Advanced probability and applications

Summary
In this course, various aspects of probability theory are considered. The first part is devoted to the main theorems in the field (law of large numbers, central limit theorems), while the second part focuses on the theory of martingales in discrete time.

Content
I. Probability
- sigma-fields, probability measures, random variables
- independence, expectation
- convergence of sequences of random variables
- laws of large numbers, central limit theorem
- concentration inequalities
- moments
II. Martingales
- conditional expectation
- definition and properties of a martingale
- stopping times, optional stopping theorem
- maximal inequalities
- convergence theorems

Keywords
probability, measure theory, martingales, convergence theorems

Learning Prerequisites
Required courses
Basic probability course
Calculus courses

Recommended courses
complex analysis

Important concepts to start the course
This course is NOT an introductory course on probability: the students should have a good understanding and practice of basic probability concepts such as: distribution, expectation, variance, independence, conditional probability.
The students should also be at ease with calculus. Complex analysis is a plus, but is not required.
On the other hand, no prior background on measure theory is needed for this course: we will go through the
basic concepts one by one at the beginning.

**Learning Outcomes**

By the end of the course, the student must be able to:

- Understand the foundations of probability theory
- Acquire a solid knowledge of martingale theory

**Teaching methods**

Ex cathedra + exercises

**Expected student activities**

active participation to exercise sessions

**Assessment methods**

Midterm 20%, homeworks 20%, exam 60%

**Resources**

**Bibliography**


**Ressources en bibliothèque**

- A Second Course in Probability / Ross
- A First Look at Rigorous Probability Theory / Rosenthal
- Probability and Random Processes / Grimmett
- Probability: Theory and Examples / Durrett

**Notes/Handbook**

available on the course website

**Websites**


**Prerequisite for**

Advanced classes requiring a good knowledge of probability