1. Consider the coupon collector problem with \( n \) bins, and let \( k \) be the number of bins reached at time \( T_k \). Which of the following statements are correct?

a) If \( k = n - c \) with fixed \( c \geq 1 \), then \( E(T_k) = \Theta(n) \)

b) If \( k = n - c \) with fixed \( c \geq 1 \), then \( E(T_k) = \Theta(n \log n) \)

c) If \( k = \alpha n \) with \( 0 < \alpha < 1 \), then \( E(T_k) = \Theta(n) \)

d) If \( k = \alpha n \) with \( 0 < \alpha < 1 \), then \( E(T_k) = \Theta(n \log n) \)

e) If \( k = n^{1-\varepsilon} \) with \( 0 < \varepsilon < 1 \), then \( E(T_k) = \Theta(n^{1-\varepsilon}) \)

f) If \( k = n^{1-\varepsilon} \) with \( 0 < \varepsilon < 1 \), then \( E(T_k) = \Theta((1-\varepsilon)n \log n) \)
2. Which of the following sequences of numbers can be the beginning of a sequence of moments $m_0, m_1, m_2, m_3, \ldots$ of a random variable $X$?

a) 1, 1, 1, 1
b) 2, 2, 2, 2
c) 1, 2, 3, 4
d) 1, 2, 4, 8
e) 1, 1/2, 1/2, 1/2
f) +1, -1, +1, -1
3. Which of the following sequences of moments \((m_k, k \geq 0)\) determine uniquely their corresponding distribution?

a) \(m_0 = 1, \ m_1 = \mu \in \mathbb{R}, \ m_2 = \sigma^2 > 0, \ m_k = +\infty \text{ for } k \geq 3\)

b) \(m_k = \frac{1}{2} \left( \frac{10^k}{1} + (-1)^k \cdot 10^k \right), \ k \geq 0\)

c) \(m_k = k!, \ k \geq 0\)

d) \(m_k = \exp \left( k^2/200 \right), \ k \geq 0\)

e) \(m_k = (-1)^k \cdot C^k, \ k \geq 0 \text{ for some } C > 0\)

f) \(m_k = \frac{1}{k+1}, \ k \geq 0\)

(For each sequence, can you also guess which is (are) the corresponding distribution(s)?)
4. Recall the lemma used in Hoeffding's inequality:

If a r.v. \( Z \) is such that \( \mathbb{E}(Z) = 0 \) and
\[
|Z(\omega)| \leq 1 \ \forall \omega \in \Omega,
\]
then \( \mathbb{E}(e^{s^2}) \leq e^{s^2/2}, \ \forall s \geq 0. \)

This is proven using:

a) Jensen's inequality and the fact that \( s \mapsto e^{s^2} \) is convex \( \forall s \geq 0 \)

b) Jensen's inequality and the fact that \( z \mapsto e^{s^2} \) is convex \( \forall s \geq 0 \)

c) the fact that \( s \mapsto e^{s^2} \) is convex \( \forall s \geq 0 \) alone

d) the fact that \( z \mapsto e^{s^2} \) is convex \( \forall s \geq 0 \) alone

e) the fact that \( e^{s^2} \leq e^{s^2} \ \forall |z| \leq 1 \) and \( s \geq 0 \)
f) the fact that \( e^{s^2} \leq \frac{1}{2}(e^{-s} + e^{+s}) \ \forall |z| \leq 1 \) and \( s \geq 0 \)