IMAGE FORMATION

Light source properties

Sensor characteristics

Surface shape

Exposure

Optics

Surface reflectance properties
ANALOG IMAGES

An image can be understood as a 2D light intensity function $f(x,y)$ where:

- $x$ and $y$ are spatial coordinates
- The value of $f$ at any point $(x, y)$ is proportional to the brightness or gray value of the image at that point.

→ Cannot be stored as such on a digital computer.
A digitized image is one in which:

- Spatial and grayscale values have been made discrete.
- Intensities measured across a regularly spaced grid in x and y directions are sampled to
  - 8 bits (256 values) per point for black and white,
  - 3x8 bits per point for color images.

They are stored as a two dimensional arrays of gray-level values. The array elements are called pixels and identified by their x, y coordinates.
Projection from surfaces to 2-D sensor.

- Where: Geometry
- How bright: Radiometry
- Stored how: Sensing
PINHOLE CAMERA MODEL

Idealized model of the perspective projection:

• All rays go through a hole and form a pencil of lines.

• The hole acts as a ray selector that allows an inverted image to form.
MAGNET LIKE SLOPES

Impossible motion: magnet-like slopes
VIRTUAL IMAGE
CAMERA GEOMETRY

Pinhole geometry without image reversal
COORDINATE SYSTEMS

World, Camera, Image Coordinate Systems

World Coordinate System:

\((X_w, Y_w, Z_w)\)

Camera Coordinate System:

\((X_c, Y_c, Z_c)\)

Image Coordinate System:

\((X_i, Y_i, Z_i)\)
CAMERA COORDINATE SYSTEM

- The center of the projection coincides with the origin of the world.
- The camera axis (optical axis) is aligned with the world’s z-axis.
- To avoid image inversion, the image plane is in front of the center of projection.
\[ \frac{X_f}{f} = \frac{X_c}{Z_c} \]

\[ X_f = f \frac{X_c}{Z_c} \]
\[ X_i = f \frac{X_c}{Z_c} \]
\[ Y_i = f \frac{Y_c}{Z_c} \]
DISTANT OBJECTS APPEAR SMALLER
PARALLEL LINES MEET
VANISHING POINTS

- The projections of parallel lines all meet at one point, called the vanishing point.
- As focal length and distance to camera increase, the image remains the same size but perspective effects diminish.
ROAD FOLLOWING

Rasmussen et al., BMVC’04
Reformulate it as a linear operation.

\[ u = X_i = f \frac{X_c}{Z_c} \]

\[ v = Y_i = f \frac{Y_c}{Z_c} \]
HOMOGENEOUS COORDINATES

Homogeneous representation of 2D point:

\[ \mathbf{x} = (x_1, x_2, x_3) \text{ represents } (x_1/x_3, x_2/x_3) \]

Homogeneous representation of 3D point:

\[ \mathbf{X} = (x_1, x_2, x_3, x_4) \text{ represents } (x_1/x_4, x_2/x_4, x_3/x_4) \]

→ Projections become linear transformations.
SIMPLE PROJECTION MATRIX

\[
\begin{bmatrix}
x \\ y \\ z
\end{bmatrix} = \begin{bmatrix}
f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0
\end{bmatrix}\begin{bmatrix}
X_c \\ Y_c \\ Z_c \\ 1
\end{bmatrix}
\]

with
\[
X_i = \frac{x}{z} = f \frac{X_c}{Z_c}
\]
and
\[
Y_i = \frac{y}{z} = f \frac{Y_c}{Z_c}
\]
INTRINSIC AND EXTRINSIC PARAMETERS

- Camera may not be at the origin, looking down the z-axis
  → Extrinsic parameters
- One unit in camera coordinates may not be the same as one unit in world coordinates
  → Intrinsic parameters

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
\text{Matrix of intrinsic parameters}
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
\text{Matrix of extrinsic parameters}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]
LINEAR CAMERA MODEL

\[
\begin{bmatrix}
  x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
  \text{Matrix of intrinsic parameters} \\
\end{bmatrix} \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
  \text{Matrix of extrinsic parameters} \\
\end{bmatrix} \begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]

\[
= K \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]

where \( K \) is a \( 3 \times 3 \) matrix and \( \text{Rt} \) a \( 4 \times 4 \) matrix.
PRINCIPAL POINT

\[ u = X_i + p_u = fX/Z + p_u \]

\[ v = Y_i + p_v = fY/Z + p_v \]

\[ K = \begin{bmatrix} f & 0 & p_u \\ 0 & f & p_v \\ 0 & 0 & 1 \end{bmatrix} \]
INHOMOGENEOUS SCALING

\[
\begin{align*}
  u &= \alpha_u X_i + p_u = \alpha_u X/Z + p_u \\
  v &= \alpha_v Y_i + p_v = \alpha_v Y/Z + p_v
\end{align*}
\]

\[
K = \begin{bmatrix}
  \alpha_u & 0 & p_u \\
  0 & \alpha_v & p_v \\
  0 & 0 & 1
\end{bmatrix}
\]

The pixels are not necessarily square.
\( s \) encodes the non-orthogonality of the \( u \) and \( v \) directions. Very close to zero in modern cameras.
Rotations and translations also expressed in terms of matrix multiplications in projective space.
FULL PROJECTION MATRIX

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
a_u & s & p_u \\
0 & a_v & p_v \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

with \( T = -R\tilde{C} \) and \( R'R = I \)

\[
= \begin{bmatrix}
p_{11} & p_{12} & p_{13} & p_{14} \\
p_{21} & p_{22} & p_{23} & p_{24} \\
p_{31} & p_{32} & p_{33} & 1
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

\[
x = K[R \mid -R\tilde{C}]X = KR[I \mid -\tilde{C}]X = [M \mid m]X = PX
\]

Hartley, Chap 6.
CAMERA CALIBRATION

Internal Parameters:
• Horizontal and vertical scaling (2)
• Principal points (2)
• Skew of the axis (1)

External Parameters:
• Rotations (3)
• Translations (3)

→ 11 free parameters.
LIMITATIONS

Idealization because the hole cannot be infinitely small
- Image would be infinitely dim
- Diffraction effects
→ Use of Lenses.
An ideal lens realizes the same projection as a pinhole but gathers much more light!
THIN LENS PROPERTIES

- Any incident ray traveling parallel to the optical axis, will refract and travel through the focal point on the opposite side of the lens.
- Any incident ray traveling through the focal point on the way to the lens will be refracted and travel parallel to the principal axis.
- An incident ray which passes through the center of the lens will in effect continue in the same direction that it had when it entered the lens.
- All rays emanating from P and entering the lens will converge at P’.
CAMERA OBSCURA

- Used by painters since the Renaissance to produce perspective projections.
- Direct ancestors to the first film cameras.
He clearly knew all about the perspective transform!
SHIFTING PERSPECTIVE

China, 8th century:

- The focal point moves from one part of the image to the other.
- The characters are always seen at eye-level as the picture is unrolled.
THIN LENS EQUATION

→ Lens with focal distance $f$ equivalent to pinhole camera with similar focal distance but larger aperture.
DEPTH OF FIELD vs APERTURE

Large Aperture:
• Large blur circles
• Shallow depth of field

Small Aperture:
• Low intensity
• Long exposure time
DEPTH OF FIELD

- Range of object distances (d-d') over which the image is sufficiently well focused.
- Range for which blur circle is less than the resolution of the sensor.

Small focal length —> Large depth of field.
APERTURE

Diameter $d$ of the lens that is exposed to light.
• Simple geometry:
  \[ r_b = \frac{a}{s'} \cdot |s' - s| \]

• Thin lens equation:
  \[ \frac{1}{d} + \frac{1}{s} = \frac{1}{f} \Rightarrow s = \frac{df}{d - f} \]
  \[ \frac{1}{d'} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{d'f}{d' - f} \]
  \[ (s' - s) = \frac{f}{(d' - f)} \cdot \frac{f}{(d - f)} \cdot (d - d') \]
CHANGING APERTURE

Small aperture, long exposure.

\[ r_b = \frac{a}{s'} \left| \frac{f^2}{(d' - f)(d - f)} \right| \]

Small \( a \) \( \rightarrow \) Small \( r_b \)
CHANGING FOCAL LENGTH

Wide field of view
(small f)

Narrow field of view
(large f)

\[ r_b = \frac{a}{s'} \left| \frac{f^2}{(d' - f)(d - f)} \right| \]

Small \( f \) \( \rightarrow \) Small \( r_b \)
DISTORTIONS

The lens is not exactly a “thin lens:”

- Different wave lengths refracted differently
- Barrel Distortion
Different wavelengths are refracted differently.
RADIAL LENS DISTORTIONS

Radial distance from Image Center:

\[ r_u = r_d + k_1 r_d^3 \]
Aberrations can be minimized by aligning several lenses with well chosen
• Shapes,
• Refraction indices.
Once the image is undistorted, the camera projection can be formulated as a projective transform.

→ The pinhole camera model applies.
**Scene Radiance:** Amount of light radiation from a surface point (Watt / m² / Steradian)

**Image Irradiance:** Amount of light incident at the image of the surface point. (Watt / m²)

Fundamental Radiometric Equation:

\[
\text{Irr} = \frac{\pi}{4} \left(\frac{d}{f}\right)^2 \cos^4(\alpha) \text{Rad}
\]
Images can get darker towards their edges because some of the light does not go through all the lenses.
DE VIGNETTING

Y. Zheng, S. Lin, and S.B. Kang, CVPR’08
SENSOR ARRAY

- Photons free up electrons that are then captured by a potential well.
- Charges are transferred row by row wise to a register.
- Pixel values are read from the register.
SENSING

Conversion of the “optical image” into an “electrical image”:

\[ E(x, y) = \int_{t_0}^{t_1} \int_{0}^{\Lambda} \text{Irr}(x, y, t, \lambda) s(\lambda) dt d\lambda \]

\[ I(m, n) = \text{Quantize}(\int_{x_0}^{x_1} \int_{y_0}^{y_1} E(x, y) dx dy) \]

→ Quantization in
- Time
- Space
IN SHORT

• Camera geometry can be modeled in terms of the pinhole camera model, which is linear in projective space.

• Image radiance is roughly proportional to surface radiance and the two can be used interchangeably for our purposes.