Mécanique des composites

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Mécanique des composites

Micro et macromécanique

Tests mécaniques

Endommagement et rupture

CADFEM

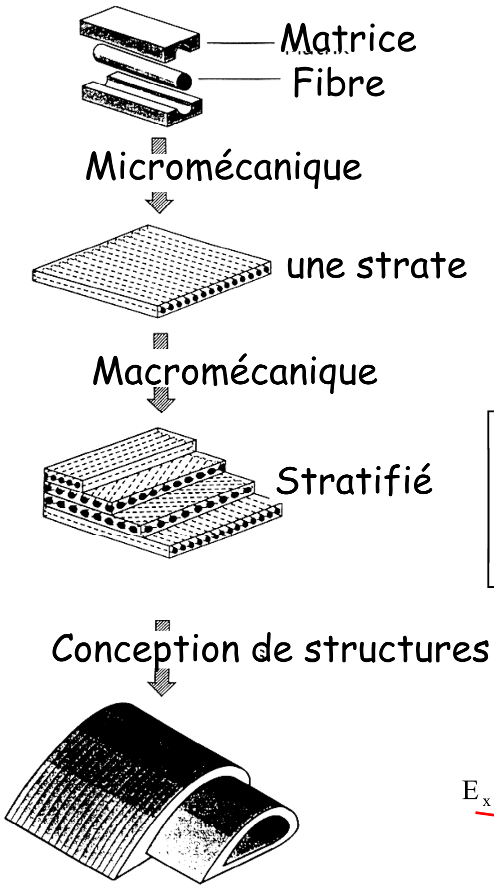
Applications

Les stratifiés

Structures sandwich

Composites textiles

De la fibre à la structure



$$E_1 = E_f V_f + E_m(1 - V_f) \qquad P = \frac{P_m(1 + \xi\,\chi V_f)}{1 - \chi V_f}$$

$$Q_{11} = \frac{E_1}{(1 - \nu_{12}\nu_{21})}$$

$$\overline{Q}_{11} = m^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + n^4 Q_{22}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \qquad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k (z_k - z_{k-1})$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \left\{ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \right\}$$

$$E_x = \frac{A_{11}A_{22} - A_{12}^2}{h\,A_{22}}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Mx élastiques
Porosité nulle/minimale
Interfaces fibre-matrice parfaites/optimales

Orientations des fibres

Orthotropie
Contraintes planes

Adhésion entre plis parfaite/optimale

Symétries d'empilement
Effets de couplage

Cas de charges
Propriétés effectives
Mécanique des matériaux

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Exemples

Orientation des fibres

Et si j'augmente le nombre de plis
tout en gardant la même épaisseur ?

Pourquoi des plis à 45 ?

Les hybrides

…

Contraintes et Rupture

Mécanique des matériaux

Contraintes principales

Critères de rupture

Rupture du premier pli

Rupture du composite

Endommagement

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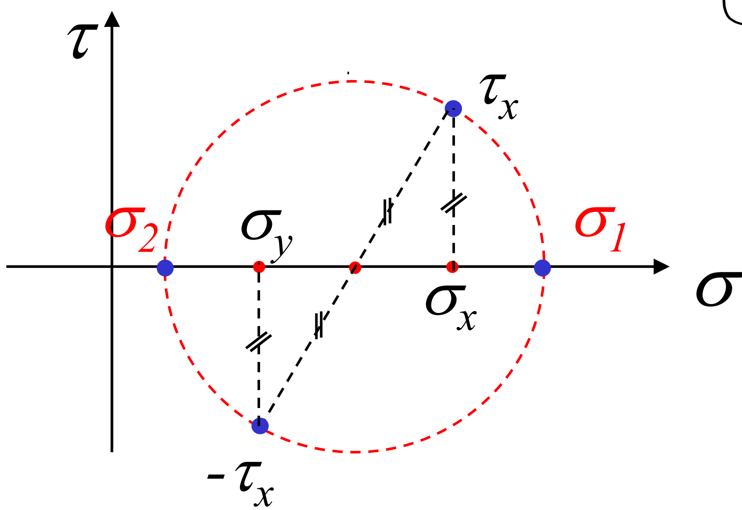
Efforts,contraintes et contraintes principales

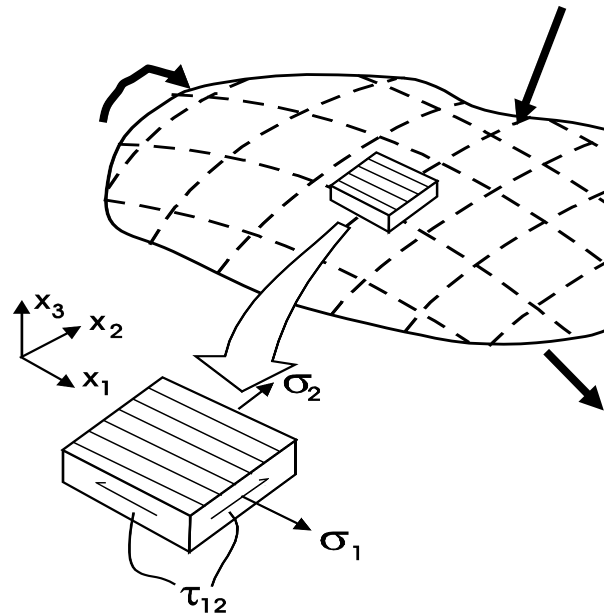
$$\sigma = \frac{N}{S}$$

$$\sigma = \frac{M}{I_y} \cdot z$$

$$\frac{1}{r_c} = \frac{M}{E \cdot I}$$

$$\tau = \frac{M_t}{I_P} \cdot r$$





Idéalement il faudrait orienter les fibres selon les isostatiques.

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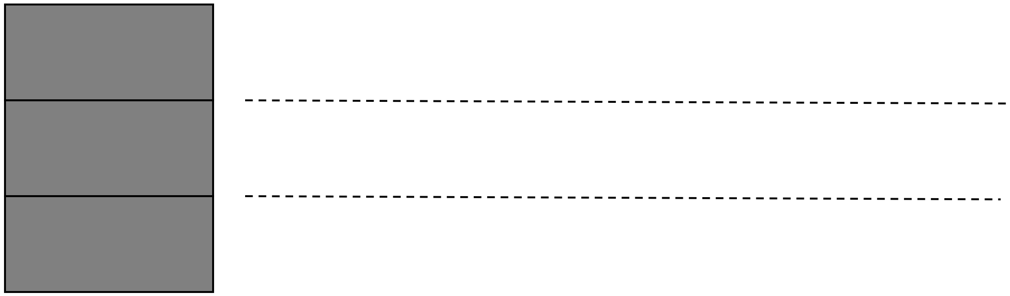
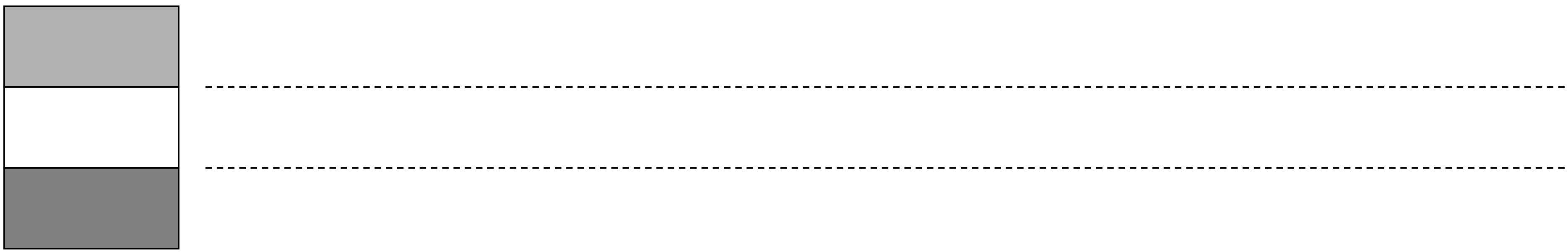
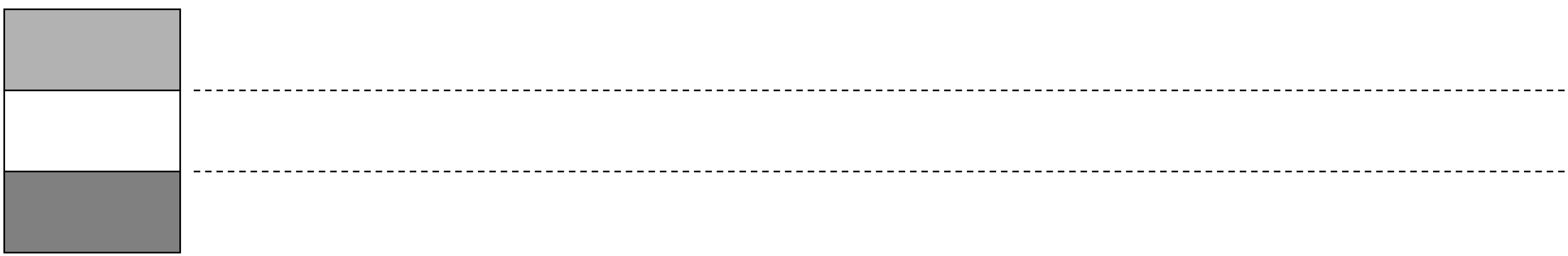
Contraintes dans les stratifiés anisotropes

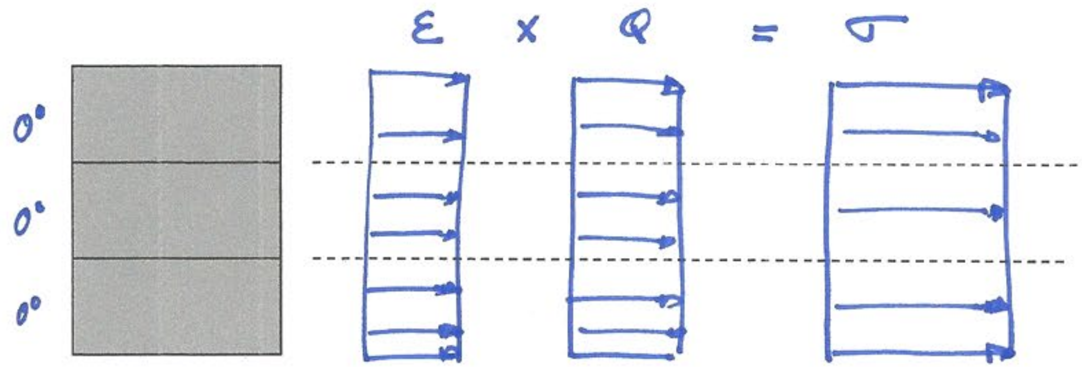


$$[\sigma] = \left[\overline{Q}\right] \left(\left\{\varepsilon^{0}\right\} + z\ \{\kappa\}\right)$$

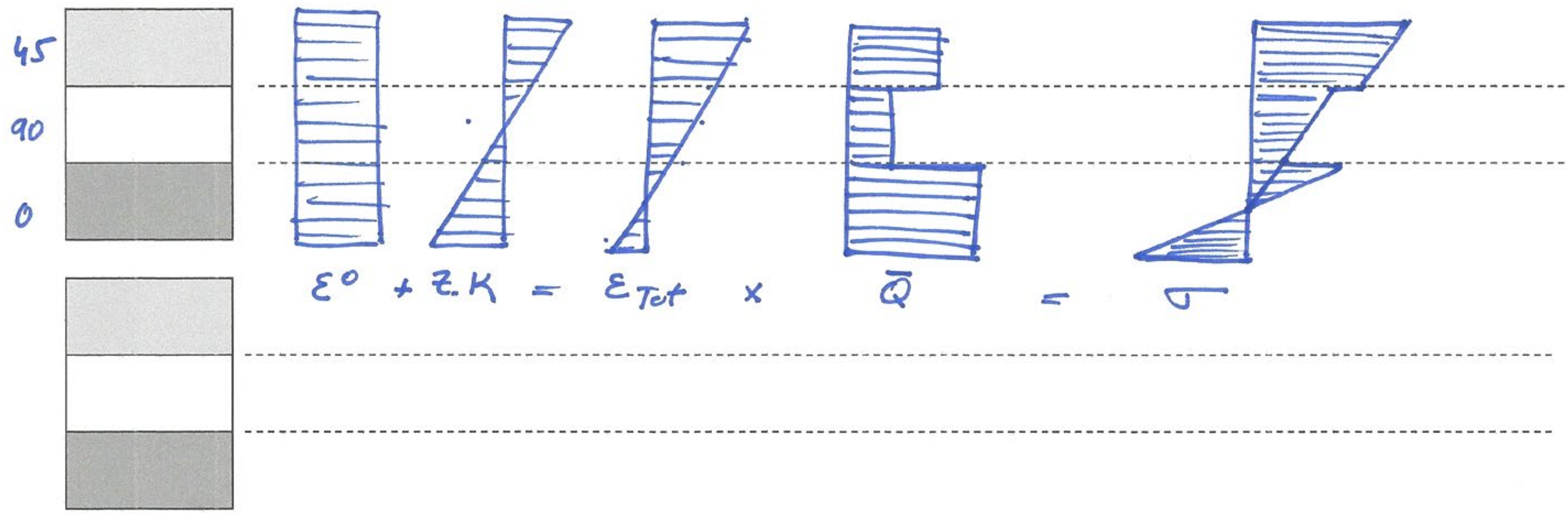




Contraintes dans les stratifiés anisotropes



$$[\sigma] = [\overline{Q}]\left(\{\varepsilon^0\} + z\ \{\kappa\}\right)$$



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Résistances ultimes

Laminate FPF analysis

Laminate : C 04590S
Modified : Sun Nov 11 17:28:27 2012

Lay-up : (0a/+45a/-45a/90a)SE   h = 1.84 mm

	Ply	t	E_1	E_2	G_12	nu_12	G_31	G_23
		mm	GPa	GPa	GPa		GPa	GPa
a	E;Epoxy;UD-.230/299/50	0.23	38	9	3.6	0.3	3.6	3.46154

	X_t	X_c	Y_t	Y_c	S	R	Q	X_eps,t	X_eps,c	Y_eps,t	Y_eps,c	S_
	MPa	MPa	MPa	MPa	MPa	MPa	MPa	%	%	%	%	
a	930	570	33	110	70	70	41.5385	2.44737	1.5	0.366667	1.22222	1.94

Load : 5kN10cm Mx125sur25cm
Modified : Sun Nov 11 21:02:20 2012
Type : Forces and moments (Var.;E)

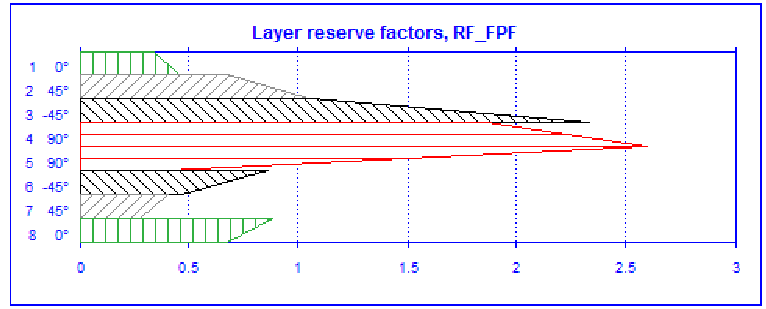
N_x	=	50000 N/m	M_x	=	500 Nm/m
N_y	=	0 N/m	M_y	=	0 Nm/m
N_xy	=	0 N/m	M_xy	=	0 Nm/m

Q_x	=	0 N/m
Q_y	=	0 N/m

Factor of safety : FoS^v = 1
Failure criterion : Tsai-Wu; Max strain; Von Mises; Out-of-plane shear; Out-of-plane s
(UD; non-UD; homogeneous; honeyc. core; foam/other core; adhe
Failure crit. param. : Tsai-Wu F_12*=-0.5
Stress/strain recovery : layer top/bottom

Laminate reserve factors

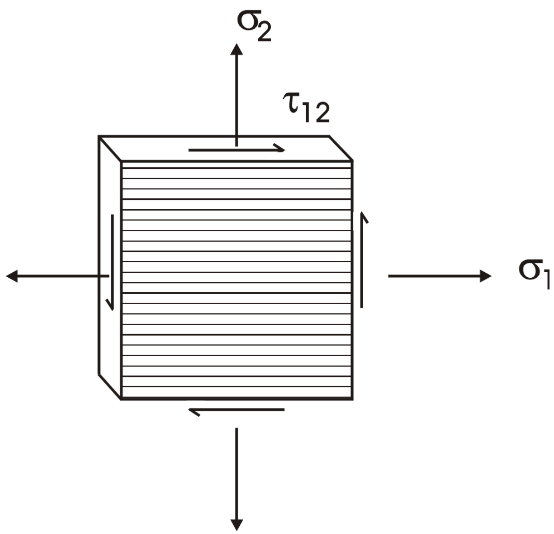
	FPF	Mode	FPF-only	Mode	Crit. layers	ILS	Crit. interf.
RF =	0.28	2t	0.28	2t	7(45°)	-	-



Layer reserve factors - FPF

	Ply	theta °			RF	
1	a	0	t	2.	0.34	1c/2t
			b		0.45	
2	a	45	t	6.	0.68	s
			b		1.05	
3	a	-45	t	7.	1.06	s
			b		2.33	
4	a	90	t	8.	1.86	2c
			b		2.60	
5	a	90	t	3.	2.60	2t
			b		0.40	
6	a	-45	t	4.	0.86	2t/s
			b		0.47	
7	a	45	t	1.	0.40	2t
			b		0.28	
8	a	0	t	5.	0.89	1t
			b		0.67	

Résistances ultimes



	V_f	$\sigma^T_{u,1}$	$\sigma^T_{u,2}$	$\sigma^C_{u,1}$	$\sigma^C_{u,2}$	τ_u
carbone / époxy (AS/3501)	0.60	1448	48	1172	248	62
Kevlar 49 /époxy	0.60	1380	27	276	65	60
verre-E / époxy (scotch ply 1002)	0.45	1100	28	620	138	83

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Les normes

Table 2 Typical mechanical properties reported on data sheets

Property	Test method	Test description	Usefulness for design purposes
Tensile strength	ASTM D 638	Property reports the stress in a standard specimen at the specified point on the stress-strain curve. Value reported could be yield or ultimate.	Depending on which value is reported, some portion of the tensile strength may be used as a design-stress limit. This is valid for gradual static loading at room temperature. For glass-reinforced materials, anisotropic properties render this property useless for design.
Elongation	ASTM D 638	Property reports percent of elongation during the tensile test.	Property is meaningful for elongations from 0 to 15%. Data may be used to set interference limits or strain limits on a design. Strain rate, environmental temperature, and molding flaws such as knit lines must be considered in an actual part. For cases in which elongation exceeds 8%, the material typically has started to yield, and necking occurs. This is well beyond the point useful for design.
Tensile modulus	ASTM D 638	Property reports the stiffness of the material. Value is the slope of the line tangent to the stress-strain curve at the origin.	Tensile modulus may be useful in calculating the deflection. Value is good only for small strains of 1% and at room temperature. Glass-reinforced materials behave anisotropically. Loads applied over extended time need to include creep.
Shear strength	ASTM D 732	Property reports the shear stress calculated by dividing the load required to blank out the specimen by the area, as calculated by the perimeter of the punch times the specimen thickness.	The shear strength may have limited value in design engineering. Stress concentrations and shear rates that vary between the test and an application will affect the accuracy of the value.
Izod impact strength	ASTM D 256	Property reports the energy absorbed when a notched specimen is subjected to a shock flexural loading.	Izod impact does not provide usable design information. It can indicate brittle behavior in a material, which would suggest certain approaches to design for impact loads.
Gardner impact	...	Property reports the energy required for a projectile to fracture a sample plaque of the material. The projectile weight, shape, and support configuration are specified.	Gardner impact has very limited design value. The speed, shape, and weight of the projectile are likely to vary from actual use conditions. Also, the manner of support for a molded part differs from that used for the test plaque.
Tensile impact strength	ASTM D 1822	Property reports the energy required to fracture a molded specimen subjected to a shock tensile load.	Tensile impact has little design value. It may indicate a brittle material that could affect the design approach employed.
Fracture toughness	...	Property reports resistance to crack propagation.	Facilitates material selection
Flexural strength	ASTM D 790	Property reports the maximum fiber stress in a simply supported bar loaded in flexure until either 5% strain is reached or the bar fractures.	Flexural strength has little design value. The value is based on classical elastic equations, which are inaccurate for large deflections, and are based on the assumption that the material obeys Hookes' Law in tension and compression. In actuality, the moduli are different. As with other strength properties, strain rate, temperature, and anisotropic properties limit the usefulness of this property in an application.
Flexural modulus	ASTM D 790	Property reports the stiffness in flexure. Value is obtained by taking the slope of the tangent to the flexural stress-strain curve at origin.	Flexural modulus is useful as an approximation for the elastic modulus in deflection equations, or to convert strain into stress. However, it can be applied only to applications at room temperature and to small deflections. It is widely used in finite element analysis (FEA). Deflection is often the biggest problem in plastic part design.
Compression strength	ASTM D 695	Property reports the maximum compressive stress carried by the test specimen during the compressive test.	Compressive strength cannot be considered significant for design when the application of the loads is widely different from the gradual test application. Fatigue and yielding are not considered and may need to be accounted for in an application.
Poisson's ratio	...	...	Poisson's ratio is needed for stress analysis, for both general deflection and FEA.

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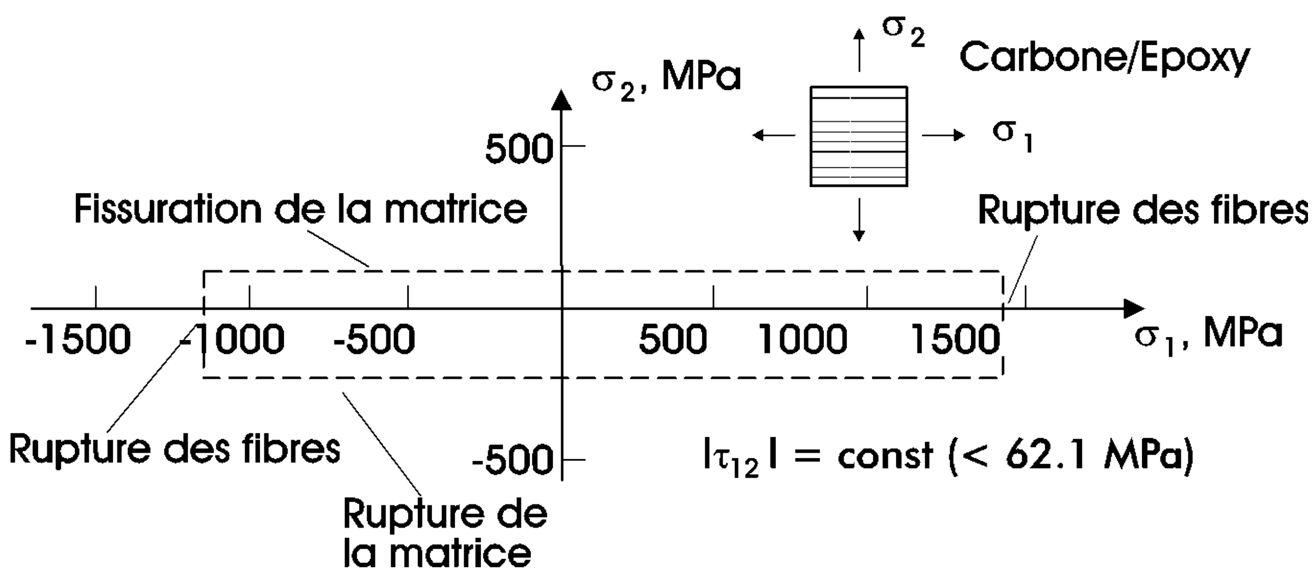
Critères de rupture

- Contraintes maximales
- Déformations maximales
- Critères d'interaction : Hill, Tsai-Hill
- Critère de Tsai-Wu
- Critères de Hashin

Critère de la contrainte maximale

Traction et cisaillement	$\sigma_1 = \sigma_{u,1}^T$	$\sigma_2 = \sigma_{u,2}^T$	$\lvert\tau_{12}\rvert = \tau_u$
Compression et cisaillement	$\lvert\sigma_1\rvert = \sigma_{u,1}^C$	$\lvert\sigma_2\rvert = \sigma_{u,2}^C$	$\lvert\tau_{12}\rvert = \tau_u$



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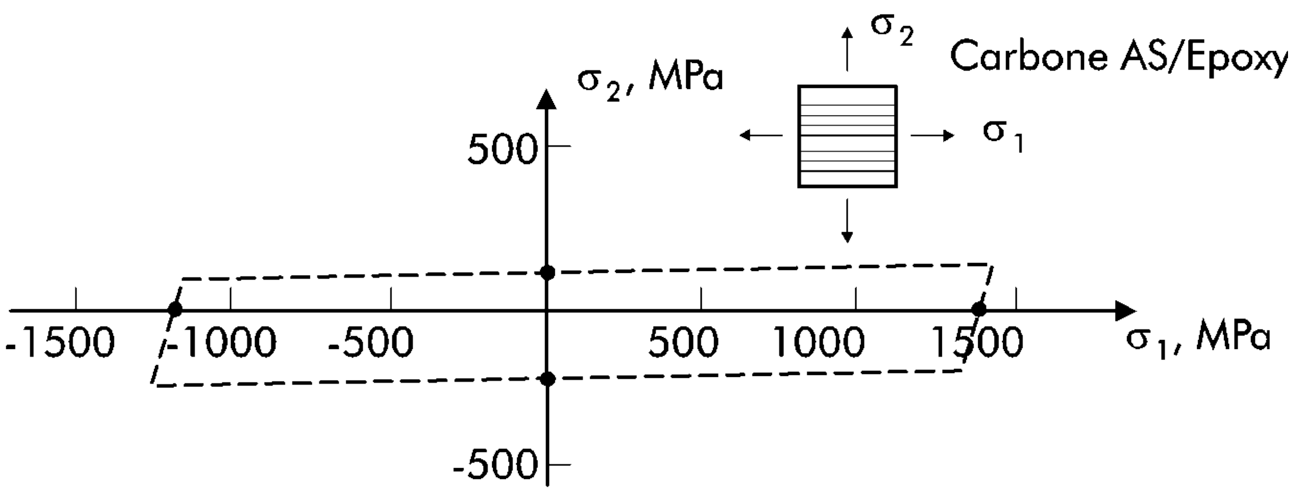
Critère de la déformation maximale

$$-\varepsilon_{u,i}^{C} < \varepsilon_i < \varepsilon_{u,i}^{T}$$



$$\sigma_{u,1} = \frac{\sigma_{u,2}^{T} - \sigma_2}{\nu_{21}} \qquad \sigma_1 > 0$$

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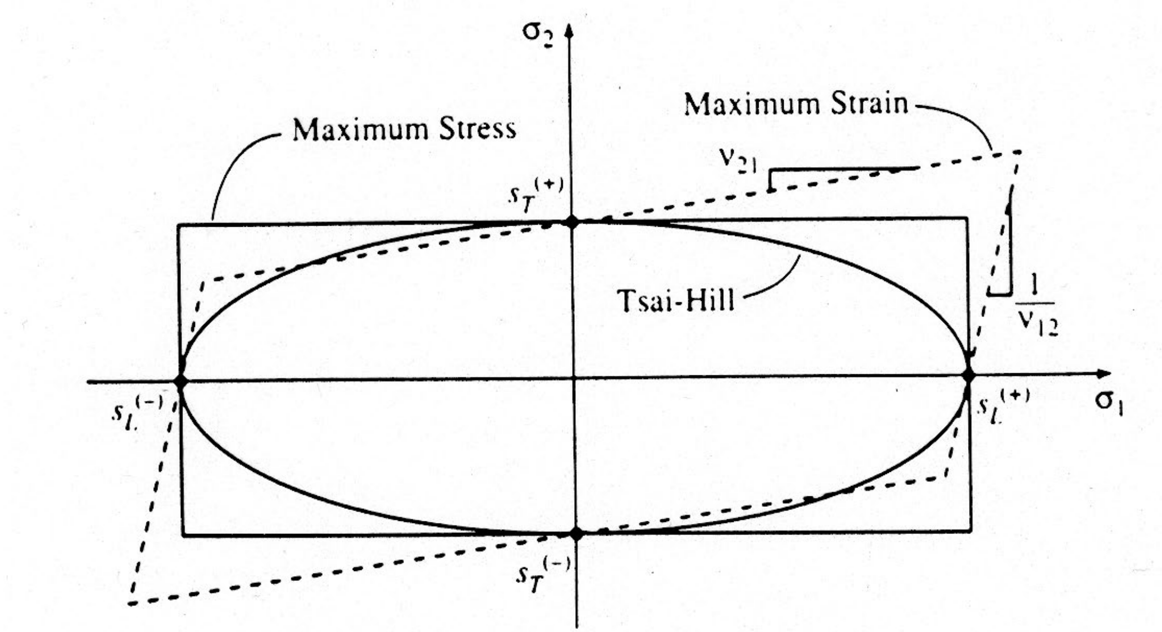
Critère de Tsai-Hill

$$\frac{\sigma_1^2 - \sigma_1\sigma_2}{\left(\sigma_{u,1}^T\right)^2} + \frac{\sigma_2^2}{\left(\sigma_{u,2}^T\right)^2} + \frac{\tau_{12}^2}{\tau_u^2} = 1$$

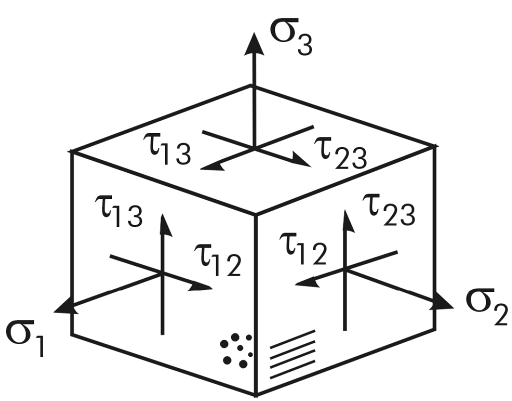


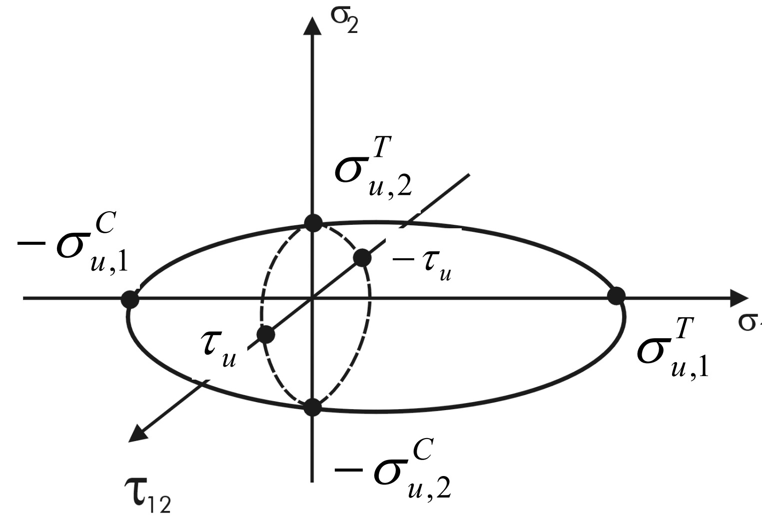
Critère de Tsai-Wu

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_1\sigma_2 = 1$$

$$F_1 = \frac{1}{\sigma_{u,1}^T} - \frac{1}{\sigma_{u,1}^C} \qquad F_{11} = \frac{1}{\sigma_{u,1}^T\sigma_{u,1}^C} \qquad F_{66} = \frac{1}{\tau_u^2} \qquad F_{12} = \frac{-\sqrt{F_{11}F_{22}}}{2}$$

$$F_2 = \frac{1}{\sigma_{u,2}^T} - \frac{1}{\sigma_{u,2}^C} \qquad F_{22} = \frac{1}{\sigma_{u,2}^T\sigma_{u,2}^C}$$





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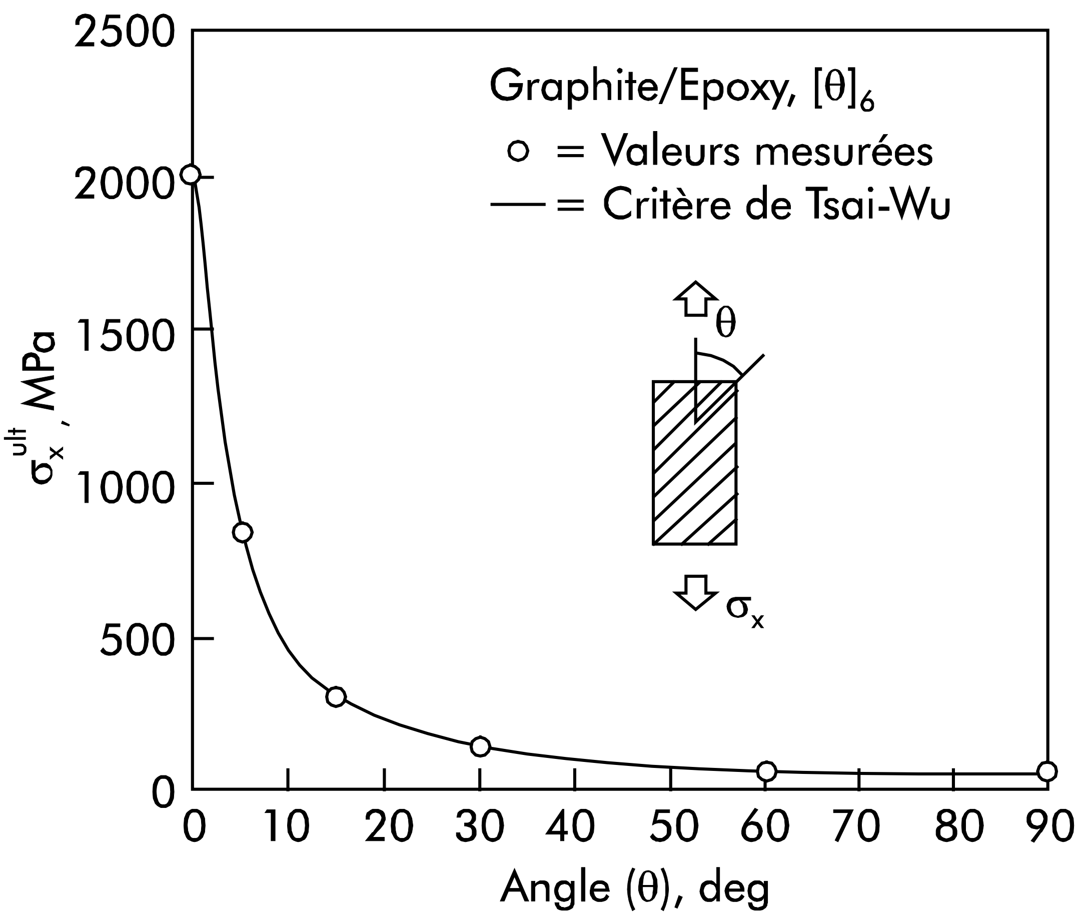
Critère de Tsai-Wu



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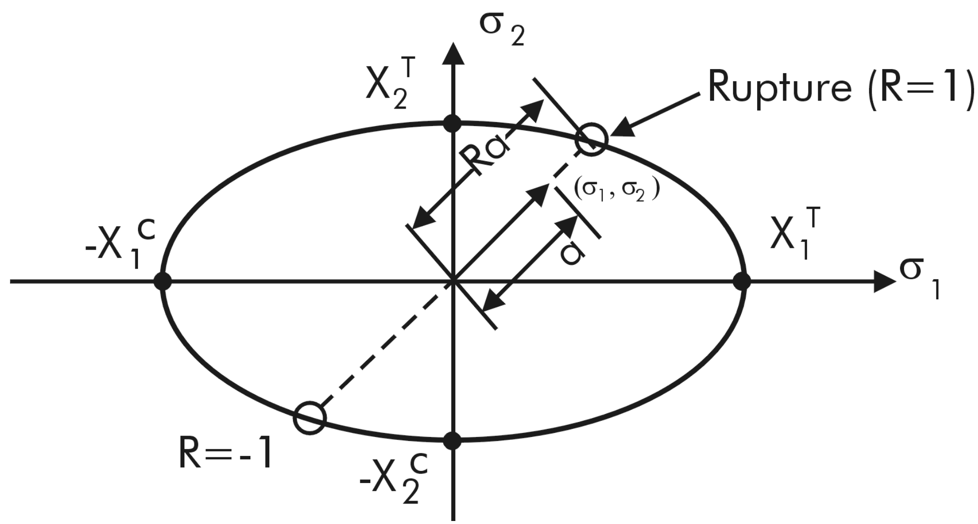
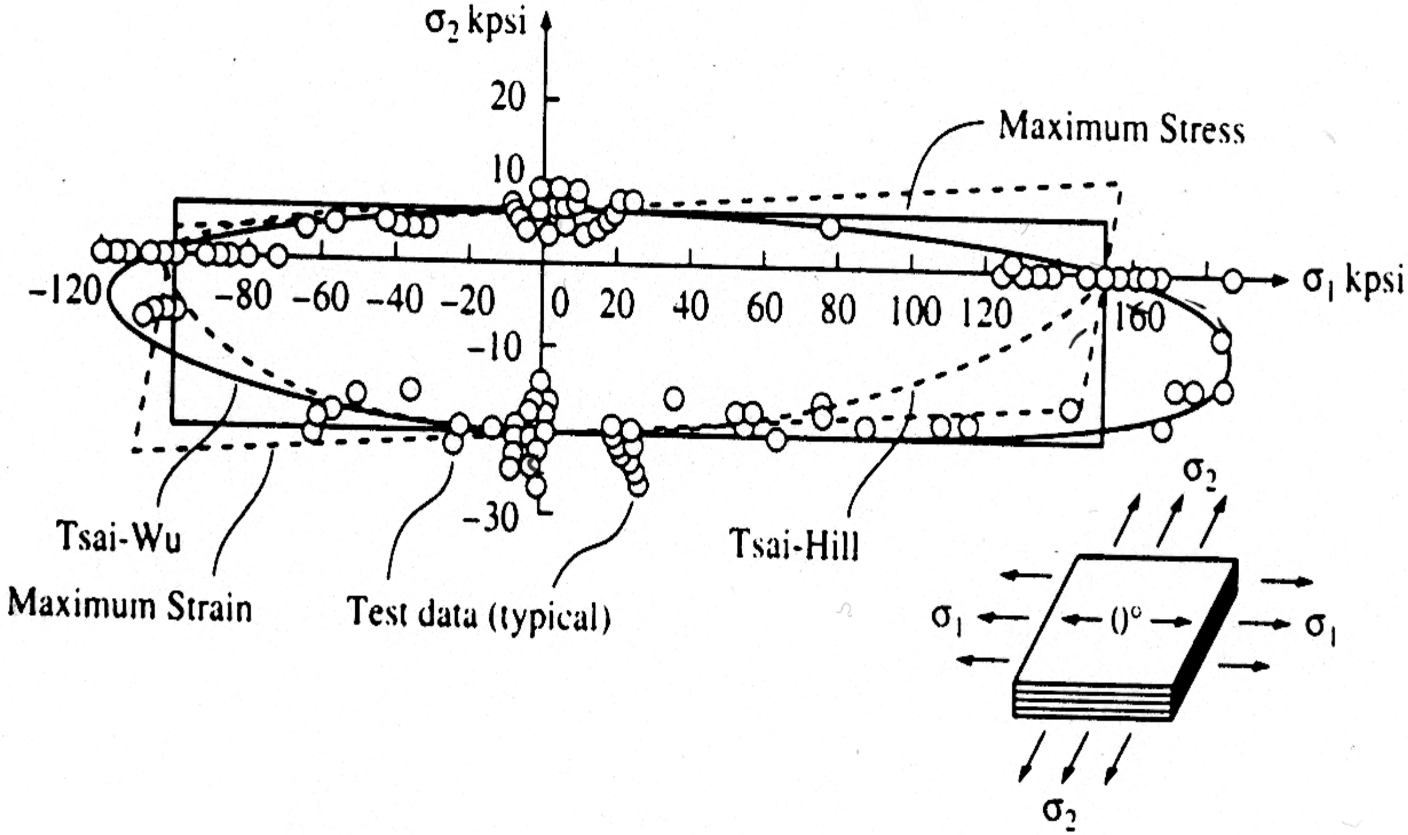
Facteur de sécurité

Reserve Factor



Mathématiquement, la valeur de R est calculée pour n'importe quel état de contraintes appliquées $(\sigma_1, \sigma_2, \tau_{12})$ par substitution de $(R\sigma_1, R\sigma_2, R\tau_{12})$ dans la théorie de la rupture adéquate et en résolvant pour R.

Critères de rupture



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Rupture du premier pli puis rupture finale

Contraintes dans les strates

$$[\sigma] = [\overline{Q}]\left(\{\varepsilon^0\} + z\ \{\kappa\}\right)$$

1

2,3…

Nouvel état de contraintes

Critère de rupture

$$\frac{\sigma_1^2 - \sigma_1\sigma_2}{\left(\sigma_{u,1}^T\right)^2} + \frac{\sigma_2^2}{\left(\sigma_{u,2}^T\right)^2} + \frac{\tau_{12}^2}{\tau_u^2} = 1$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

Rupture d'un pli

$$\left(\overline{Q_{ij}}\right)_{\text{pli rompu}} = 0$$

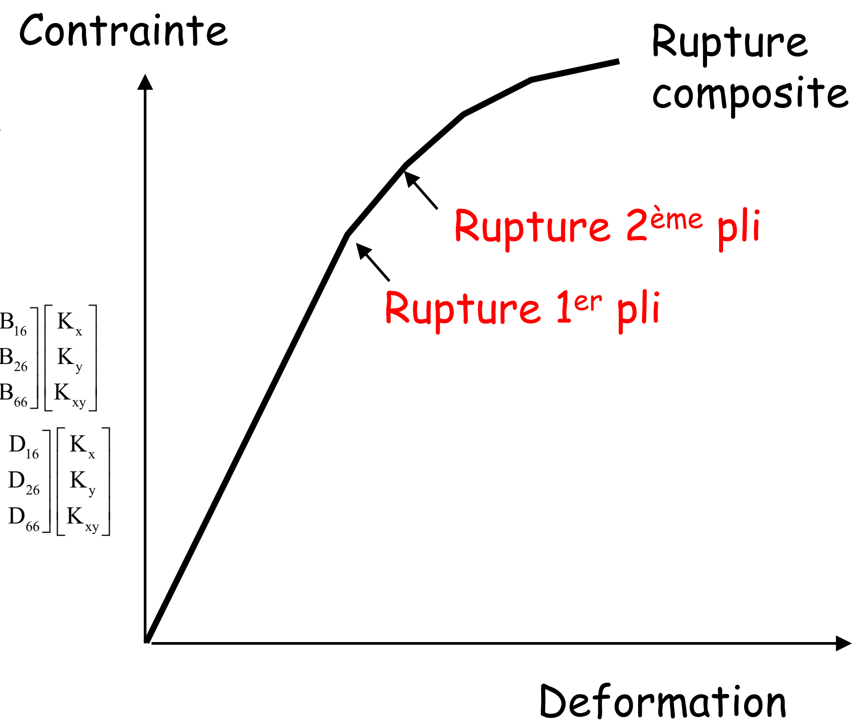
$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k (z_k^3 - z_{k-1}^3)$$

Re-calcul des constantes

First Ply Failure



Dimensionnement : propriétés effectives

Efforts connus
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

h = épaisseur du stratifié

Contraintes globales moyennes (fictives)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = 1/\mathrm{h} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = 1/\mathrm{h} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\frac{1}{\mathrm{h}} \mathrm{A}_{\mathrm{ij}} = \sum_{\mathrm{k}=1}^{\mathrm{N}} \left(\overline{\mathrm{Q}_{\mathrm{ij}}}\right)_{\mathrm{k}} \frac{\text{épaisseur}_{\mathrm{k}}}{\mathrm{h}} = \sum_{\mathrm{k}=1}^{\mathrm{N}} \left(\overline{\mathrm{Q}_{\mathrm{ij}}}\right)_{\mathrm{k}} \text{pourcentage}_{\mathrm{k}}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \mathrm{h}. \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Modules effectifs

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Dimensionnement : déformation et épaisseur minimale

Choisir des pourcentages initiaux

Calculer $$\frac{1}{h}A_{ij} = \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \text{pourcentage}_k$$

Inverser pour obtenir modules effectifs E_x, E_y …. et les déformations

Critère de rupture pour le pli k en entrant dans le critère de rupture $h\,\sigma_x$ qui est connu car = N_x

$$\frac{(h\sigma_1)^2 - h\sigma_1 h\sigma_2}{\left(\sigma_{u,1}^T\right)^2} + \frac{(h\sigma_2)^2}{\left(\sigma_{u,2}^T\right)^2} + \frac{(h\tau_{12})^2}{\tau_u^2} = h_k^{\ 2}$$

h pour éviter la rupture du pli k est donc connu

idem pour autres plis

Choisir le h = sup(h_k)

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Pourcentages initiaux technologiques

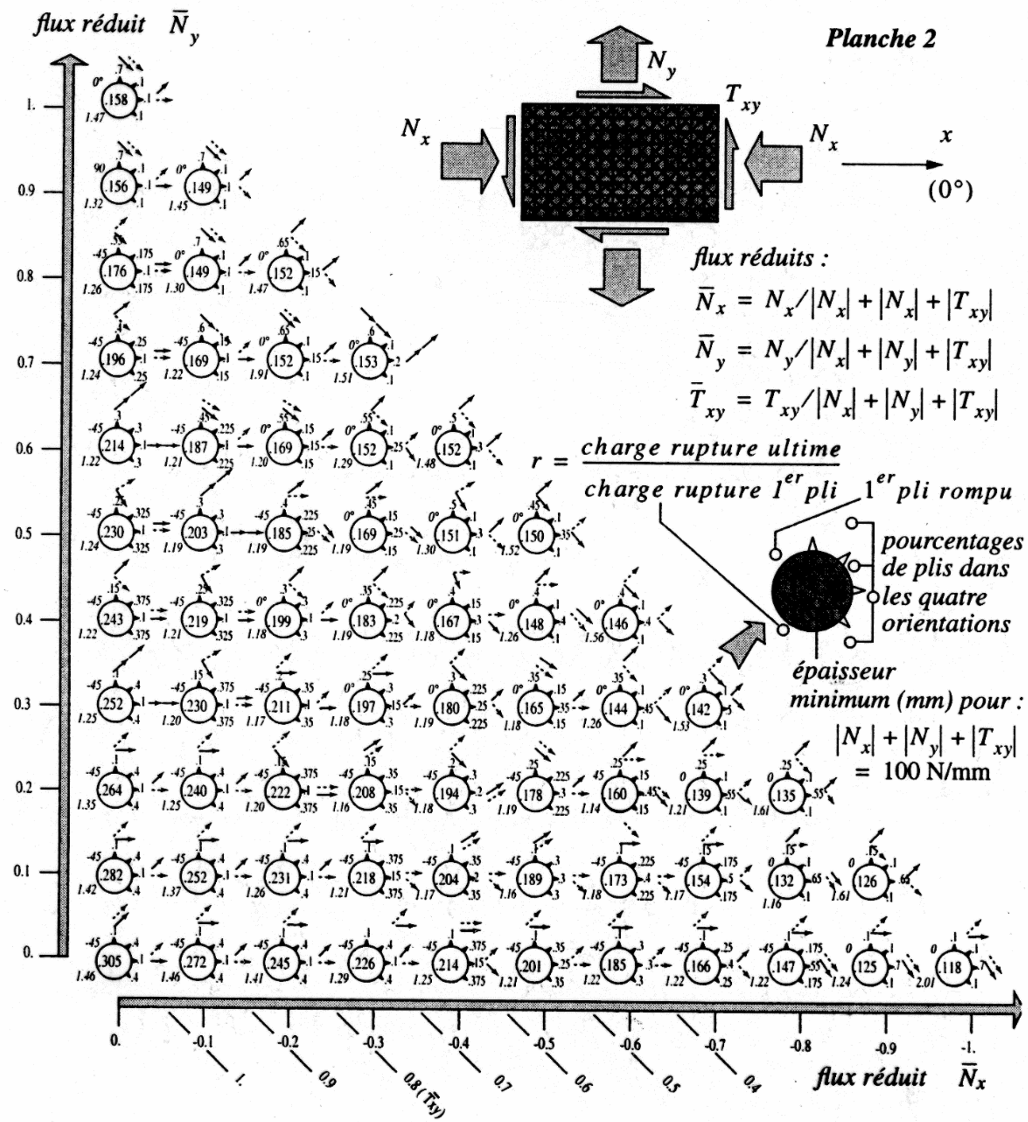
- Proportion minimum de 5 à 10% dans chaque direction 0, 90, +45, -45
- Epaisseur minimale du stratifié de l'ordre du mm (8 couches UD ou 3 à 4 couches de tissu équilibré)
- Considérer les directions des efforts principaux
- Plis à 90 placés en surface, puis plis à 45, puis à 0 lorsque les efforts prépondérants sont à 0
- Pas plus de 4 plis consécutifs dans une même direction

Guides de conception

Composition optimum d'un stratifié carbone/époxyde

V_f = 0,6 ; caractéristiques du pli : *cf.* annexe 1 ou paragraphe 3.3.3.

10 % minimum de plis dans chaque direction 0°, 90°, + 45°, – 45°



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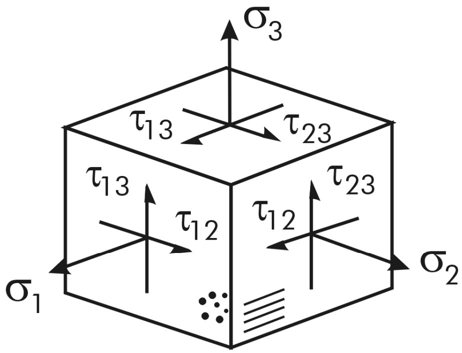
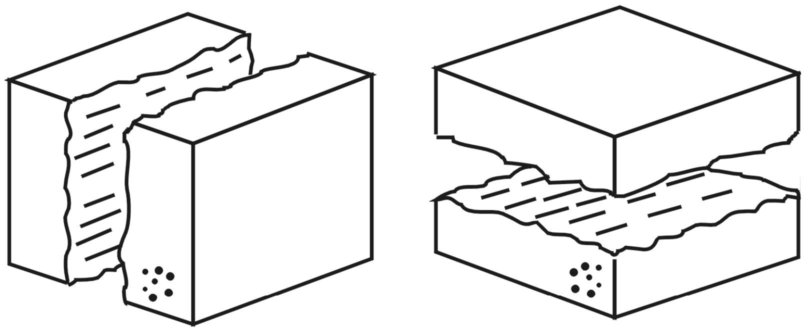
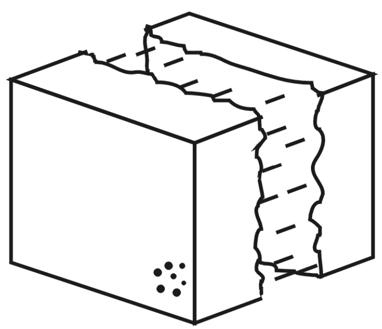
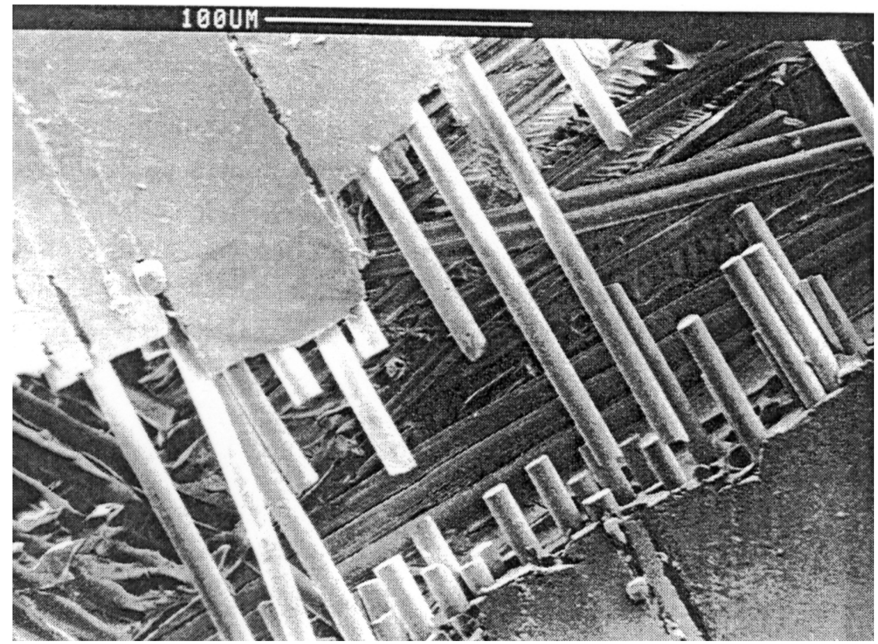
Endommagement





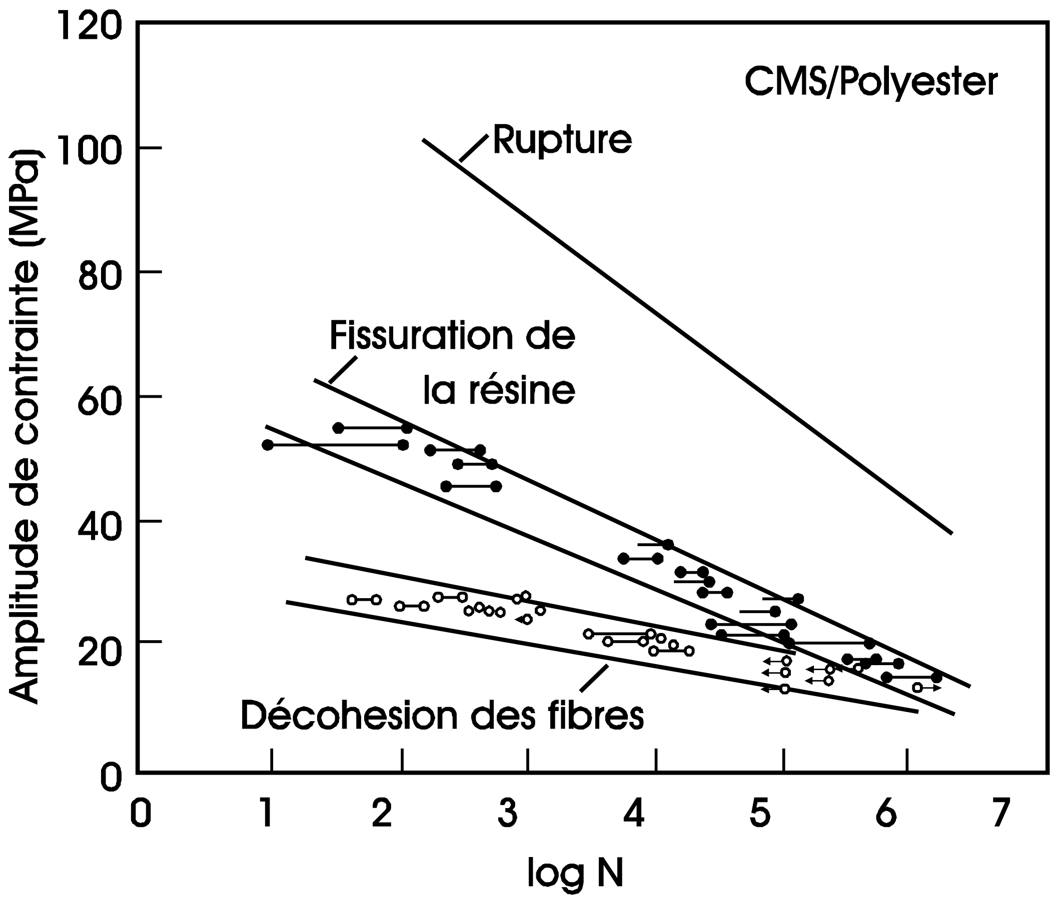
(b) Rupture de la matrice



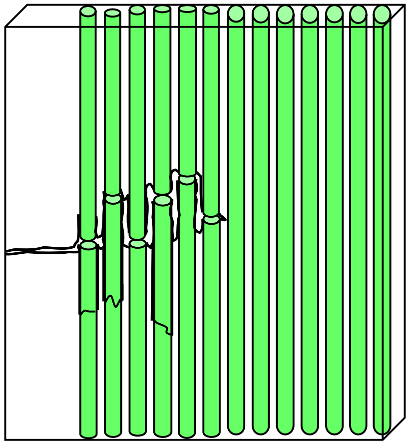


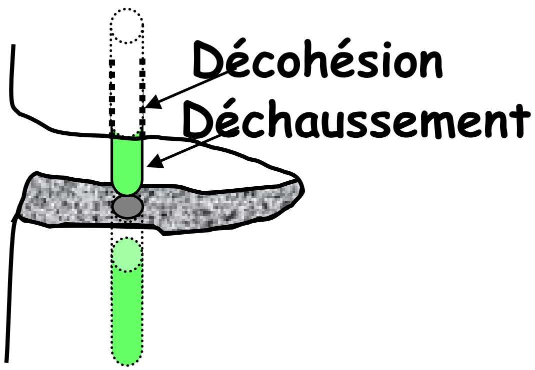
(c) Arrachage/rupture des fibres

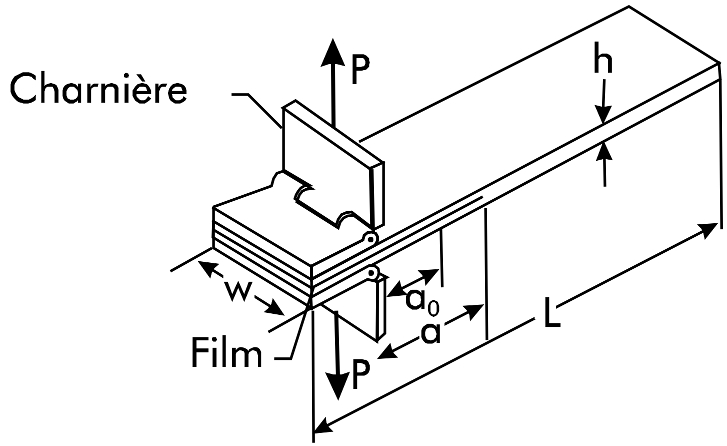
Fatigue des composites



Propagation de fissures







Approche G_{IC}, courbes R

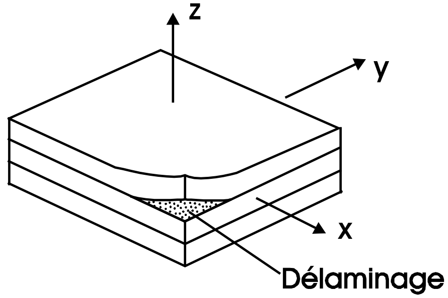
https://mediaspace.epfl.ch/media/DLL-MAT-M%C3%A9canique-rupture+-+Quelle+%C3%A9nergie+propage+une+fissure+F/0_ftatg5g5/29020

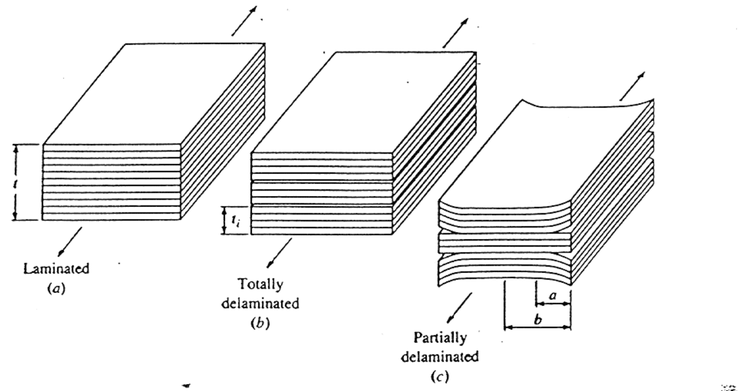
Energie de rupture typique d 'une matrice thermodurcie: ~100 J/m^2

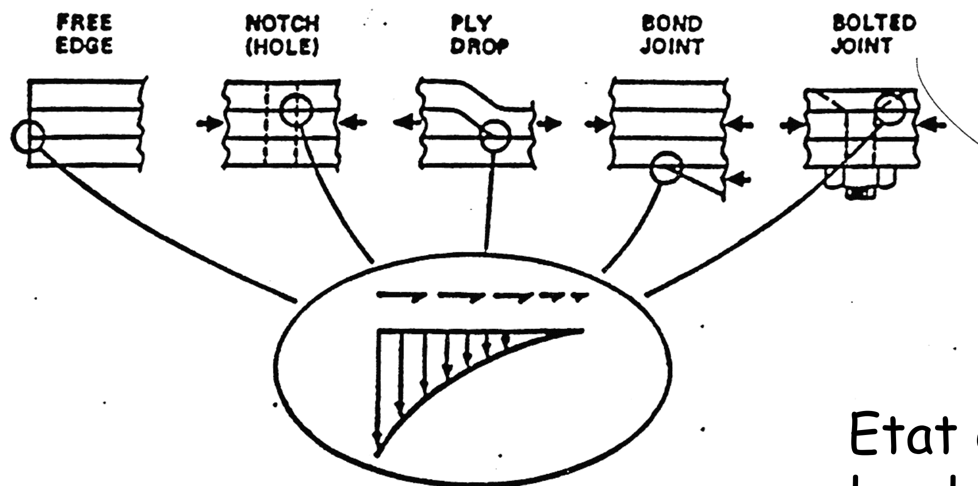
des fibres de verre: ~10 J/m^2

d 'un composite: ~ 10000 J/m^2

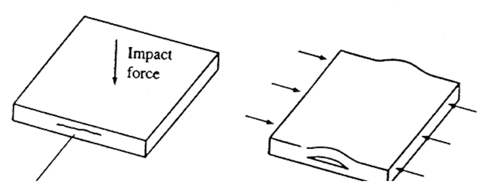
Le délaminage







Etat de contraintes
locales complexe



Mécanique des composites

Micro et macromécanique

Tests mécaniques

Endommagement et rupture

CADFEM

Applications

Les stratifiés

Structures sandwich

Composites textiles

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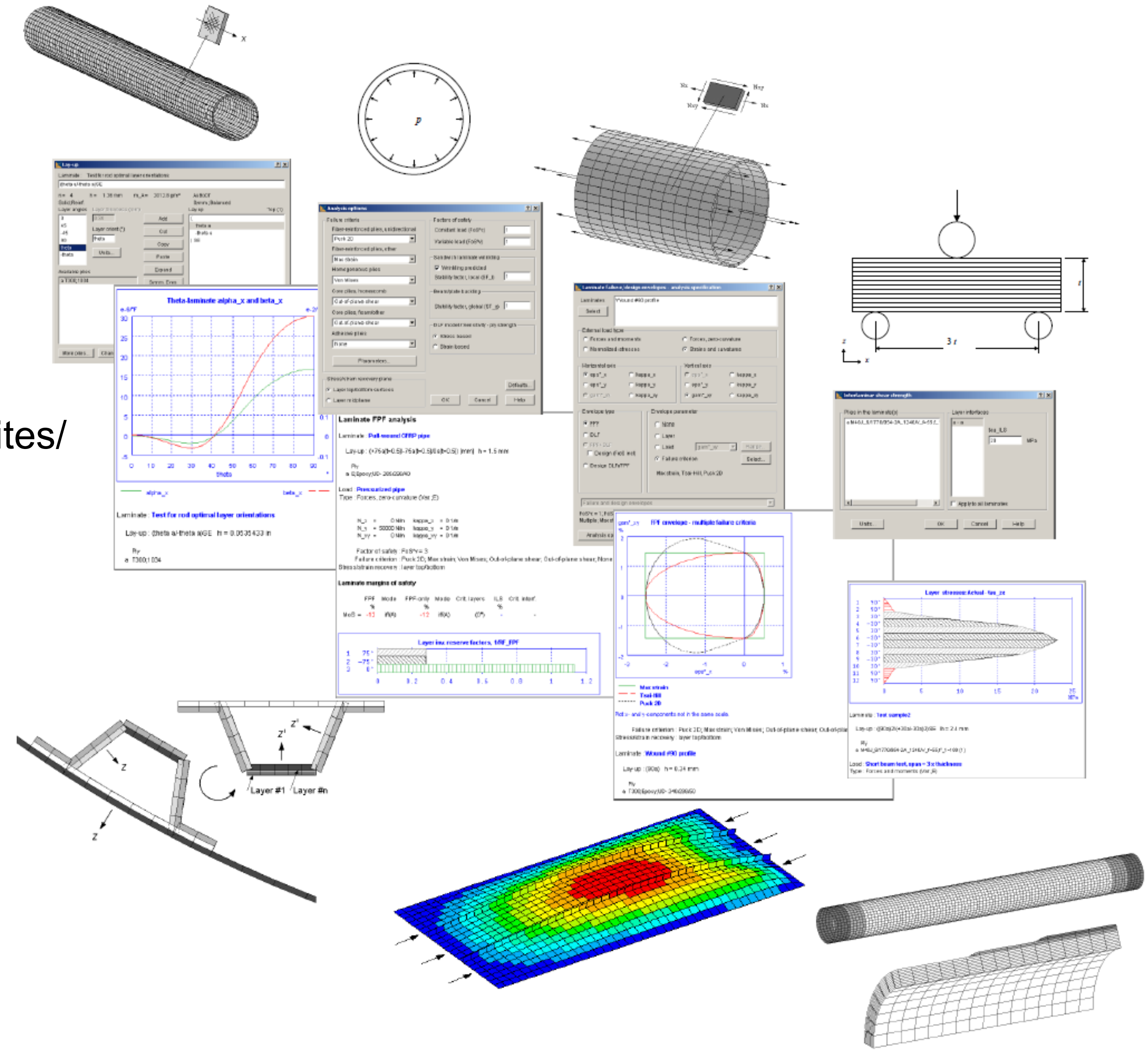
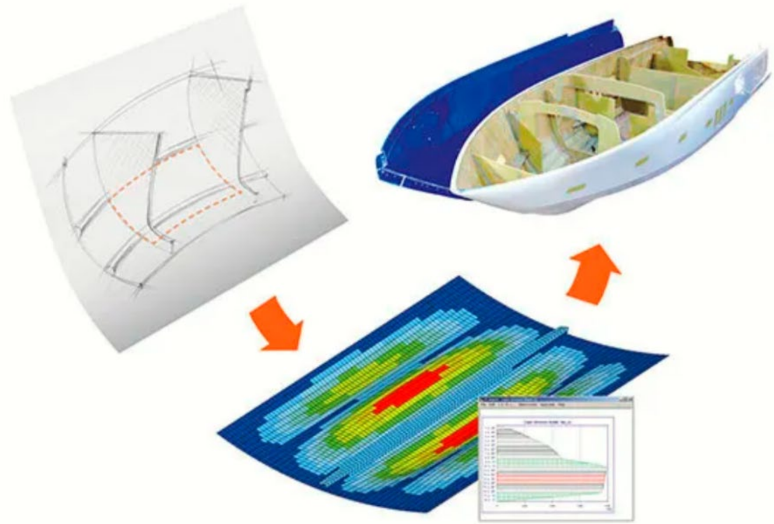
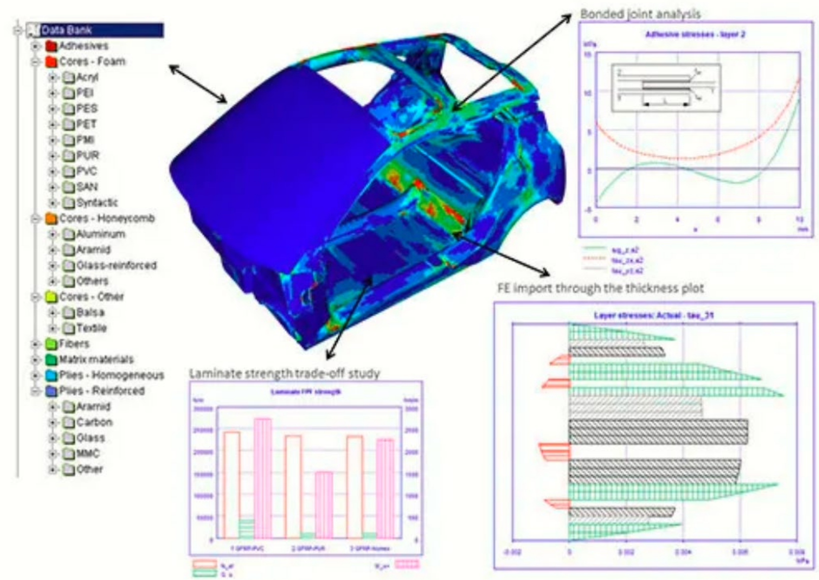
https://www.altair.com/composites/

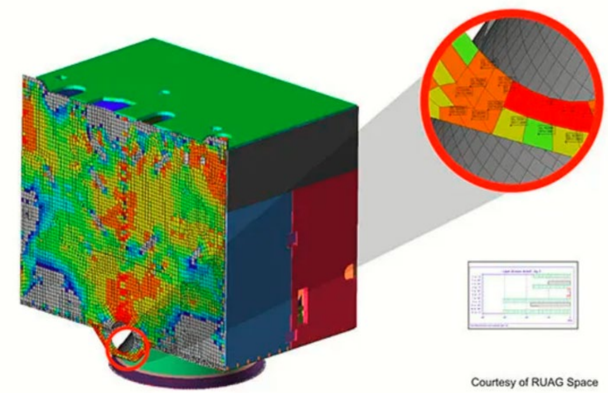


FEM



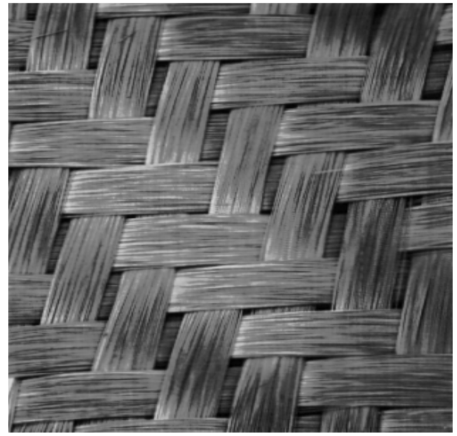
https://filecr.com/windows/esacomp/?id=13436849765

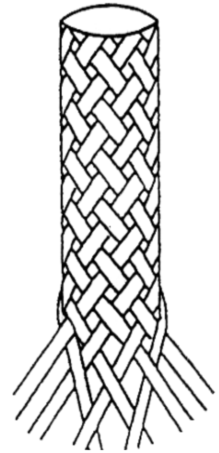


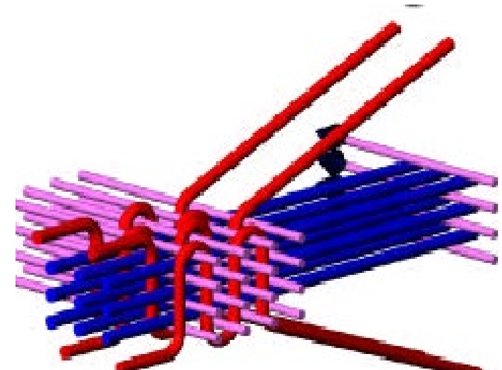


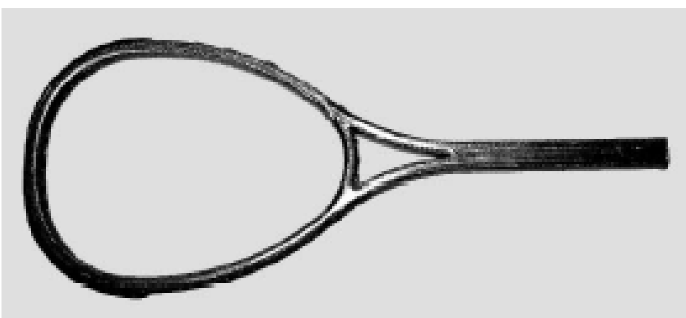
Courtesy of RUAG Space

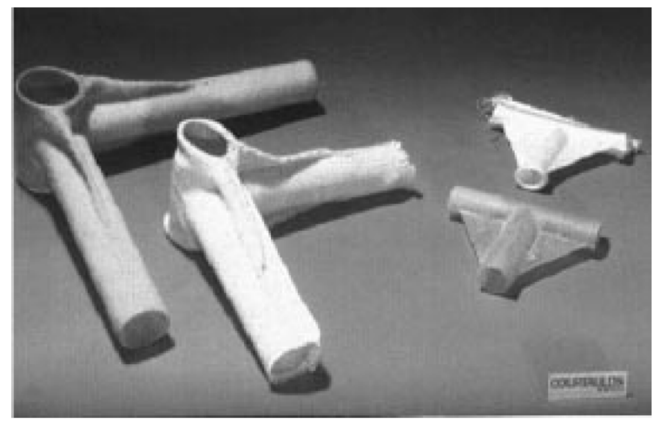
Les composites textiles



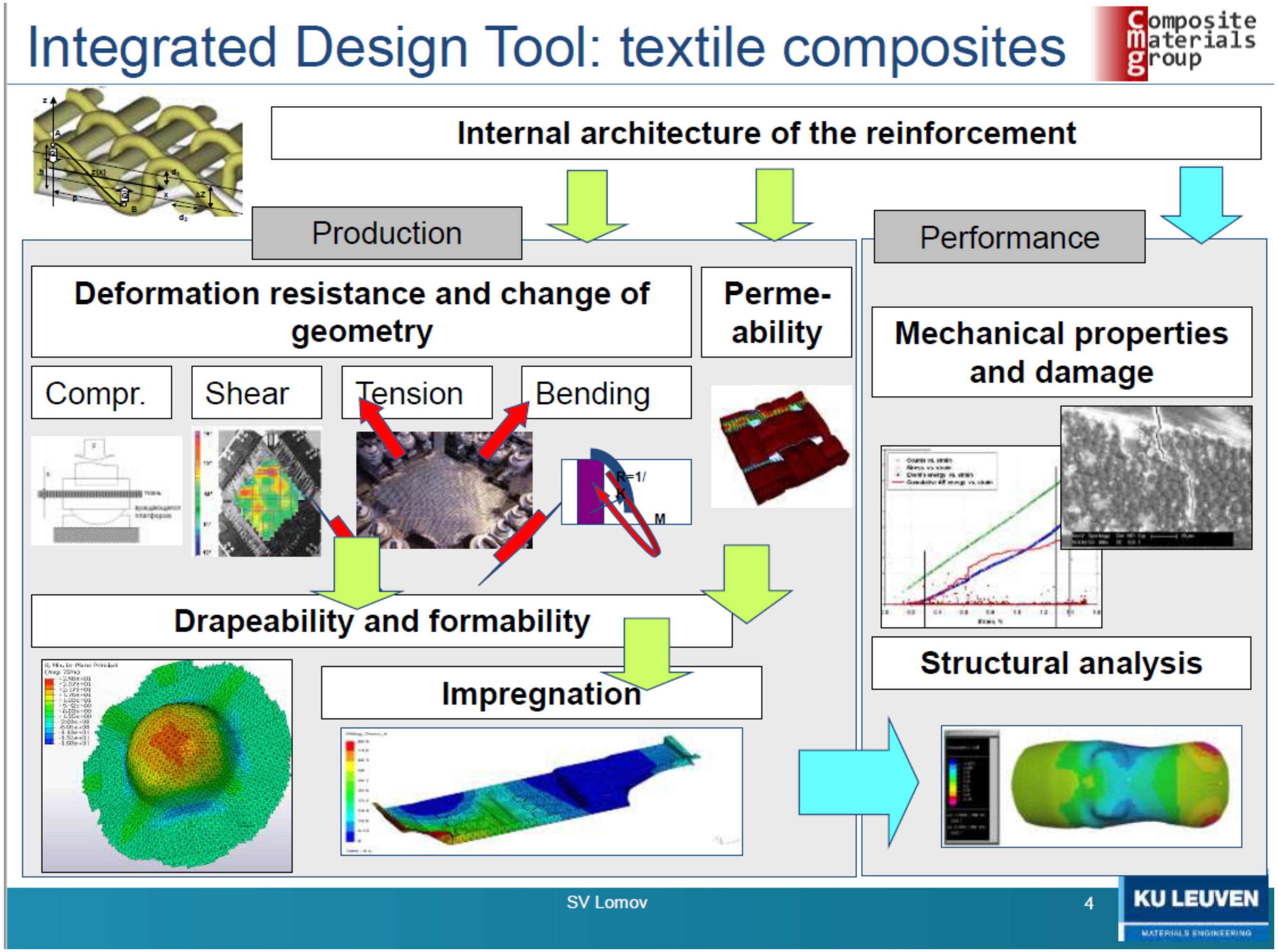






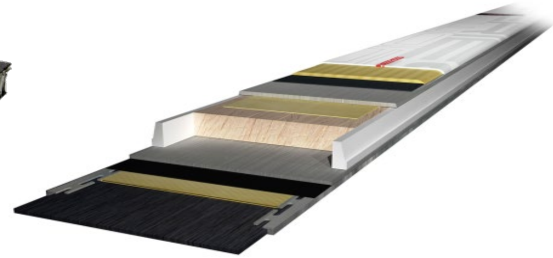


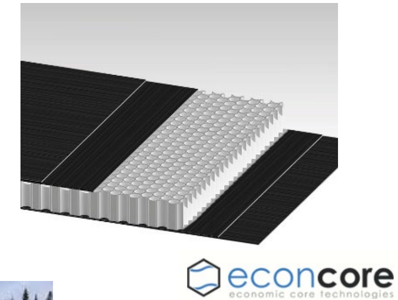
Les composites textiles

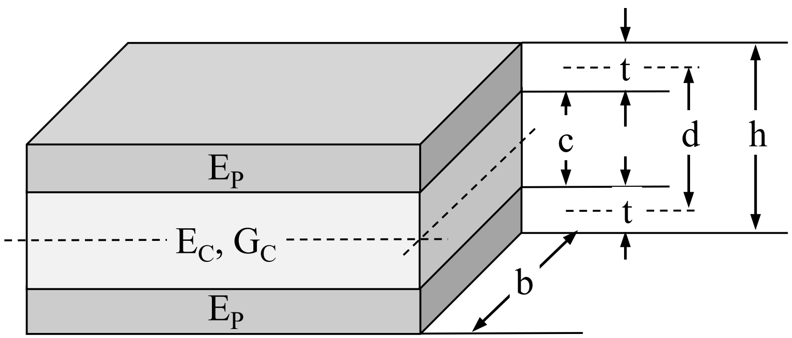


https://www.mtm.kuleuven.be/onderzoek/scalint/Composites/software/wisetex

Structures sandwich







$$D \approx D' = E_P \frac{b\,t\,d^2}{2}$$

Révision et examen

https://docs.google.com/spreadsheets/d/1vMpbdes4FlINZgwfZNScfrZTd3DPYVLIwxQKzwLknu0/edit#gid=0

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