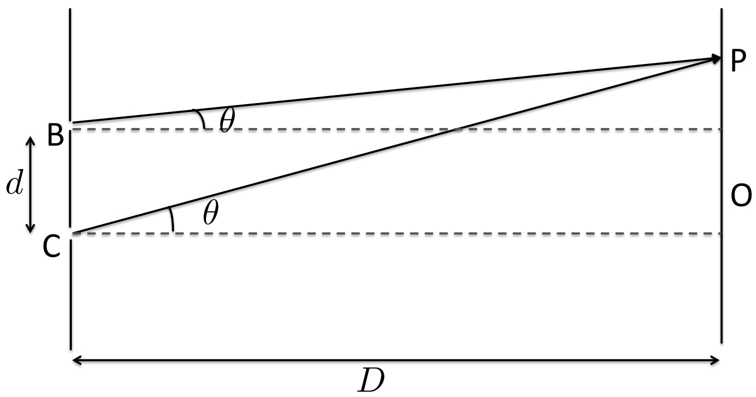
Homework 1
Traitement Quantique de l'Information

Exercise 1 *The Young double slit experiment (1803)*

In this exercise we want to calculate the form of the Young interference fringes. A beam of monochromatic light of wave length λ is sent through a double slit, and the light is reflected on a screen at a distance D. The distance between the two slits is d.



We assume that the waves diffracted by each slit have a spherical shape (λ the wavelength and ν the frequency):

$$\phi_B(\vec{r}) = A\frac{e^{i\left(\frac{2\pi}{\lambda}|\vec{r}_B-\vec{r}|-2\pi\nu t\right)}}{|\vec{r}_B-\vec{r}|}, \quad \phi_C(\vec{r}) = A\frac{e^{i\left(\frac{2\pi}{\lambda}|\vec{r}_C-\vec{r}|-2\pi\nu t\right)}}{|\vec{r}_C-\vec{r}|}.$$

The total wave function at P on the screen is

$$\psi(\vec{r}_P) = \phi_B(\vec{r}_P) + \phi_C(\vec{r}_P).$$

We will use the plane wave approximation for $D >> d$:

$$\psi(\vec{r}_P) \simeq \frac{A}{D}e^{-2\pi i\nu t}\left(e^{\frac{2\pi i}{\lambda}|\vec{r}_B-\vec{r}_P|} + e^{\frac{2\pi i}{\lambda}|\vec{r}_C-\vec{r}_P|}\right).$$

1) Show that the intensity at P on the screen is equal to

$$|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right).$$

Hint: evaluate first the path difference $|\vec{r}_C-\vec{r}_P| - |\vec{r}_B-\vec{r}_P|$ for $D >> d$.

2) Find the condition on $\sin\theta$ which leads to minima and maxima of the intensity on the screen.

3) Let ρ be the coordinate on the screen measured from O. We have $\tan\theta \approx \frac{\rho}{D}$ and since θ is small $\theta \approx \frac{\rho}{D}$. Compute the distance between two successive minima of the intensity pattern on the screen.
   Let $d = 0.25$mm, $D = 10$m and $\lambda = 652$nm (red light). What is the distance between two successive minima?

Exercise 2 *Modern Young's experiment*

Young's double slit experiment has been reformed with Carbon 60 molecules, C_{60} in 1999. Surprisingly, these molecules behave like waves when they are well isolated from their environment. The more recent experiments have evidenced such a wave-like behavior for bigger molecules with 400 to 1000 atoms.

The diameter of C_{60} (this molecule has the form of a sphere and contains 60 carbon atoms) is approximately 0.7 nm and a mole containing $N_A = 6.022 \times 10^{23}$ carbon atoms weights 12 grams.

1) Compute the De Broglie wavelength of molecules produced in an oven which have an average velocity of 220m/s. Compare with the size of individual molecules.

2) We perform a Young's experiment with $d = 100$nm and $D = 1.25m$. What do we observe on the screen assuming a wave like behavior?

3) A football weights approximately 450g and the initial velocity of a professional shoot can attain 100km/h. Estimate the De Broglie wavelength.

Exercise 3 *Photoelectric effect*

The maximal wavelength to extract a photoelectron from tungsten is 230nm (ultraviolets). What is the necessary wavelength of light to extract electrons with kinetic energy 1.5eV? What is the speed of these electrons?

<u>Useful Constant</u>
$c = 2.997 \times 10^8$ m/s (speed of light)
$\hbar = 1.054 \times 10^{-34}$ J $\cdot$ s ($\frac{h}{2\pi}$ where h is Planck's constant)
$m = 9.109 \times 10^{-34}$kg (mass of an electron)
1eV $= 1.6 \times 10^{-19}$J