\textbf{NCAA Lecture 4: Quiz Solutions}

1) \text{b) } \nu=(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \text{ is the closest to } \mu=(\frac{1}{2}, 0, \frac{1}{2}) : \quad \|\nu - \mu\|_\text{TV} = \frac{1}{2}(\frac{1}{3} - \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{2}) = \frac{1}{3} \\
\text{d) } \nu=(0,1,0) \text{ is the farthest to } \mu=(\frac{1}{2}, 0, \frac{1}{2}) : \quad \|\mu - \nu\|_\text{TV} = 1 \quad [\text{and a), c) are both at distance } \frac{1}{2} \text{ }]

2) \text{c) } A=\{1,2\} \text{ is the answer: } \quad \|\mu - \nu\|_\text{TV} = \frac{1}{2}(\frac{1}{2} + 0 + \frac{1}{2}) = \frac{1}{2} \quad \text{and } \quad \mu(A) - \nu(A) = \frac{1}{2} - 0 = \frac{1}{2} \\
(\text{Note that A}=\{2\} \text{ would also work, and that A}=\{0,2\} \text{ gives } \mu(A) - \nu(A) = \frac{1}{2} - 1 = -\frac{1}{2}, \text{ so } \|\mu(A) - \nu(A)\| = \|\mu - \nu\|_\text{TV})
3) (a), (d), (e) and (f) are couplings of \( M \) & \( \nu 

(a) is the "grand coupling": \( \Pr (X = Y) = 1 \)

(e) is the "statistical coupling": \( X \) & \( Y \) are independent

(d) is something in between (positive correlation between \( X \) & \( Y \))

(f) is the case where \( X, Y \) are the most negatively correlated as possible

For (b) and (c), computing the marginals \( \mu_0 = \rho_{00} + \rho_{01} \) etc. does not lead to the desired values for \( M \) & \( \nu \).

Subsidiary question: If \( \mu - \nu \perp \nu_{TV} = 0 \), and the only coupling for which \( \Pr (X \neq Y) = 0 \) is the "grand coupling" (a).