Cours MSE 340 Composites Polymères 2024,
Exo A avec ESACOMP : stratifiés, charges, profils de contraintes
Exemples de solutions
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3 stratifiés

Laminate stiffness and compliance matrices

Laminate : A: 30
Modified : Sun Nov 11 17:21:24 2012

Lay-up : ((+30a/-30a)2)SE h = 1.84 mm

Ply
a E;Epoxy;UD-.230/299/50

Stiffness matrices

[A] (N/m)	4.81157e+007	1.47728e+007	0
	1.47728e+007	2.08546e+007	0
	0	0	1.63206e+007
[B] (N)	0	0	0
	0	0	0
	0	0	0
[D] (Nm)	13.5751	4.1679	1.84121
	4.1679	5.88379	0.656602
	1.84121	0.656602	4.60458

Compliance matrices

[a] (m/N)	2.65596e-008	-1.8814e-008	0
	-1.8814e-008	6.12783e-008	-0
	0	-0	6.12723e-008
[b] (1/N)	0	0	0
	-0	-0	-0
	0	0	0
[d] (1/(Nm))	0.0979937	-0.0660948	-0.0297592
	-0.0660948	0.217286	-0.00455559
	-0.0297592	-0.00455559	0.229724

Symétrique et balancé donc B=0 et A16=A26=0

Laminate stiffness and compliance matrices

Laminate : B:04590
Modified : Sun Nov 11 17:25:51 2012

Lay-up : (90a/-45a/+45a/0a) h = 0.92 mm

Ply
a E;Epoxy;UD-.230/299/50

Stiffness matrices

[A] (N/m)	1.88587e+007	5.7703e+006	0
	5.7703e+006	1.88587e+007	0
	0	0	6.5442e+006
[B] (N)	2351.27	0	391.878
	0	-2351.27	391.878
	391.878	391.878	0
[D] (Nm)	1.50115	0.236015	0
	0.236015	1.50115	0
	0	0	0.290601

Compliance matrices

[a] (m/N)	7.69678e-008	-2.48905e-008	8.48491e-009
	-2.48905e-008	7.69678e-008	-8.48491e-009
	8.48491e-009	-8.48491e-009	1.58464e-007
[b] (1/N)	-0.00011924	-2.2454e-005	-7.02268e-005
	2.2454e-005	0.00011924	-7.02268e-005
	-4.72315e-005	-4.72315e-005	-4.98616e-021
[d] (1/(Nm))	0.879527	-0.090782	0.130517
	-0.090782	0.879527	-0.130517
	0.130517	-0.130517	3.63055

Laminate stiffness and compliance matrices

Laminate : C 04590S
Modified : Sun Nov 11 17:28:27 2012

Lay-up : (0a/+45a/-45a/90a)SE h = 1.84 mm

Ply
a E;Epoxy;UD-.230/299/50

Stiffness matrices

[A] (N/m)	3.77174e+007	1.15406e+007	0
	1.15406e+007	3.77174e+007	0
	0	0	1.30884e+007
[B] (N)	0	0	0
	0	0	0
	0	0	0
[D] (Nm)	15.3096	2.91402	0.721056
	2.91402	6.65696	0.721056
	0.721056	0.721056	3.35071

Compliance matrices

[a] (m/N)	2.92515e-008	-8.95025e-009	0
	-8.95025e-009	2.92515e-008	-0
	0	-0	7.64036e-008
[b] (1/N)	0	0	0
	-0	-0	-0
	0	0	0
[d] (1/(Nm))	0.0715112	-0.0303438	-0.00885904
	-0.0303438	0.166679	-0.0293388
	-0.00885904	-0.0293388	0.306665

le premier stratifié n'est pas symétrique alors que le deuxième l'est, donc B=0

le deuxième est quasi-isotrope A11=A22

Application d'une charge de traction Nx

Layer stresses/strains

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Lay-up : ((+30a/-30a)2)SE h = 1.84 mm

Ply
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Load : **5kN sur 10 cm**
Modified : Sun Nov 11 18:15:35 2012
Type : Forces and moments (Var.;E)

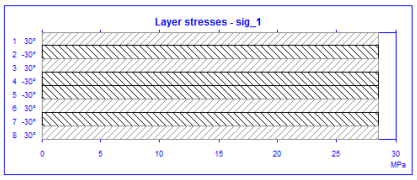
N_x = 50000 N/m M_x = 0 Nm/m
N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

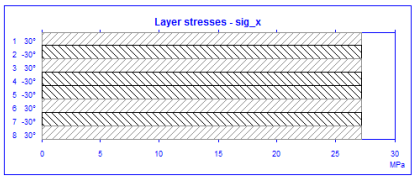
Actual stress, Actual (=Eq.) strain

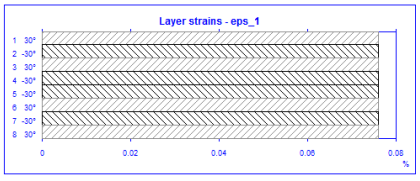
	Ply	theta		sig_1	sig_2	tau_12	eps_1	eps_2	gam_12	sig_x	sig_y	tau_xy	eps_x	eps_y	gam_xy
		°		MPa	MPa	MPa	%	%	%	MPa	MPa	MPa	%	%	%
1	a	30	t	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
2	a	-30	t	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
3	a	30	t	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
4	a	-30	t	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
5	a	-30	t	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
6	a	30	t	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
7	a	-30	t	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	7.07	0.0761	-0.0374	0.1965	27.17	0.00	-9.39	0.1328	-0.0941	0.0000
8	a	30	t	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000
			b	28.51	-1.34	-7.07	0.0761	-0.0374	-0.1965	27.17	0.00	9.39	0.1328	-0.0941	0.0000

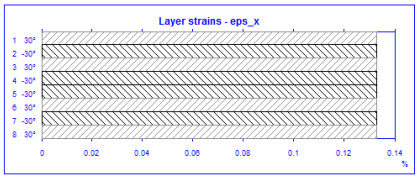
Actual stress, Actual (=Eq.) strain



Actual stress, Actual (=Eq.) strain







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Load : **5kN sur 10 cm**
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Type : Forces and moments (Var.;E)

N_x = 50000 N/m M_x = 0 Nm/m
N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

Laminate : **A: 30**
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N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

Layer stresses/strains

Laminate : **B:04590**
Modified : Sun Nov 11 17:28:40 2012

Lay-up : (90a/-45a/+45a/0a) h = 0.92 mm

Ply
a E;Epoxy;UD-.230/299/50

Load : **5kN sur 10 cm**
Modified : Sun Nov 11 18:15:35 2012
Type : Forces and moments (Var.;E)

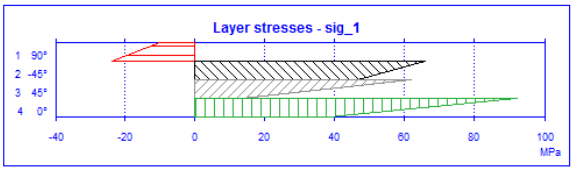
N_x	= 50000 N/m	M_x	= 0 Nm/m
N_y	= 0 N/m	M_y	= 0 Nm/m
N_xy	= 0 N/m	M_xy	= 0 Nm/m

Q_x	= 0 N/m
Q_y	= 0 N/m

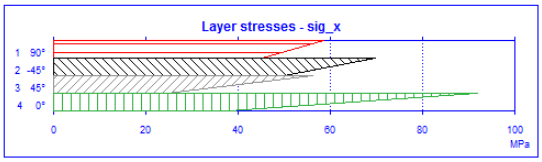
Actual stress, Actual (=Eq.) strain

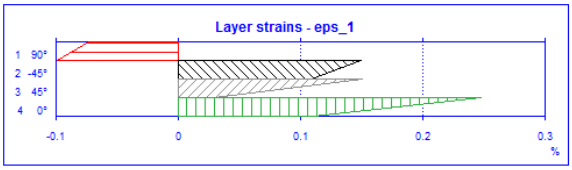
	Ply	theta		sig_1	sig_2	tau_12	eps_1	eps_2	gam_12	sig_x	sig_y	tau_xy	eps_x	eps_y	gam_xy
		°		MPa	MPa	MPa	%	%	%	MPa	MPa	MPa	%	%	%
1	a	90	t	-10.09	58.60	-7.34	-0.0728	0.6591	-0.2039	58.60	-10.09	7.34	0.6591	-0.0728	0.2039
			b	-23.90	45.28	-4.43	-0.0986	0.5220	-0.1232	45.28	-23.90	4.43	0.5220	-0.0986	0.1232
2	a	-45	t	65.81	29.27	22.34	0.1501	0.2733	0.6206	69.88	25.20	-18.27	0.5220	-0.0986	0.1232
			b	46.49	16.93	18.33	0.1090	0.1514	0.5093	50.05	13.38	-14.78	0.3848	-0.1245	0.0424
3	a	45	t	61.79	14.20	-18.33	0.1514	0.1090	-0.5093	56.33	19.66	23.80	0.3848	-0.1245	0.0424
			b	13.35	7.06	-14.33	0.0296	0.0679	-0.3980	24.53	-4.13	3.14	0.2477	-0.1503	-0.0383
4	a	0	t	92.04	-6.99	-1.38	0.2477	-0.1503	-0.0383	92.04	-6.99	-1.38	0.2477	-0.1503	-0.0383
			b	38.08	-13.14	-4.29	0.1106	-0.1761	-0.1191	38.08	-13.14	-4.29	0.1106	-0.1761	-0.1191

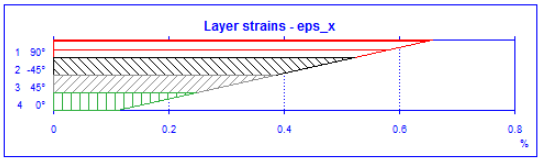
Actual stress, Actual (=Eq.) strain



Actual stress, Actual (=Eq.) strain







Laminate : B:04590
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Lay-up : (90a/-45a/+45a/0a) h = 0.92 mm

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N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

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N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

Rappel: 1 est la direction des fibres de chaque pli, x celle de la charge
les fibres du pli à 90 sont en compression
le pli a 90 se déforme le plus dans la direction de la charge Nx

Layer stresses/strains

Laminate : **C 04590S**
Modified : Sun Nov 11 17:28:27 2012

Lay-up : (0a/+45a/-45a/90a)SE h = 1.84 mm

Ply
a E;Epoxy;UD-.230/299/50

Load : **5kN sur 10 cm**
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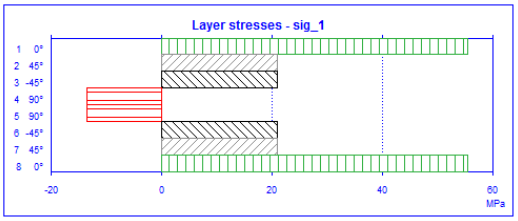
N_x = 50000 N/m M_x = 0 Nm/m
N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

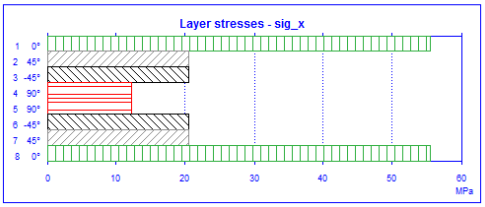
Actual stress, Actual (=Eq.) strain

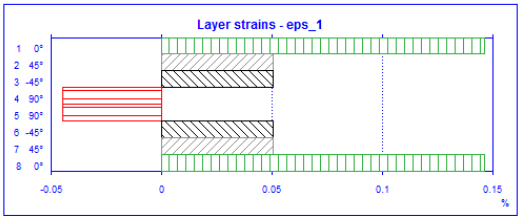
	Ply	theta		sig_1	sig_2	tau_12	eps_1	eps_2	gam_12	sig_x	sig_y	tau_xy	eps_x	eps_y	gam_xy
		°		MPa	MPa	MPa	%	%	%	MPa	MPa	MPa	%	%	%
1	a	0	t	55.55	-0.08	0.00	0.1463	-0.0448	0.0000	55.55	-0.08	0.00	0.1463	-0.0448	0.0000
			b	55.55	-0.08	0.00	0.1463	-0.0448	0.0000	55.55	-0.08	0.00	0.1463	-0.0448	0.0000
2	a	45	t	21.11	6.07	-6.88	0.0508	0.0508	-0.1910	20.46	6.71	7.52	0.1463	-0.0448	0.0000
			b	21.11	6.07	-6.88	0.0508	0.0508	-0.1910	20.46	6.71	7.52	0.1463	-0.0448	0.0000
3	a	-45	t	21.11	6.07	6.88	0.0508	0.0508	0.1910	20.46	6.71	-7.52	0.1463	-0.0448	0.0000
			b	21.11	6.07	6.88	0.0508	0.0508	0.1910	20.46	6.71	-7.52	0.1463	-0.0448	0.0000
4	a	90	t	-13.34	12.22	0.00	-0.0448	0.1463	0.0000	12.22	-13.34	0.00	0.1463	-0.0448	0.0000
			b	-13.34	12.22	0.00	-0.0448	0.1463	0.0000	12.22	-13.34	0.00	0.1463	-0.0448	0.0000
5	a	90	t	-13.34	12.22	0.00	-0.0448	0.1463	0.0000	12.22	-13.34	0.00	0.1463	-0.0448	0.0000
			b	-13.34	12.22	0.00	-0.0448	0.1463	0.0000	12.22	-13.34	0.00	0.1463	-0.0448	0.0000
6	a	-45	t	21.11	6.07	6.88	0.0508	0.0508	0.1910	20.46	6.71	-7.52	0.1463	-0.0448	0.0000
			b	21.11	6.07	6.88	0.0508	0.0508	0.1910	20.46	6.71	-7.52	0.1463	-0.0448	0.0000
7	a	45	t	21.11	6.07	-6.88	0.0508	0.0508	-0.1910	20.46	6.71	7.52	0.1463	-0.0448	0.0000
			b	21.11	6.07	-6.88	0.0508	0.0508	-0.1910	20.46	6.71	7.52	0.1463	-0.0448	0.0000
8	a	0	t	55.55	-0.08	0.00	0.1463	-0.0448	0.0000	55.55	-0.08	0.00	0.1463	-0.0448	0.0000
			b	55.55	-0.08	0.00	0.1463	-0.0448	0.0000	55.55	-0.08	0.00	0.1463	-0.0448	0.0000

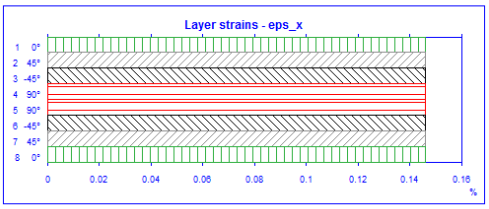
Actual stress, Actual (=Eq.) strain



Actual stress, Actual (=Eq.) strain







Laminate : **C 04590S**
Modified : Sun Nov 11 17:28:27 2012

Lay-up : (0a/+45a/-45a/90a)SE h = 1.84 mm

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Q_x = 0 N/m
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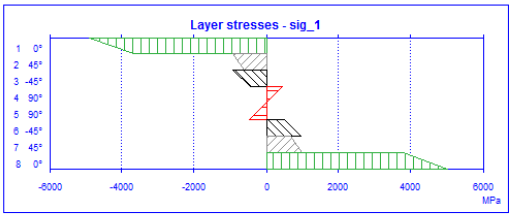
N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

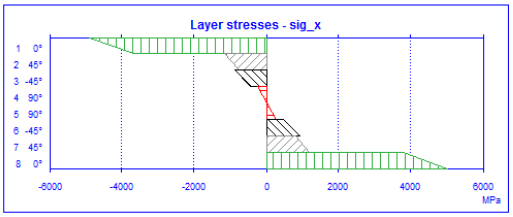
tous les plis se déforment de la même quantité dans la direction de la charge

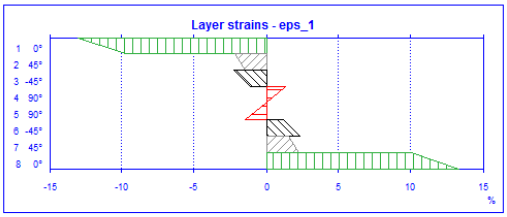
Un moment Mx est superposé à la charge axiale:

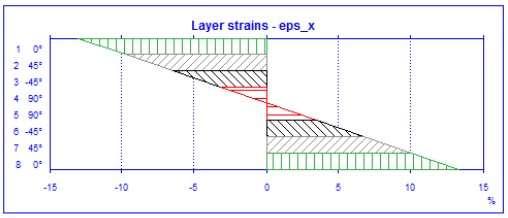
Actual stress, Actual (=Eq.) strain



Actual stress, Actual (=Eq.) strain







Laminate : C 04590S
Modified : Sun Nov 11 17:28:27 2012

Lay-up : (0a/+45a/-45a/90a)SE h = 1.84 mm

Ply
a E;Epoxy;UD-.230/299/50

Load : 5kN sur 10 cm et Mx500Nm sur 25 cm
Modified : Sun Nov 11 19:20:02 2012
Type : Forces and moments (Var.;E)

N_x = 50000 N/m M_x = 2000 Nm/m
N_y = 0 N/m M_y = 0 Nm/m
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

Laminate : C 04590S
Modified : Sun Nov 11 17:28:27 2012

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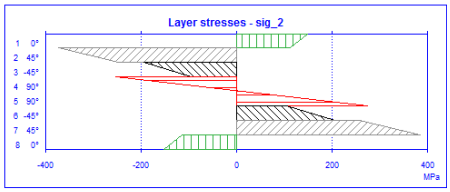
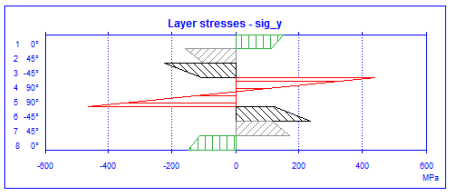
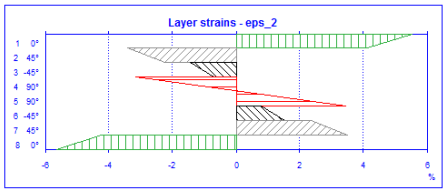
Q_x = 0 N/m
Q_y = 0 N/m

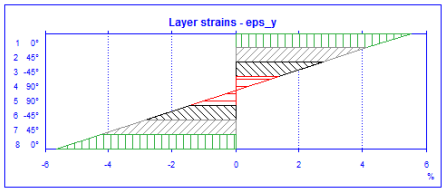
Actual stress, Actual (=Eq.) strain



Actual stress, Actual (=Eq.) strain







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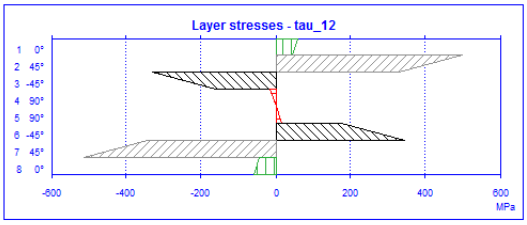
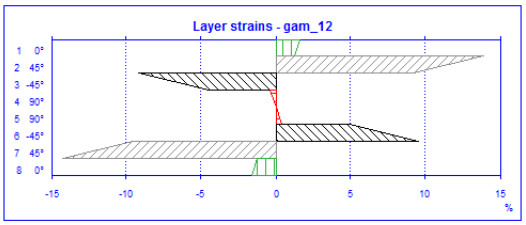
N_xy = 0 N/m M_xy = 0 Nm/m

Q_x = 0 N/m
Q_y = 0 N/m

Distributions des contraintes et déformations : selon 1 direction des fibres, 2 perpendiculaires aux fibres et selon x,y direction des charges appliquées

Actual stress, Actual (=Eq.) strain





Laminate : **C 04590S**
Modified : Sun Nov 11 17:28:27 2012

Lay-up : (0a/+45a/-45a/90a)SE h = 1.84 mm

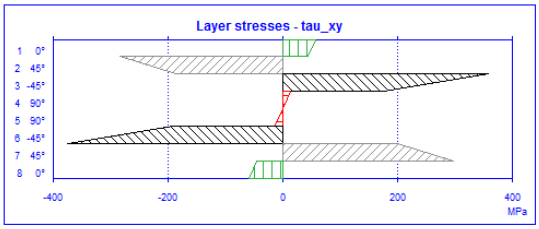
Ply
a E;Epoxy;UD-.230/299/50

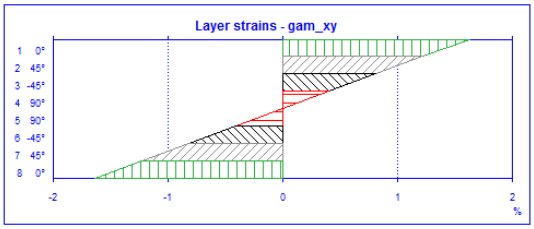
Load : **5kN sur 10 cm et Mx500Nm sur 25 cm**
Modified : Sun Nov 11 19:20:02 2012
Type : Forces and moments (Var.;E)

N_x	= 50000 N/m	M_x	= 2000 Nm/m
N_y	= 0 N/m	M_y	= 0 Nm/m
N_xy	= 0 N/m	M_xy	= 0 Nm/m

Q_x	= 0 N/m
Q_y	= 0 N/m

Actual stress, Actual (=Eq.) strain





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Q_x	= 0 N/m
Q_y	= 0 N/m

Contraintes et déformation de cisaillement selon 1,2 et x,y