1) On $S = \{0, 1, 2\}$, which of the following distributions has the smallest/largest total variation distance from $\mu = (\frac{1}{2}, 0, \frac{1}{2})$?

   a) $\nu = (1, 0, 0)$
   b) $\nu = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
   c) $\nu = (\frac{1}{2}, \frac{1}{2}, 0)$
   d) $\nu = (0, 1, 0)$

2) On $S = \{0, 1, 2\}$, let $\mu = (\frac{1}{2}, 0, \frac{1}{2})$ and $\nu = (1, 0, 0)$. For which $A \subseteq S$ does it hold that $\|\mu - \nu\|_{TV} = \mu(A) - \nu(A)$?

   a) $A = \{0, 1\}$
   b) $A = \{0, 2\}$
   c) $A = \{1, 2\}$
   d) $A = \{0, 1, 2\}$
3) On $S=\{0,1\}$, let $\mu = \nu = \left(\frac{1}{4}, \frac{3}{4}\right)$. Let also $(x,y)$ be a couple of random variables with values in $S \times S$ such that $\Pr(x=y=0) = p_{00}$, $\Pr(x=0, y=1) = p_{01}$, $\Pr(x=1, y=0) = p_{10}$ and $\Pr(x=y=1) = p_{11}$. In which case(s) is $(x,y)$ a coupling of $\mu$ and $\nu$?

a) $p_{00} = \frac{1}{4}$, $p_{11} = \frac{3}{4}$, $p_{01} = p_{10} = 0$

b) $p_{00} = \frac{1}{8}$, $p_{11} = \frac{3}{8}$, $p_{01} = p_{10} = \frac{1}{4}$

c) $p_{00} = \frac{1}{8}$, $p_{11} = \frac{3}{8}$, $p_{01} = \frac{1}{8}$, $p_{10} = \frac{3}{8}$

d) $p_{00} = \frac{1}{8}$, $p_{11} = \frac{5}{8}$, $p_{01} = p_{10} = \frac{1}{8}$

e) $p_{00} = \frac{1}{16}$, $p_{11} = \frac{9}{16}$, $p_{01} = p_{10} = \frac{3}{16}$

f) $p_{00} = 0$, $p_{11} = \frac{1}{2}$, $p_{01} = p_{10} = \frac{1}{2}$

Subsidiary question:

For which coupling(s) is it the case that $\Pr(\mu - \nu \mid \tau_{uv}) = \Pr(x \neq y)$?