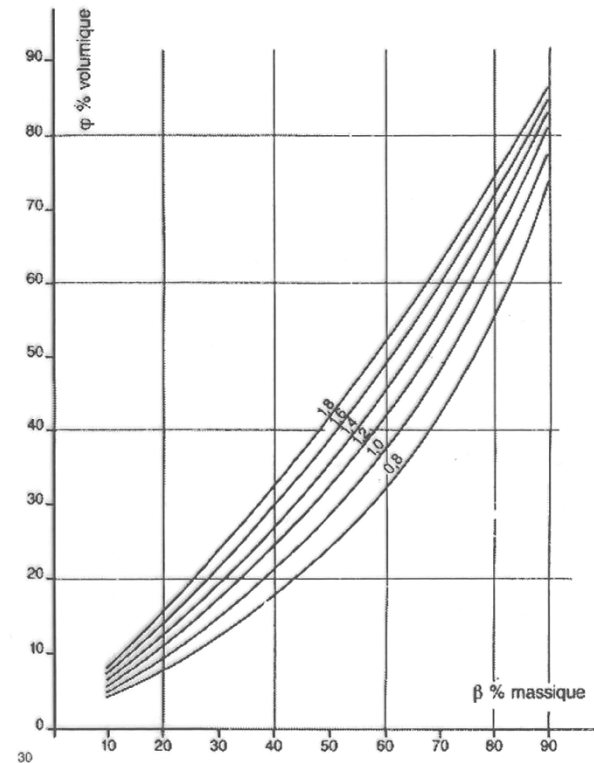
$$V_m = 1 - V_f$$

$$!Hypo : V_{porosité} \cong 0$$

$$\rho = \frac{M}{V} \quad \rho_c = \rho_f V_f + \rho_m V_m$$

$$M_f = \frac{\rho_f V_f}{\rho_f V_f + \rho_m V_m}$$

Correspondance % massique % volumique
en fonction de la densité de la résine



Importance des fibres

$$\frac{P_f}{P_m} \approx \frac{\sigma_f V_f}{\sigma_m V_m} \approx \frac{E_f V_f}{E_m V_m}$$

$$\frac{P_f}{P_c} \approx \frac{E_f V_f}{E_f V_f + E_m V_m}$$

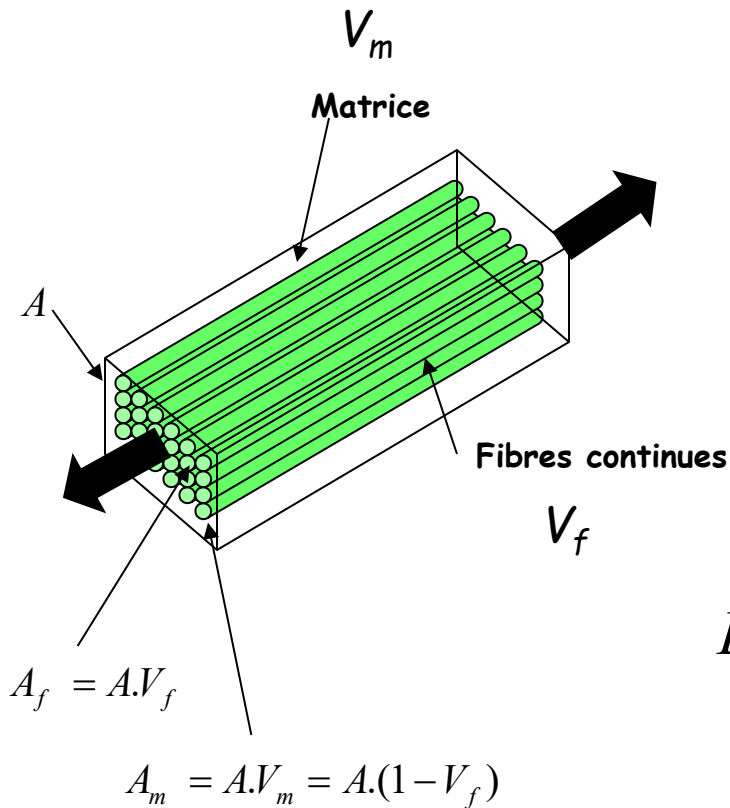
Module fibres de verre: 72 GPa
Module matrice epoxy : 3.6 GPa

V_f	0.1	0.5
$\frac{P_f}{P_c}$	?	?

Module fibres de carbone: 430 GPa
Module matrice epoxy : 3.6 GPa

V_f	0.1	0.5
$\frac{P_f}{P_c}$	?	?

Fibres continues: propriétés longitudinales

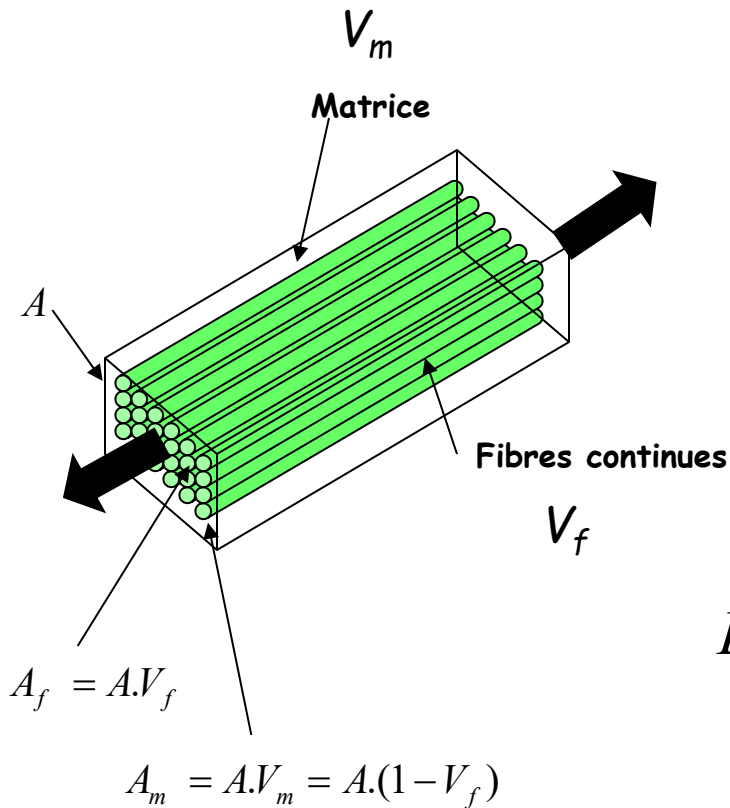


$$F = F_f + F_m$$

avec $\varepsilon_f = \varepsilon_m = \varepsilon_{cL}$

$$E_{cLongitudinal} = E_1 = E_f V_f + E_m (1 - V_f)$$

Fibres continues: propriétés longitudinales



Hypo1: interfaces parfaites

$$F = F_f + F_m$$

$$\sigma.A = \sigma_f.A.V_f + \sigma_m.A.V_m$$

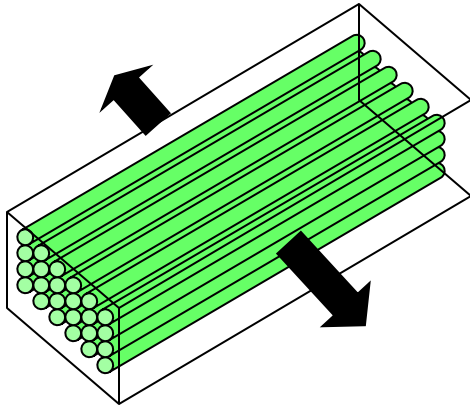
$$\text{avec } \sigma = E.\varepsilon \quad \text{Hypo2: mx élastiques linéaires}$$

$$E_c \varepsilon_c = E_f V_f \varepsilon_f + E_m (1 - V_f) \varepsilon_m$$

$$\text{avec } \varepsilon_f = \varepsilon_m = \varepsilon_{cL} \quad \text{Hypo3: isodéformation}$$

$$E_{cLongitudinal} = E_1 = E_f V_f + E_m (1 - V_f)$$

Fibres continues: propriétés transversales



$$a = a_f + a_m$$

avec $\sigma_f = \sigma_m = \sigma_{cT}$

$$E_{cT} = E_2 = \frac{E_m E_f}{E_f(1 - V_f) + E_m V_f}$$

Fibres continues: propriétés transversales

Hypo1: interfaces parfaites

$$a = a_f + a_m$$

$$\varepsilon_c = \varepsilon_f V_f + \varepsilon_m (1 - V_f)$$

$$\text{avec } \varepsilon = \frac{\sigma}{E}$$

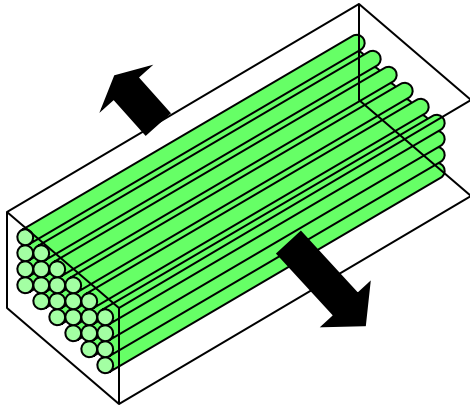
Hypo2: mx élastiques linéaires

$$\frac{\sigma_c}{E_c} = \frac{\sigma_f}{E_f} V_f + \frac{\sigma_m}{E_m} (1 - V_f)$$

$$\text{avec } \sigma_f = \sigma_m = \sigma_{cT}$$

Hypo3: isocontraintes

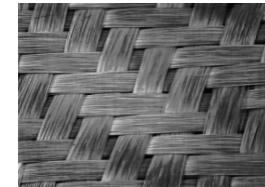
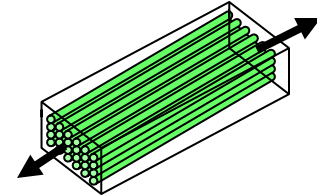
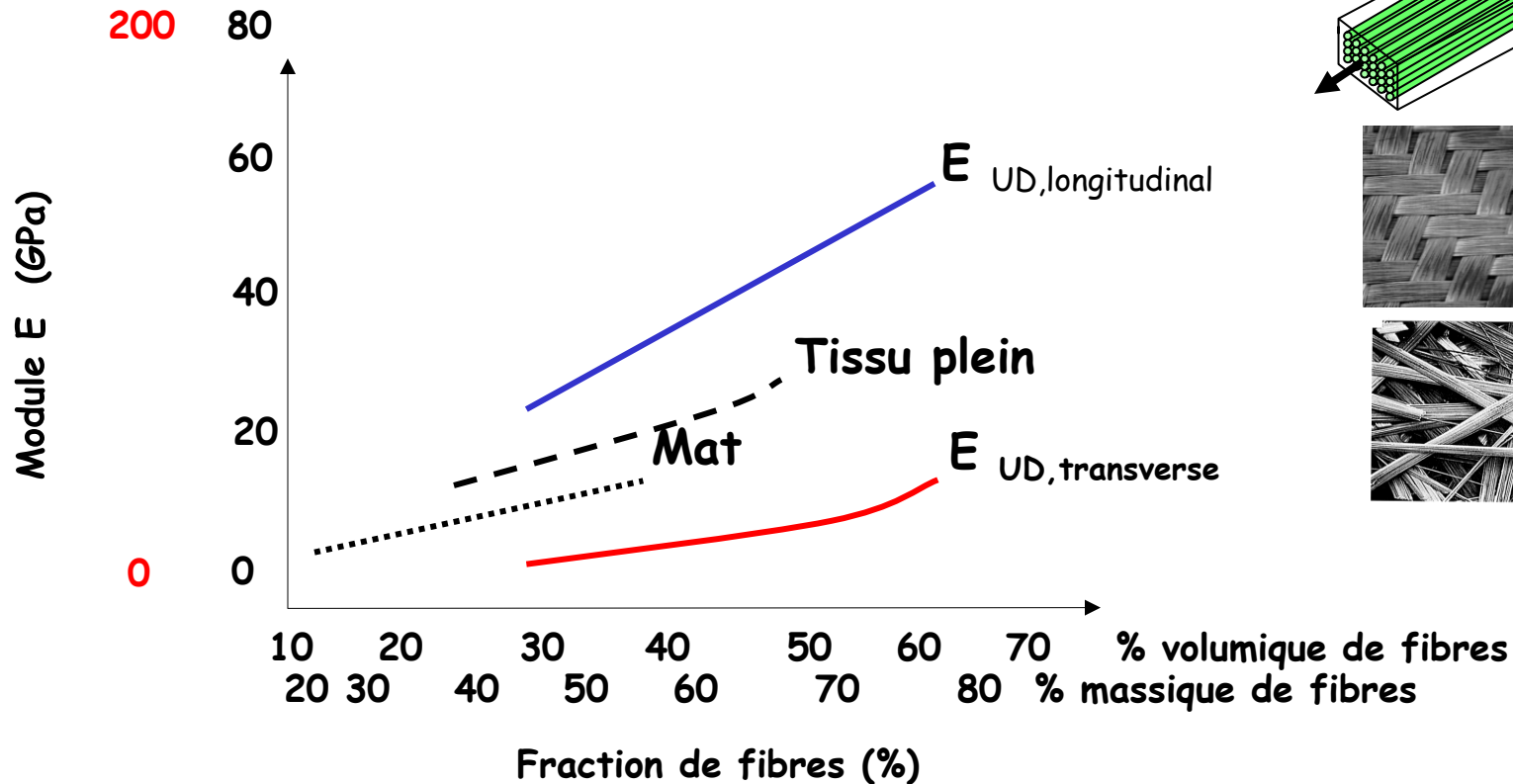
$$E_{cT} = E_2 = \frac{E_m E_f}{E_f (1 - V_f) + E_m V_f}$$



Rigidités à volonté

Fibres de carbone

Fibres de verre



empilement hexagonal



$$A_f = 3 \cdot \frac{\pi r^2}{6} = \frac{\pi r^2}{2}$$

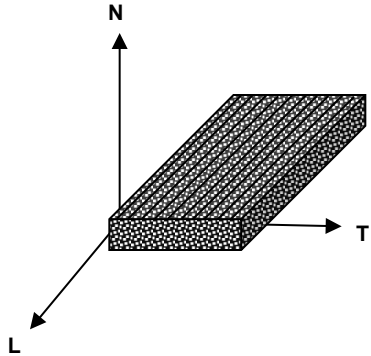


$$A = r \cdot 2r \sin \frac{\pi}{6} = r^2 \sqrt{3}$$

$$\sqrt{f_{\max hwa}} = \frac{A_f}{A} = \frac{\pi r^2}{2 r^2 \sqrt{3}} = \frac{\pi}{2\sqrt{3}} = 0,9$$

$$\sqrt{f_{\text{cubique}}} = \frac{\pi r^2}{4 r^2} = 0,78$$

Propriétés des composites unidirectionnels



	Verre S Epoxy	Kevlar 49 Epoxy	Carbone HR Epoxy	Carbone HM Epoxy	Bore Epoxy
vf	65 %	65 %	65 %	65 %	55 %
Densité ρ	2.04 g/cm ³	1.36 g/cm ³	1.56 g/cm ³	1.5 g/cm ³	1.97 g/cm ³
E_L	56 GPa	86 GPa	145 GPa	270 GPa	220 GPa
E_T	16 GPa	5.6 GPa	10 GPa	7 GPa	2.3 GPa
ν_{LT}	0.26	0.32	0.29	0.3	0.26
G_{LT}	7 GPa	2.5 GPa	5.5 GPa	5.7 GPa	6.9 GPa
σ_{rL} traction	1.75 GPa	1.5 GPa	1.2 GPa	0.95 GPa	1.3 GPa
σ_{rT} traction	0.04 GPa	0.03 GPa	0.08 GPa	0.035 GPa	0.065 GPa
σ_{rL} compression	0.9 GPa	0.28 GPa	1 GPa	0.75 GPa	2.85 GPa
σ_{rT} compression	0.15 GPa	0.14 GPa	0.25 GPa	0.2 GPa	0.03 GPa
γ_{rLT} cisaillement	0.06 GPa	0.05 GPa	0.1 GPa	0.055 GPa	0.06 GPa

Equations de Halpin-Tsai

$$P = \frac{P_m (1 + \xi \chi V_f)}{1 - \chi V_f}$$

$$\chi = \frac{P_f - P_m}{P_f + \xi P_m}$$

où P est la propriété étudiée (E_T ou G_{LT}) et P_f et P_m sont respectivement les propriétés des fibres et de la matrice. Le paramètre ξ est déterminé empiriquement et définit l'efficacité du renforcement, en général $\xi(E_T) = 2$ et $\xi(G_{LT}) = 1$.

Coefficients d'expansion thermique

Fibres anisotropes

Résistance longitudinale lors de rupture de fibres continues alignées

σ_{c1}^R

$\varepsilon_f^R < \varepsilon_m^R$: TP, MMC



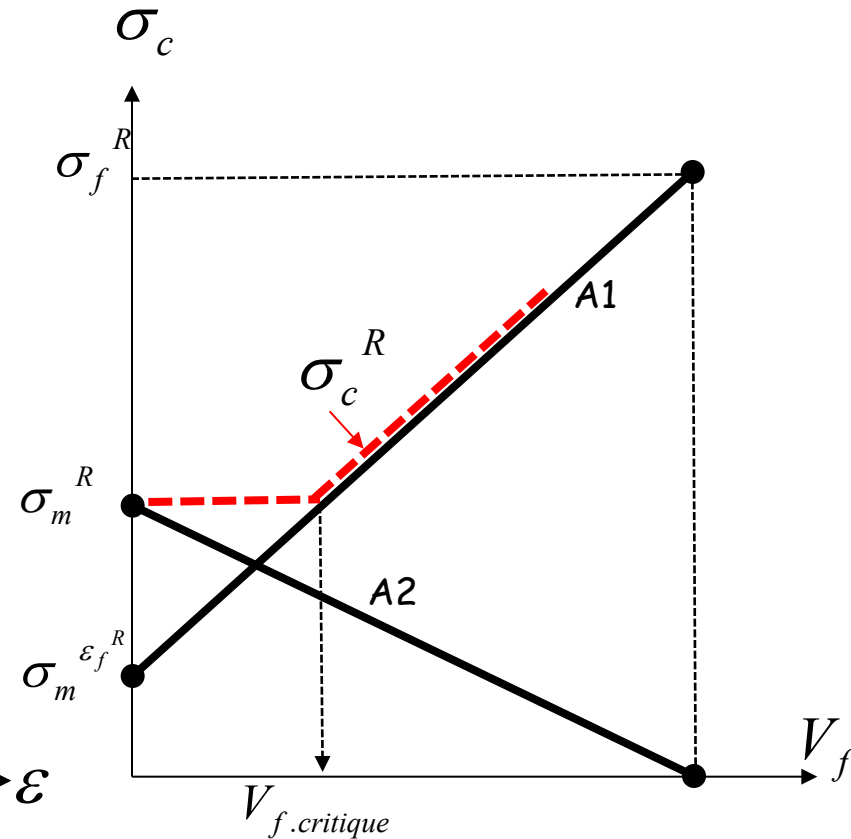
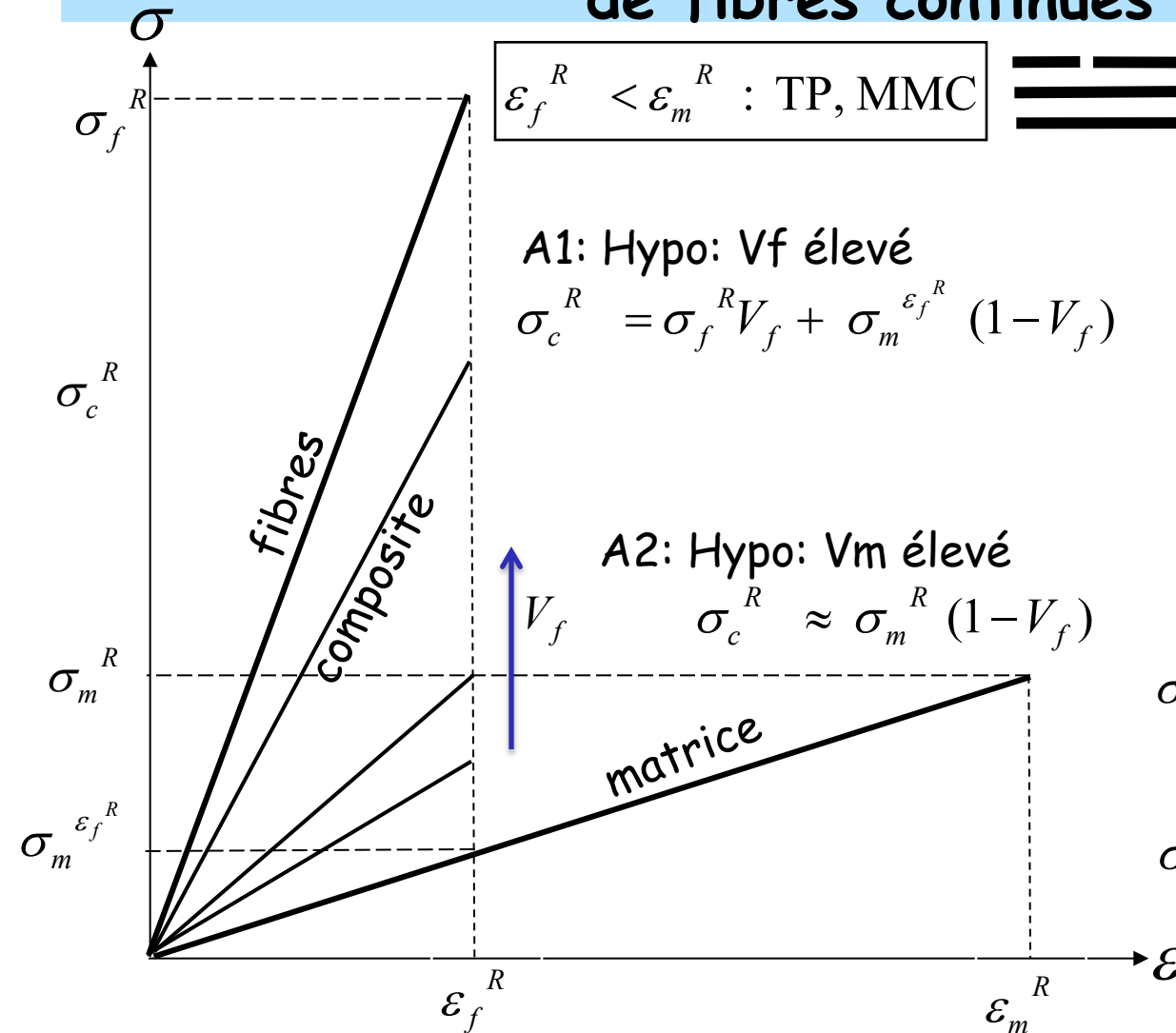
$$\sigma_c = \sigma_f V_f + \sigma_m (1 - V_f)$$

A1: Hypo: V_f élevé

$$\sigma_c^R = \sigma_f^R V_f + \sigma_m^{\varepsilon_f^R} (1 - V_f)$$

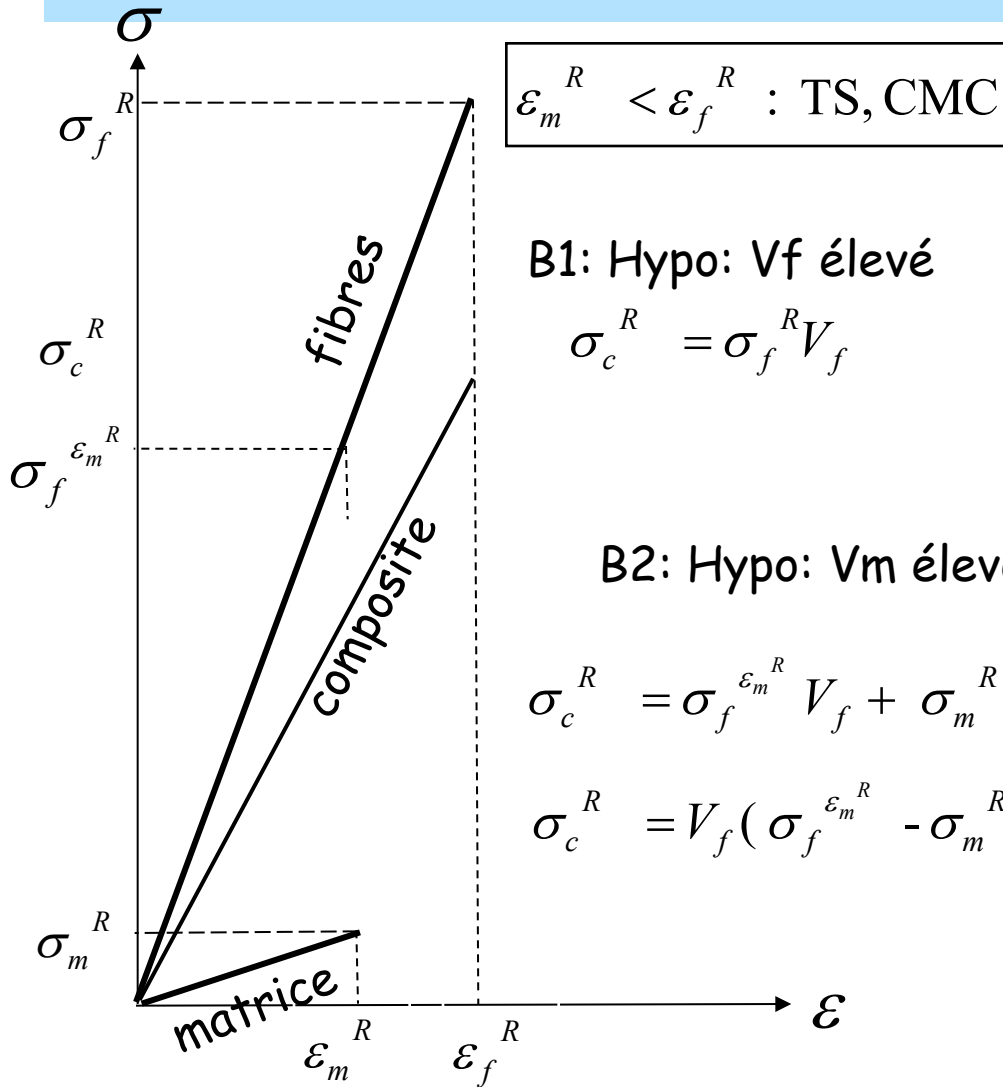
A2: Hypo: V_m élevé

$$\sigma_c^R \approx \sigma_m^R (1 - V_f)$$



$$V_{f.critique} = \frac{\sigma_m^R - \sigma_m^{\varepsilon_f^R}}{\sigma_f^R - \sigma_m^{\varepsilon_f^R}}$$

Résistance longitudinale lors de rupture de la matrice



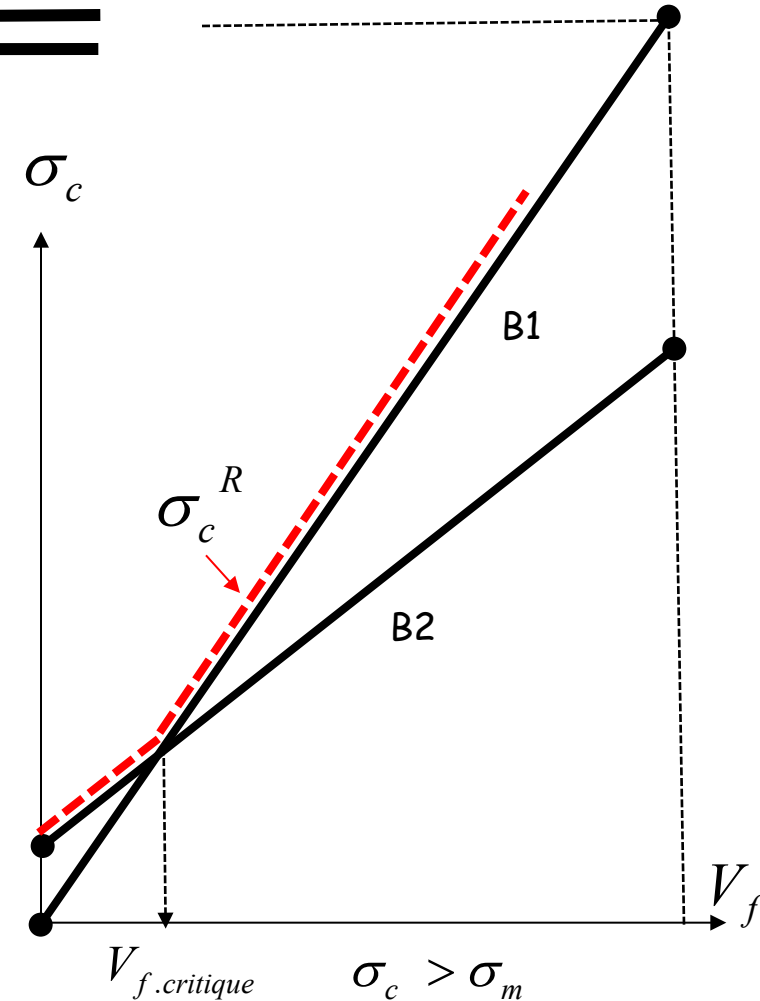
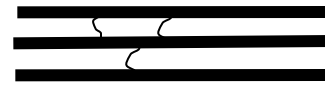
B1: Hypo: V_f élevé

$$\sigma_c^R = \sigma_f^R V_f$$

B2: Hypo: V_m élevé

$$\sigma_c^R = \sigma_f^{\varepsilon_m^R} V_f + \sigma_m^R (1 - V_f)$$

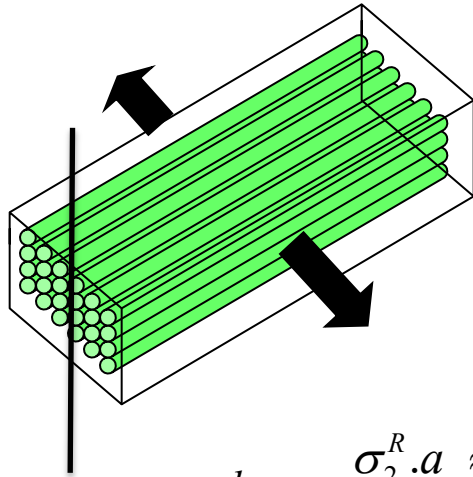
$$\sigma_c^R = V_f (\sigma_f^{\varepsilon_m^R} - \sigma_m^R) + \sigma_m^R$$



$$V_{f.critique} = \frac{\sigma_m^R}{(\sigma_f^R - \sigma_f^{\varepsilon_m^R}) + \sigma_m^R}$$

Résistance transversale : Cooper-Kelly

$$\sigma_{c2}^R$$



$$\sigma_2^R \cdot a \approx \sigma_m \cdot (a - d_f)$$

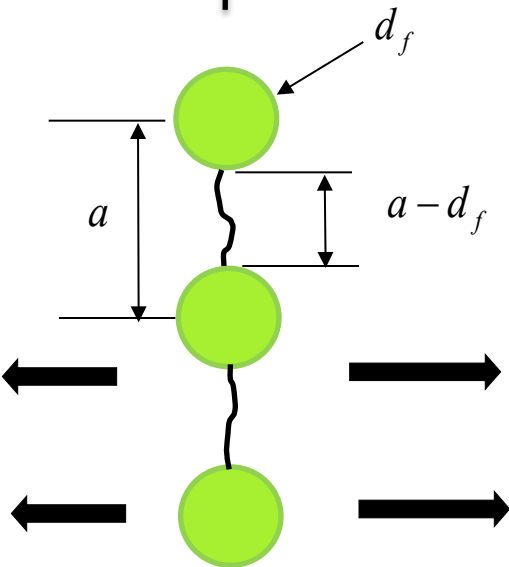
Hypo1: résistance plus faible des interfaces

$$\frac{\sigma_2^R}{\sigma_m} \approx \frac{(a - d_f)}{a} = 1 - \frac{d_f}{a}$$

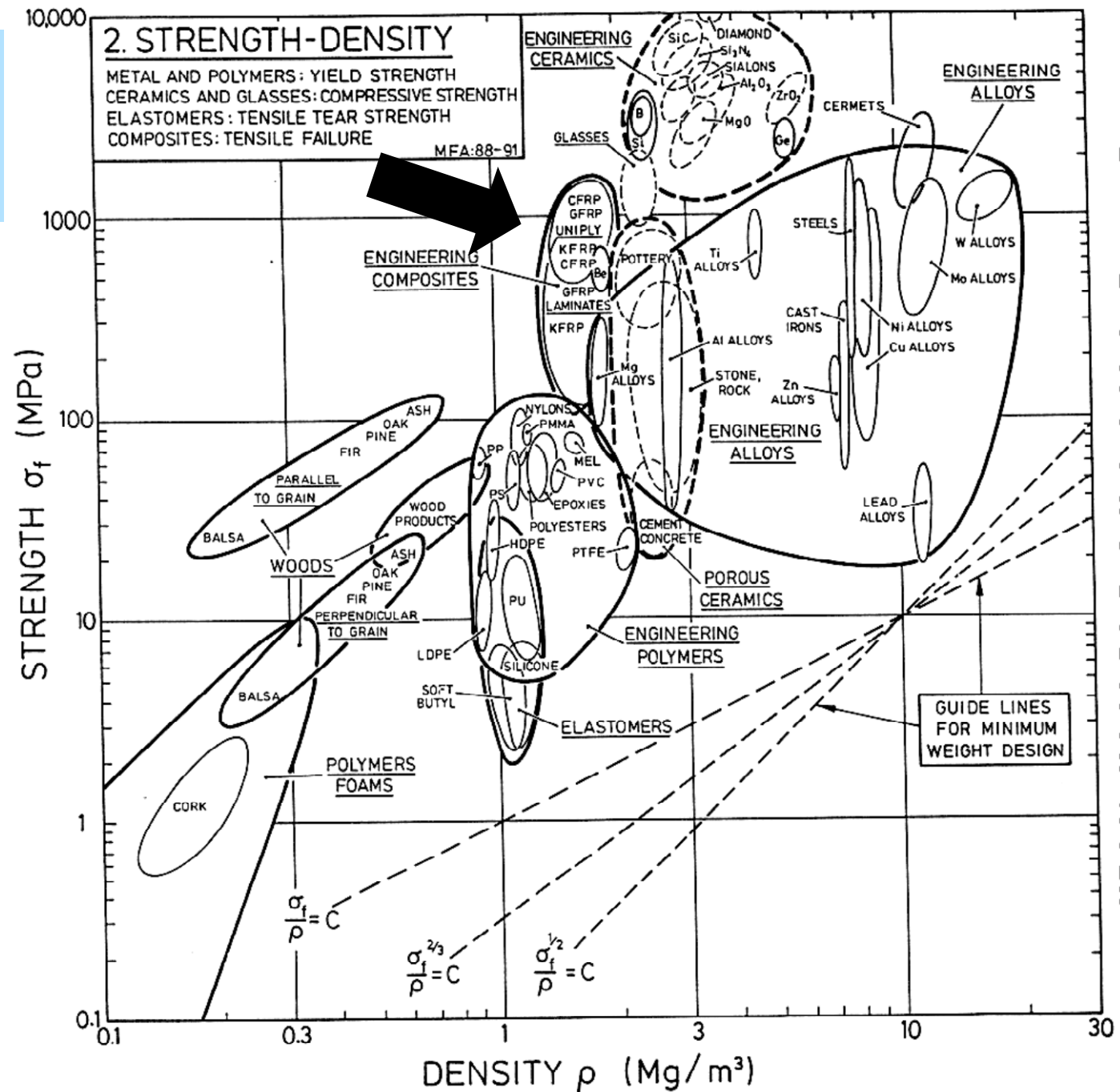
Hypo2: arrangement cubique $V_f = \frac{\pi \cdot d_f^2}{4a^2}$ donc $\frac{d_f}{a} = \sqrt{\frac{4V_f}{\pi}}$

$$\sigma_2^R = \sigma_m \cdot \left(1 - \sqrt{\frac{4V_f}{\pi}}\right)$$

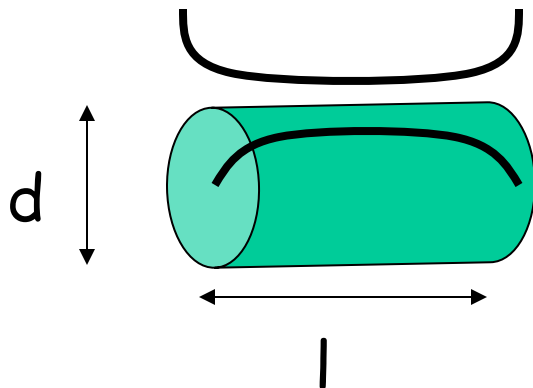
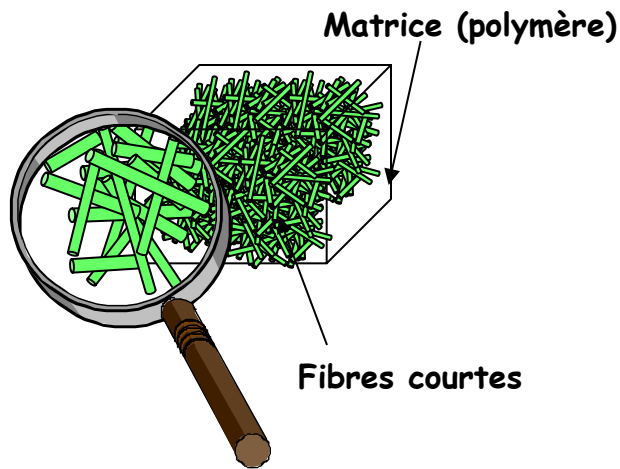
si une résistance σ_i de l'interface existe : $\sigma_2^R \approx \sigma_m \cdot \left(1 - \sqrt{\frac{4V_f}{\pi}}\right) + \sigma_i \cdot \sqrt{\frac{4V_f}{\pi}}$



Résistance spécifique



Composites à fibres discontinues



Facteur de formes l/d

Fraction volumique V_f

Qualité des interfaces

Distribution statistique de l'orientation des fibres

Distribution de la longueur des fibres

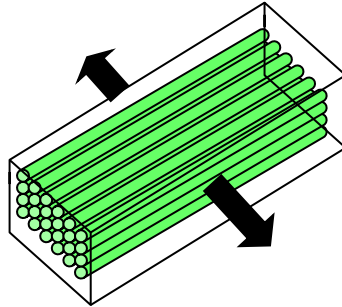
Halpin Tsai

$$E = \eta_{\text{orientation}} \eta_{\text{longueur fibre}} E_f V_f + E_m (1 - V_f)$$

Equations de Halpin-Tsai

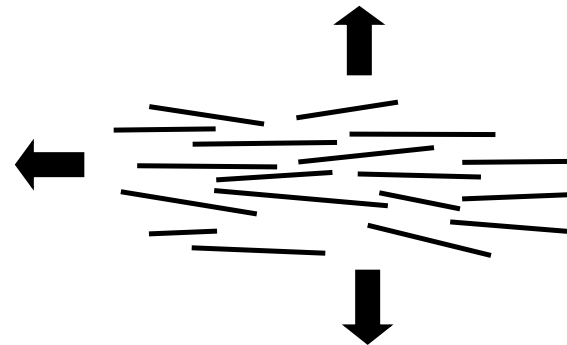
$$P = \frac{P_m (1 + \xi \chi V_f)}{1 - \chi V_f}$$

$$\chi = \frac{P_f - P_m}{P_f + \xi P_m}$$



$$\xi(E_T) = 2$$

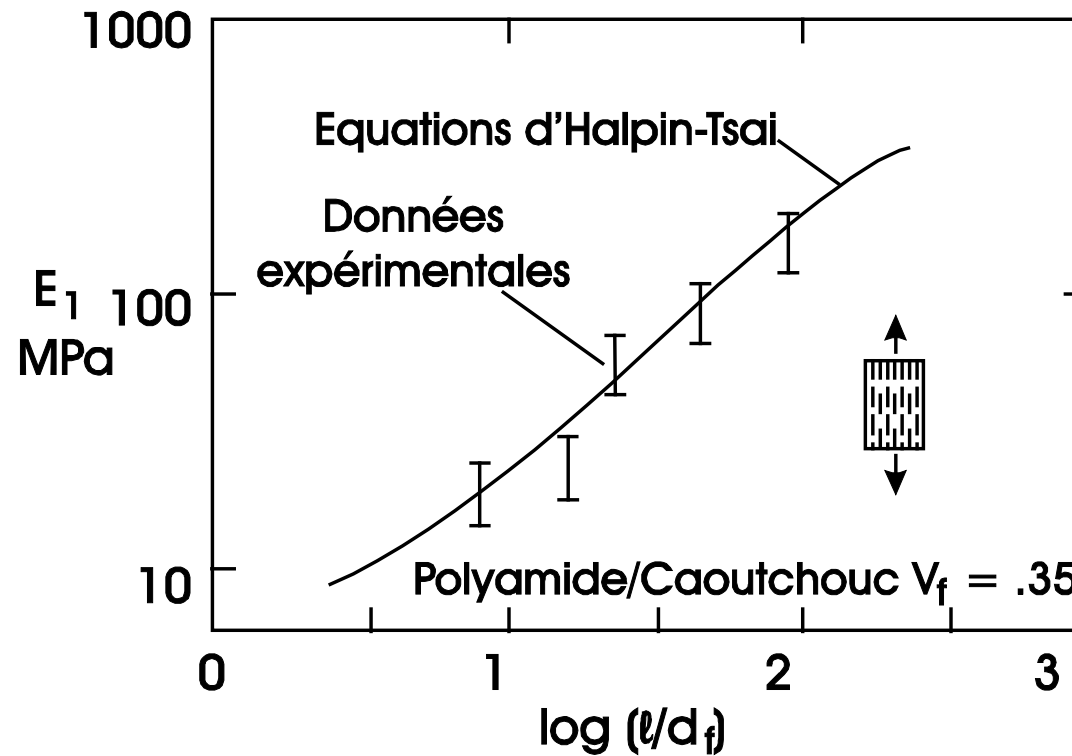
$$\xi(G_{LT}) = 1$$



$$\xi = 2 + 40 V_f^{10}$$

$$\xi = \frac{21}{d} + 40 V_f^{10}$$

Composites à fibres courtes



$$\frac{\pi d_f^2}{4} [(\sigma + d\sigma) - \sigma] - \pi d_f \tau dx_1 = 0$$

$$\frac{d\sigma}{dx_1} = \frac{4\tau}{d_f} \quad \sigma(x_1) = \frac{4}{d_f} \int_0^{x_1} \tau dx_1$$

$\tau = c(u_f - u_m)$

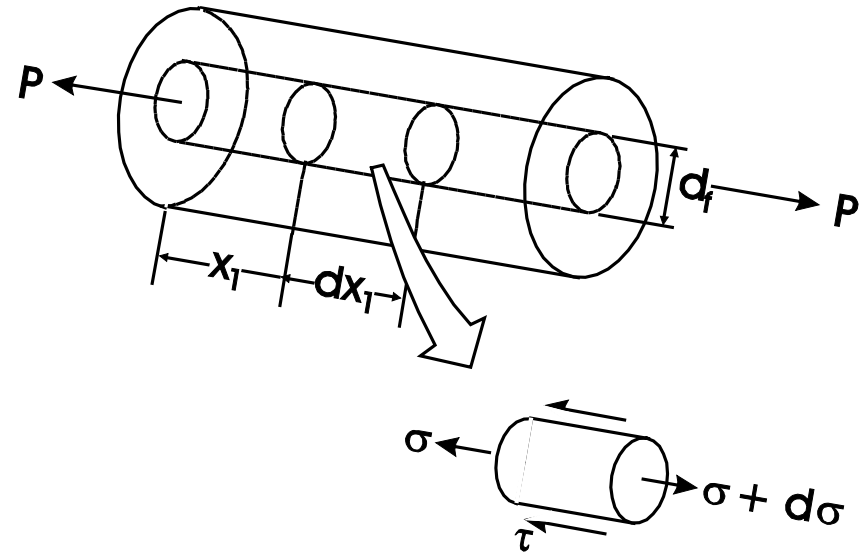
$$\frac{d\sigma}{dx_1} = \frac{4c(u_f - u_m)}{d_f}$$

$$\frac{d^2\sigma}{dx_1^2} = \frac{4c}{d_f} \left(\frac{du_f}{dx_1} - \frac{du_m}{dx_1} \right)$$

$\frac{du_f}{dx_1} = \frac{\sigma}{E_f}$
 $\frac{du_m}{dx_1} = \epsilon_m$

$$\frac{d^2\sigma}{dx_1^2} - \beta^2 \sigma = \frac{-4c}{d_f} \epsilon_m \quad \text{avec} \quad \beta^2 = \frac{4c}{d_f E_f}$$

$$\sigma(x_1) = E_f \epsilon_m \left[1 - \frac{\cosh \beta (\ell/2 - x_1)}{\cosh (\ell \beta/2)} \right]$$



Composites à fibres courtes alignées

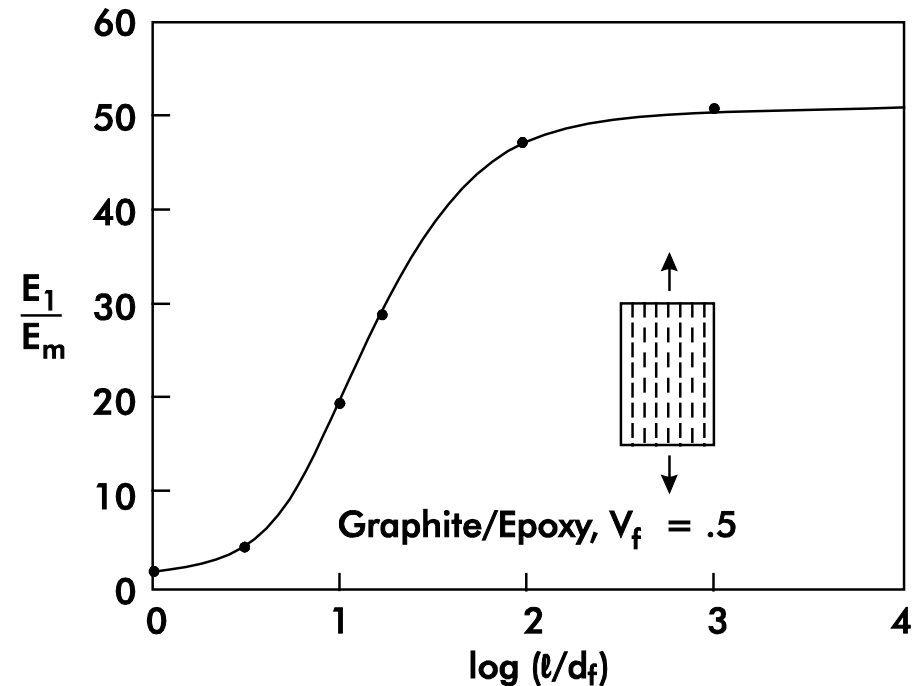
$$\sigma_f = E_f \varepsilon_m \left[1 - \frac{\tanh(\beta \ell/2)}{\beta \ell/2} \right]$$

$$\sigma_c = \sigma_f V_f + \sigma_m V_m$$

$$\sigma_c = E_f \varepsilon_m \left[1 - \frac{\tanh(\beta \ell/2)}{\beta \ell/2} \right] V_f + \sigma_m V_m$$

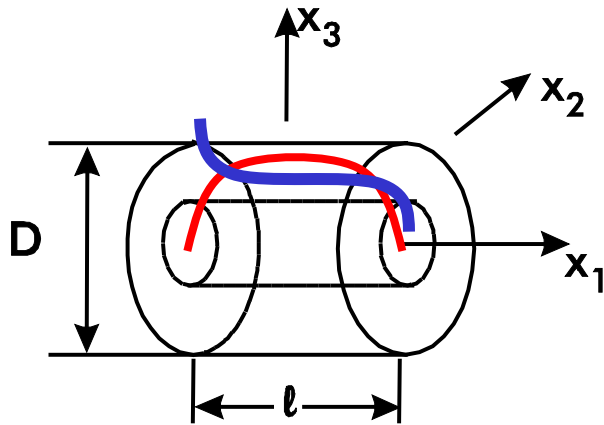
$$E_I = E_f V_f \left[1 - \frac{\tanh(\beta \ell/2)}{\beta \ell/2} \right] + E_m V_m$$

$$E = \eta_{\text{orientation}} \eta_{\text{longueur fibre}} E_f V_f + E_m (1 - V_f)$$



$\ell/d_f > 100$: loi des mélanges

Composites à fibres courtes alignées



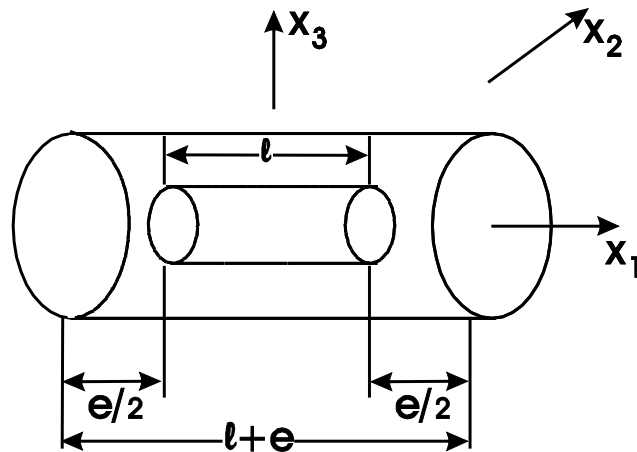
Cox

$$\tau = c(u_f - u_m)$$

$$E_I = E_f V_f \left[1 - \frac{\tanh(\beta \ell/2)}{\beta \ell/2} \right] + E_m V_m$$

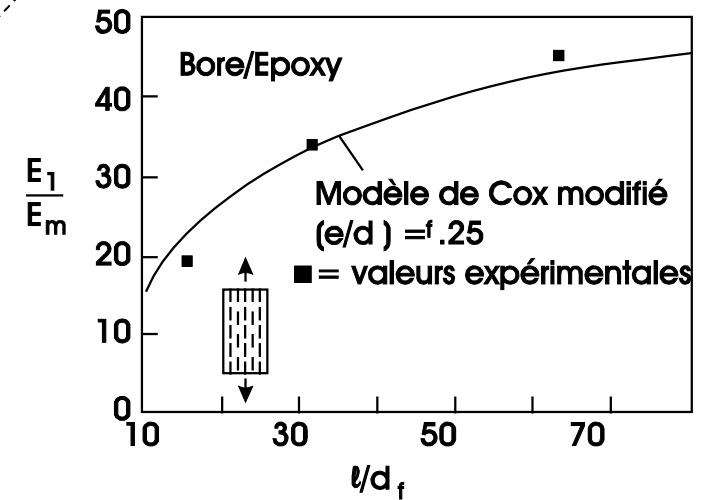
Cox-Kelly

$$\tau = \frac{d_f E_f \varepsilon_m \beta}{4} \left[\frac{\sinh(\beta(\ell/2 - x_1))}{\cosh(\beta \ell/2)} \right]$$



Hwang-Gibson

$$\frac{1}{E_c} = \frac{\ell / (\ell + e)}{E_I} + \frac{e / (\ell + e)}{E_m}$$



Composites à fibres courtes alignées

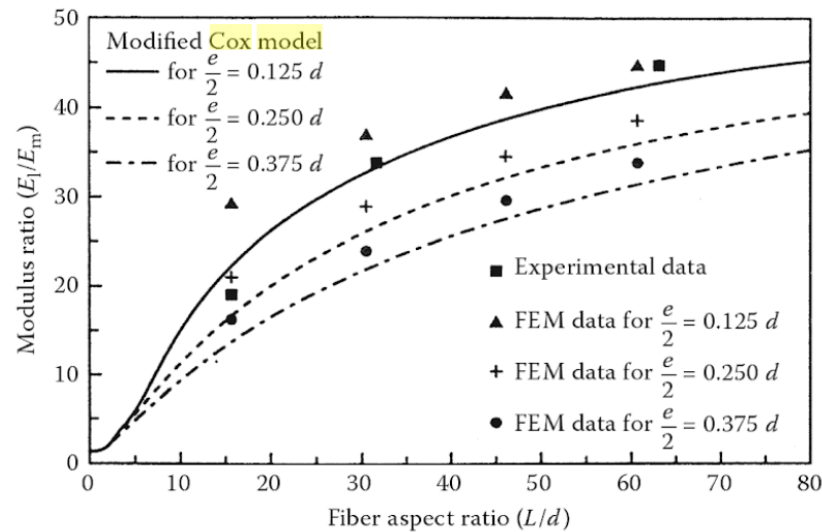
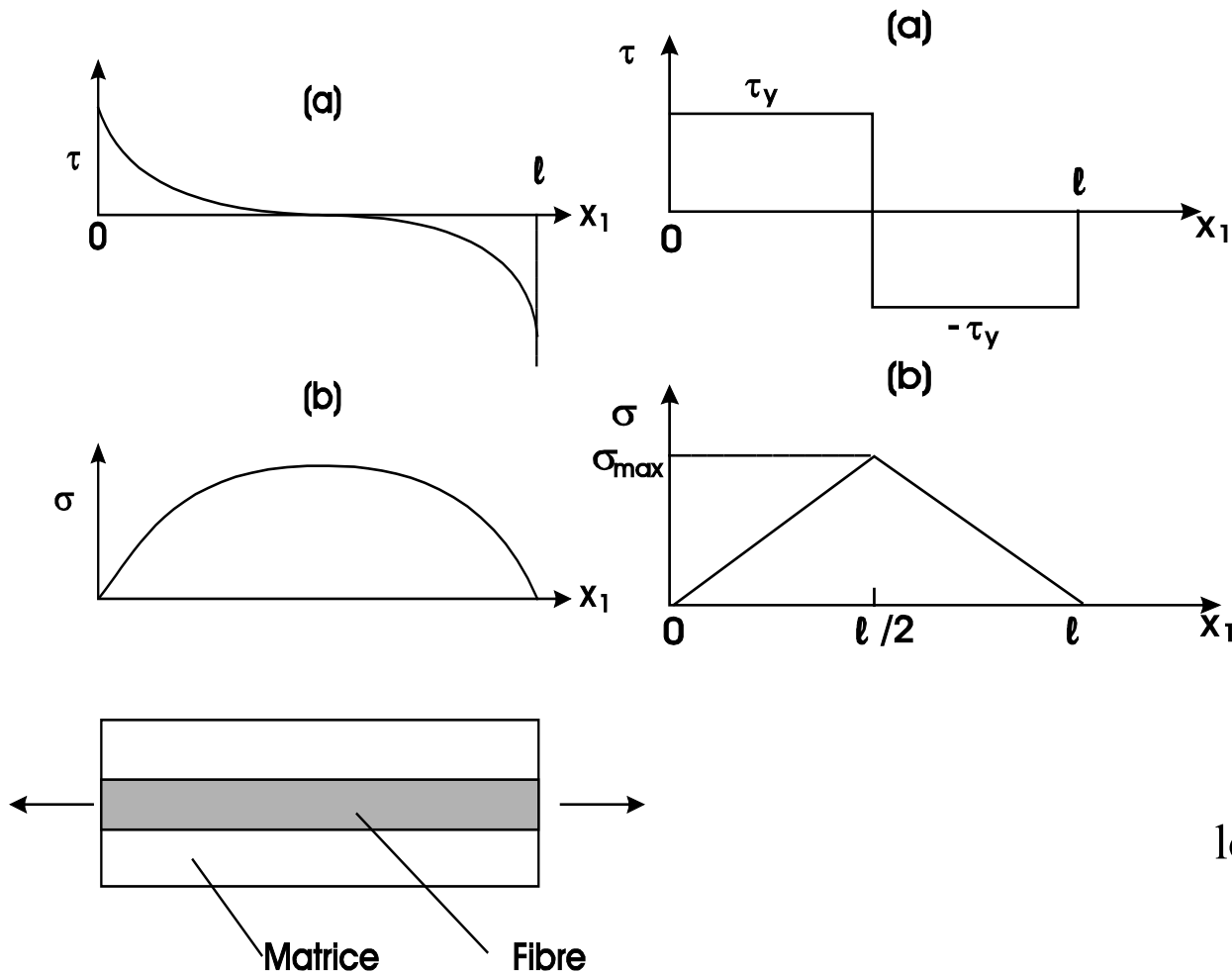


FIGURE 6.17

Comparison of predictions from the modified Cox model and FEA with experimental data for boron/epoxy-aligned discontinuous fiber composite at different fiber aspect ratios. (From Hwang, S. J. and Gibson, R. F. 1987. *Journal of Engineering Materials and Technology*, 109, 47–52. Reprinted with permission from ASME.)

Modèle de Kelly-Tyson



$$\tau = \tau_y = \text{cste}$$

$$\sigma = \frac{4}{d_f} \tau_y x_1$$

$$\sigma_{\max} = \frac{2 \tau_y \ell}{d_f}$$

La contrainte max dans les fibres augmente avec la longueur des fibres

$$\text{longueur critique} = \ell_c = \frac{d_f \sigma_{f, \text{ult}}}{2 \tau_y}$$

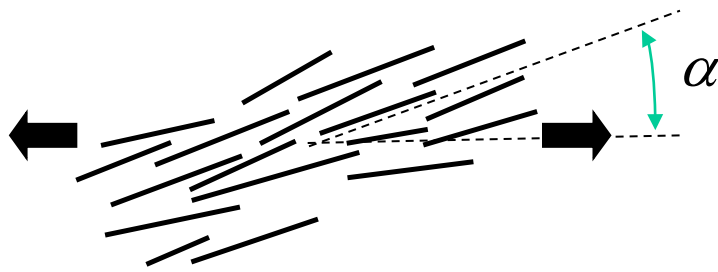
! Uniquement pour composites à fibres courtes

Composites à fibres courtes orientées

$$E = \eta_{\text{orientation}} \eta_{\text{longueur fibre}} E_f V_f + E_m (1 - V_f)$$

$p. =$

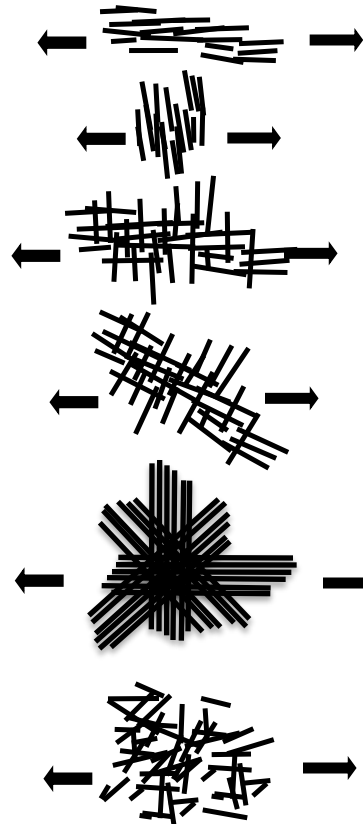
$\eta_o =$



$$\eta_{\text{orientation}} \approx \cos^4 \alpha$$

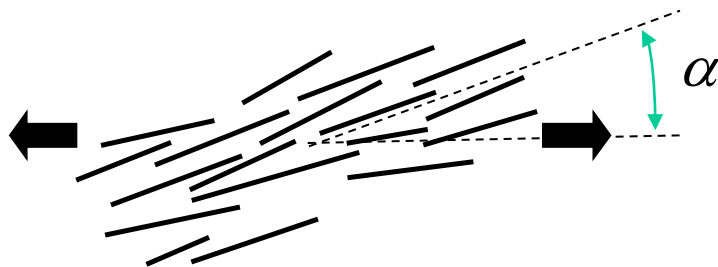
$$\eta_{\text{orientation}} \approx \sum_n p_n \cos^4 \alpha_n$$

$$\sum_n p_n = 1$$



Composites à fibres courtes orientées

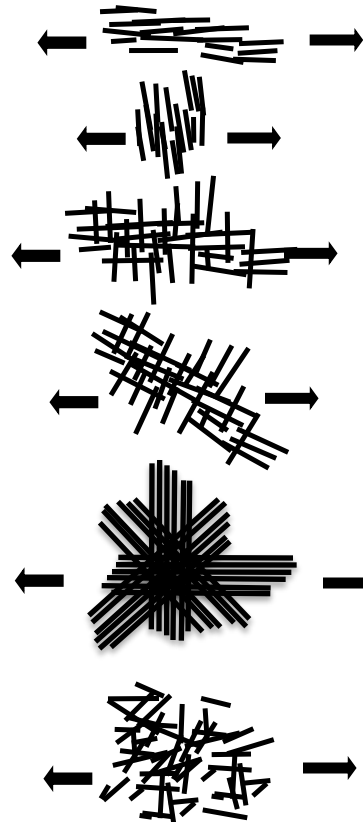
$$E = \eta_{\text{orientation}} \eta_{\text{longueur fibre}} E_f V_f + E_m (1 - V_f)$$



$$\eta_{\text{orientation}} \approx \cos^4 \alpha$$

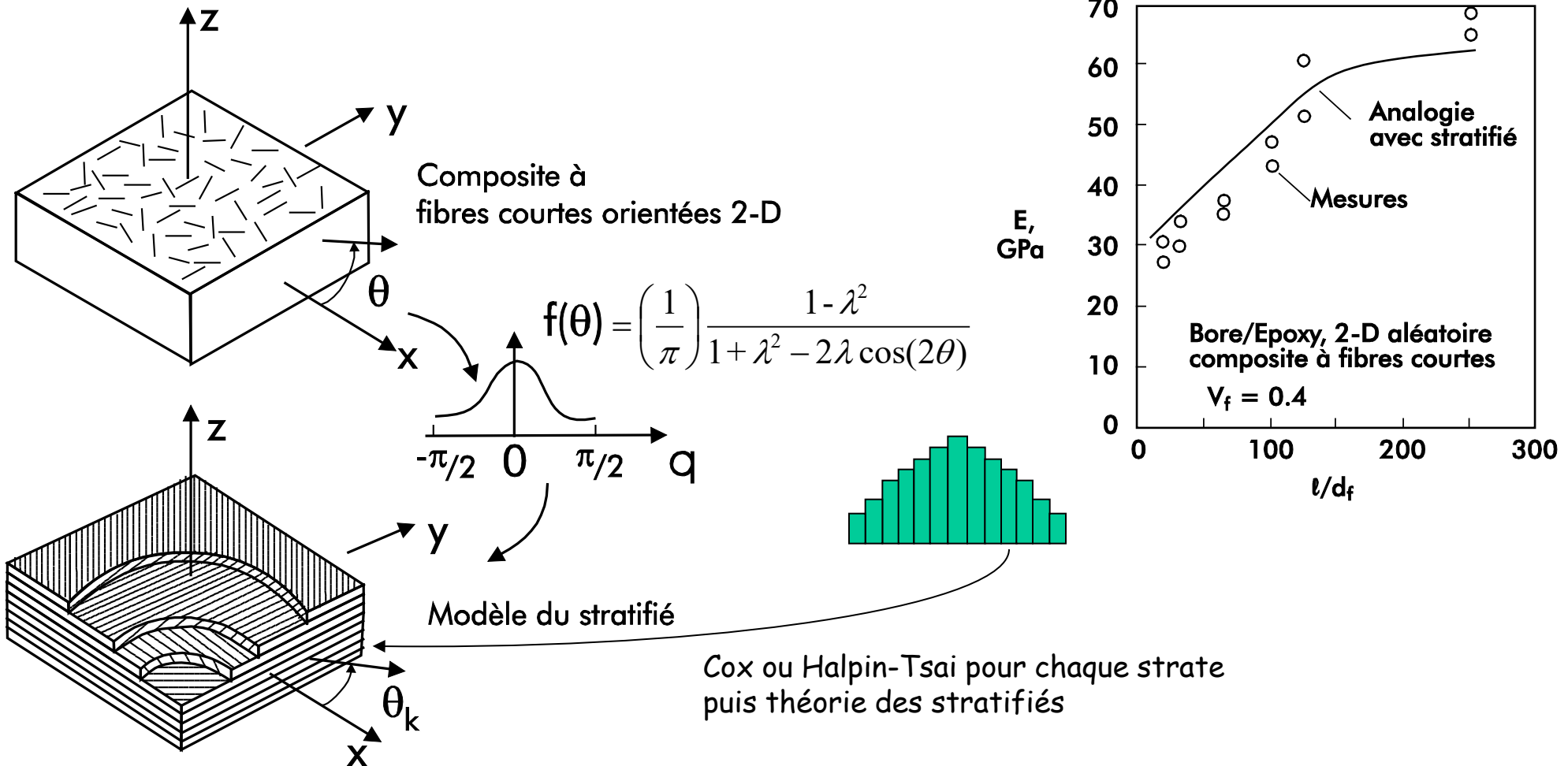
$$\eta_{\text{orientation}} \approx \sum_n p_n \cos^4 \alpha_n$$

$$\sum_n p_n = 1$$



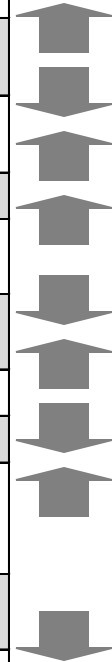
$p_n =$	$\eta_o =$
$p_n = 1$	$\eta_o = 1 \cos^4 0 = 1$
$p_n = 1$	$\eta_o = 1 \cos^4 90 = 0$
$p_1 = p_2 = \frac{1}{2}$	$\eta_o = \frac{1}{2} \cos^4 0 + \frac{1}{2} \cos^4 90 = \frac{1}{2}$
$p_1 = p_2 = \frac{1}{2}$	$\eta_o = \frac{1}{2} \cos^4 45 + \frac{1}{2} \cos^4 (-45) = \frac{1}{4}$
$p_1 = p_2 = p_3 = p_4 = \frac{1}{4}$	$\eta_o = \frac{1}{4} (\cos^4 0 + \cos^4 90 + \cos^4 45 + \cos^4 (-45)) = \frac{3}{8}$
Aléatoire 3dim	$\eta_o = \frac{1}{5}$
Aléatoire 2dim	$E = \frac{3}{8} E_0 + \frac{5}{8} E_{90}$

Composites à fibres courtes orientées



Propriétés des polymères chargés de fibres courtes

Propriété à 23°C	Zytel®		Zytel® 40% fibres courtes		Zytel® 50% fibres courtes	
	0 % HR	50% HR	0 % HR	50% HR	0 % HR	50% HR
Contrainte au seuil plastique σ_y (MPa)	84 MPa	48 MPa	205 MPa	135 MPa	230 MPa	155 MPa
Allongement à la rupture ε_y (%)	50 %	>300 %	3 %	6 %	2 %	5 %
Module de flexion E	2.7 GPa	0.9 GPa	10.5 GPa	6.5 GPa	23.5 GPa	8.5 GPa
Résistance au choc entaillé Izod	50 J/m	200 J/m	160 J/m	214 J/m	180 J/m	270 J/m
Résistance au choc Charpy	Pas de rupture		60 kJ/m ²		65 kJ/m ²	
Densité ρ	1.14 g/cm ³		1.45 g/cm ³		1.58 g/cm ³	
Point de fusion	245°C		233 °C		233 °C	
Température de flexion sous charge de 1.8 MPa	65 °C		224 °C			
Absorbion d'eau en 24h (immersion)	1.6 %					
Retrait au moulage	1.3 %		0.18 %		0.16 %	



Comparaison des propriétés d'un copolymère PA66/6 (Nylon)
non chargé et chargé avec des fibres courtes

