1. Which of the following functions \( g: \mathbb{R} \to \mathbb{R} \) are bounded? 

(guaranteeing therefore that \( E(g(X)) \) is well defined) 

(for any random variable \( X \))

a) \( g(x) = x \)
b) \( g(x) = \max \left( \min(x, a), b \right) \) where \( a, b \in \mathbb{R} \)
c) \( g(x) = \int_1^x \frac{1}{t} \, dt \) for \( x \geq 1 \) & \( g(x) = 0 \) for \( x < 1 \)
d) \( g(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \)
e) \( g(x) = \sum_{k \geq 1} \frac{1}{k} \, \mathbf{1}_{[k-1, k]}(x) \)
f) \( g(x) = \frac{1}{(x-1)^2} \)
2. Which of the following random variables $X$ have a finite expectation $\mathbb{E}(|X|) < +\infty$?

a) $P(\{X=n^2\}) = \frac{C}{n+1}$, $n \geq 1$, where $C = \sum_{n=1}^{\infty} \frac{1}{n+1}$

b) $P(\{X=n^3\}) = \frac{C}{n^2}$, $n \geq 1$, where $C = \sum_{n=1}^{\infty} \frac{1}{n^2}$

c) $P(\{X=n^3\}) = 2^{-n}$, $n \geq 1$

d) $X$ is continuous and $p_X(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$, $x \in \mathbb{R}$

e) $X$ is continuous and $p_X(x) = \frac{1}{\pi(1+x^2)}$, $x \in \mathbb{R}$

f) $X = 1/U$, where $U$ is a uniform random variable on $[0,1]$.
3. Let $X, Y$ be two square-integrable random variables. Which of the following statements is correct?

a) $\text{Var}(X) \leq E(X)^2$

b) If $X \geq 0$ and $Y \geq 0$, then $\text{Cov}(X, Y) \geq 0$

c) If $X, Y$ are uncorrelated, then $E(X^2Y^2) = E(X^2)E(Y^2)$

d) If $X, Y$ are independent, then $\text{Var}(XY) = \text{Var}(X) \cdot \text{Var}(Y)$

e) If $X, Y$ are centered, then $\text{Cov}(X, Y) = 0$

f) If $X \geq 0$ a.s. and $\text{Cov}(X, Y) < 0$, then $Y \leq 0$ a.s.
4. Let $X$ be a random variable and $\phi_x$ be its characteristic function. Which of the following statements are correct?

a) $t \mapsto \phi_x(t)$ is a decreasing function on $\mathbb{R}_+$.

b) If $X \geq 0$ a.s., then $\phi_x(t) \geq 0 \ \forall t \in \mathbb{R}_+$.

c) If $\int_{\mathbb{R}} |\phi_x(t)| \, dt < +\infty$, then $X$ is a continuous r.v.

d) If $\phi_x(0) = 2$, then $\mathbb{P}(\exists x \geq 0) = \frac{1}{2}$.

e) If $X$ is bounded, then $\phi_x$ is differentiable.

f) If $\phi_{x+X}(t) = \phi_x(t) \cdot \phi_x(t) \ \forall t \in \mathbb{R}$, then $X \ \perp \ \perp X$. 