Multi-period optimisation problems

- Type of problems
  - Periodic
    - same environment, same requirement but independent
    - defined by:
      - conditions(p), requirement(p) for duration(p)
        - ex. typical days, day/night, summer/winter
    - optimal operating conditions for each period
    - storage is not considered
  - Operation horizon
    - defined by:
      - conditions(t), requirement(t) @ time(t)
        - ex. daily profiles, batch processes recipe
    - Scheduling: when to produce what, where
    - Optimal management inc. storage
Targeting the integration of multi-period utility systems for site scale process integration

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Problem definition

- Industrial site

Existing steam network

Import export

Boilers

Turbines

5 process sections
Process Requirements

- 5 processes:

  P1 Heating: 0 kW
  Cooling: 3562 kW

  P2 Heating: 25932 kW
  Cooling: 0 kW

  SITE reference
  Heating: 22560 kW
  (38510 kW)
  Cooling: 17732 kW
  (33682 kW)

  P3 Heating: 0 kW
  Cooling: 5491 kW

  P4 Heating: 0 kW
  Cooling: 24629 kW

  P5 Heating: 12578 kW
  Cooling: 0 kW
Total site composite

Integrated SITE
Heating : 9565 kW (22560 kW)
Cooling : 4050 kW (17732 kW)
Refrigeration : 686 kW
Interest of integration

- Representation with all the hot and cold streams
  - System sub-divisions
  - No abstraction of pockets potentials

Heat pump not useful for P3

Heat pump saving potential for total site: 2957 kW (30%)
Production levels variations

Market conditions
Productions shifts
Batch
Efficiency variations
Maintenance
Cleaning procedures
Process requirement

Nominal
- Heating: 9565 kW
- Cooling: 4050 kW
- Refrigeration: 686 kW

Average
- Heating: 10270 kW
- Cooling: 386 kW
- Refrigeration: 261 kW

Heat pump?  CHP?  Refrigeration?
Utility system selection

Gas turbine

Process Heat + CHP - Heat pump?

Integrated SITE
- Heating: 9565 kW
- Cooling: 4050 kW
- Refrigeration: 686 kW
Multi-period principle

• Process annual operation defined by a set of operating periods
  – Limited number of sets: \( n_p \)
  – For each set we have to define the operating conditions of utility system

• Definition of operating period \( p \)
  – \( L_{j,p} \): production levels or conditions for process \( j \)
  – \( t_p \): operating time (h/years)

• Assume no heat storage
  – Sequence of operations not constraining
Load scenarios calculation

\[
\min_{L_{j,p}} \sum_{t=1}^{n_t} \left\{ \min_{p=1, \ldots, n_p} \sum_{j=1}^{n_j} \left( \frac{L_{j,p} - q_{j,t}}{\omega_j} \right)^2 \right\}
\]

\begin{align*}
np &= 5 \\
n_t &= 8760 \\
n_j &= 10
\end{align*}
Load scenario model

- **Problem characteristics**
  - Discontinuous
  - Non-linear
  - Multi-modal

- **Solved by a Evolutionary Algorithm**
  - Easy implementation
  - Robustness
  - Multi-objectives
    - Validity for each process
    - Trade-off
      - Number of representing levels
      - Individual errors
      - Overall error
  - ? Computation time
# Multi-period levels

Minimum energy requirements and levels of operations in the different periods

<table>
<thead>
<tr>
<th></th>
<th>MER hot (kW)</th>
<th>MER cold (kW)</th>
<th>MER frg (kW)</th>
<th>time h/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>22581</td>
<td>0</td>
<td>155</td>
<td>432</td>
</tr>
<tr>
<td>L2</td>
<td>0</td>
<td>18119.7</td>
<td>1027.8</td>
<td>48</td>
</tr>
<tr>
<td>L3</td>
<td>19814</td>
<td>987</td>
<td>347</td>
<td>192</td>
</tr>
<tr>
<td>L4</td>
<td>10146</td>
<td>1411</td>
<td>380</td>
<td>264</td>
</tr>
<tr>
<td>L5</td>
<td>27771</td>
<td>0</td>
<td>0</td>
<td>720</td>
</tr>
<tr>
<td>L6</td>
<td>13311</td>
<td>4897</td>
<td>751</td>
<td>144</td>
</tr>
<tr>
<td>L7</td>
<td>0</td>
<td>10526</td>
<td>677</td>
<td>240</td>
</tr>
<tr>
<td>L8</td>
<td>12252</td>
<td>173</td>
<td>311</td>
<td>1224</td>
</tr>
<tr>
<td>L9</td>
<td>7061.7</td>
<td>831</td>
<td>310</td>
<td>2904</td>
</tr>
<tr>
<td>L10</td>
<td>8539</td>
<td>577</td>
<td>278</td>
<td>2592</td>
</tr>
<tr>
<td>Average</td>
<td>10270</td>
<td>385.6</td>
<td>261</td>
<td>8760</td>
</tr>
</tbody>
</table>
Multi-period model principle

Objective function = \( \sum_i y_i \ (\text{Inv}_i(fsize_i)) + \sum_i \sum_t f_{ti} \ * C_{t1} \ * \text{Time}_t \)

\( f_{size_i} > f_{ti} \ \forall \ t \)

\( y_i > y_{ti} \ \forall \ t \)

Utility i

Use in period t: \( y_{it} \)

Flow in period t: \( f_{it} \)

- List_A
  - Time = 8000 h/years

- List_stopped
  - Time = 500 h/years

- List_min
  - Time = 1 h

- List_max
  - Time = 1 h
Multi-period problem

Generic formulation

\[
\begin{align*}
Min & \sum_{t=1}^{n_t} t_t * c(x_t, s) + I(y, s) \\
\text{Submitted to:} & \\
& h_t(x_t, s) = 0, \quad \forall t = 1, \ldots, n_t \\
& g_t(x_t, s) \geq 0, \quad \forall t = 1, \ldots, n_t \\
& y_t \leq y, \quad x_t \leq x, \quad \forall t = 1, \ldots, n_t \\
& y_t, y \in \{0, 1\}, \quad \forall t = 1, \ldots, n_t
\end{align*}
\]

Only s (size => investment) and y (investment decisions) are shared between periods
**Multi-period MILP problem formulation**

\[
\begin{align*}
\text{Operating cost} & \quad \min_{R^p, y^p, f^p, y^w, f^w} \sum_{p=1}^{n_p} \sum_{w=1}^{n_w} \left( C^2_w f^p_w + C_{ei}^p W_e^p - C_{el}^p W_{el}^p \right) t_p \\
& \quad + \sum_{w=1}^{n_w} \left( C_{1_w} y^w + \frac{1}{T} \sum_{w=1}^{n_w} (ICF_w y^w + ICP_w f^w) \right)
\end{align*}
\]

**For each period**

**Heat Cascade**

\[
\sum_{w=1}^{n_w} f^p_w q_{w,r} + \sum_{i=1}^{n} Q_{i,r} \cdot L_{i,p} + R^p_{r+1} - R^p_r = 0 \\
R^p_1 = 0, R^p_{n_r+1} = 0, R^p_r \geq 0
\]

**Electricity Consumption**

\[
\sum_{w=1}^{n_w} f^p_w w_w + W_e^p - L_{c,p} \cdot W_c \geq 0
\]

**Electricity Production**

\[
\sum_{w=1}^{n_w} f^p_w w_w + W_e^p - W_{el}^p - L_{c,p} \cdot W_c = 0 \\
W_e^p \geq 0, W_{el}^p \geq 0
\]

**Part load operation**

\[
f_{min} y^p_w \leq f^p_w \leq f_{max} y^p_w \\
y^p_w \in [0, 1]
\]

**Size**

\[
f^w_{max} - f^p_w \geq 0
\]

**Decision**

\[
y^w_{max} - y^p_w \geq 0
\]
Application

- 5 Processes (60 streams)
- 10 operation levels
- 4 Gas turbines
- Refrigeration system
- Steam network
- Heat pump

- MILP problem characteristics
  - 1900 constraints
  - 1600 variables
  - 50 decision variables
# Results

Table 4  
Results with different gas turbines, Natural gas price : 0,135 €/kg, Electricity : 35€/MWh(high)  

<table>
<thead>
<tr>
<th></th>
<th>Electricity (MWh)</th>
<th>Fuel (MWh)</th>
<th>Total cost (€/year)</th>
<th>CHP Eff (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GT_3_{low}$</td>
<td>116494</td>
<td>248996</td>
<td>263893</td>
<td>78.6</td>
<td>GT shut down</td>
</tr>
<tr>
<td>$GT_1_{int}$</td>
<td>90303</td>
<td>210622</td>
<td>-226811</td>
<td>82.2</td>
<td>Min post combustion</td>
</tr>
<tr>
<td>$GT_3_{int}$</td>
<td>143949</td>
<td>304960</td>
<td>-78213</td>
<td>70.5</td>
<td>Min post combustion</td>
</tr>
<tr>
<td>$GT_4_{int}$</td>
<td>113220</td>
<td>243344</td>
<td>-220005</td>
<td>79.4</td>
<td>Min post combustion</td>
</tr>
<tr>
<td>$GT_1_{high}$</td>
<td>139690</td>
<td>356721</td>
<td>-377481</td>
<td>54.7</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_2_{high}$</td>
<td>137665</td>
<td>354075</td>
<td>-209181</td>
<td>54.4</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_3_{high}$</td>
<td>192503</td>
<td>451327</td>
<td>-196827</td>
<td>54.9</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_4_{high}$</td>
<td>166219</td>
<td>400155</td>
<td>-381427</td>
<td>55.5</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_{1_{int}}^{hpmp}$</td>
<td>88428</td>
<td>204451</td>
<td>-220910</td>
<td>85.3</td>
<td>Min post combustion +hpmp</td>
</tr>
</tbody>
</table>

$$CHP\ eff = \frac{E_{l}(MWh) - \frac{M_{er}(MWh)}{\eta_{th}}}{Fuel(MWh)}$$ with $\eta_{th} = 95\%$ the efficiency of a conventional boiler
## Solution with heat pump

Details of the solution computed with $GT_{int}^{hpmp}$

<table>
<thead>
<tr>
<th></th>
<th>CND</th>
<th>HPMP</th>
<th>GT</th>
<th>CHP</th>
<th>FUEL</th>
<th>EFF</th>
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<tbody>
<tr>
<td></td>
<td>kmol/s</td>
<td>kW</td>
<td>kW</td>
<td>kW</td>
<td>kW</td>
<td>%</td>
</tr>
<tr>
<td>L1</td>
<td>0.013</td>
<td>OFF</td>
<td>5515.0</td>
<td>11455.0</td>
<td>36640.8</td>
<td>89.0</td>
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<tr>
<td>L2</td>
<td>0.349</td>
<td>45.7</td>
<td>5515.0</td>
<td>13679.0</td>
<td>16849.8</td>
<td>81.2</td>
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<tr>
<td>L3</td>
<td>OFF</td>
<td>130.2</td>
<td>5515.0</td>
<td>10359.0</td>
<td>31916.7</td>
<td>93.7</td>
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<tr>
<td>L4</td>
<td>OFF</td>
<td>110.1</td>
<td>5515.0</td>
<td>7967.3</td>
<td>19829.7</td>
<td>87.1</td>
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<tr>
<td>L5</td>
<td>0.020</td>
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<td>5515.0</td>
<td>12404.0</td>
<td>43015.1</td>
<td>90.0</td>
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<tr>
<td>L6</td>
<td>OFF</td>
<td>327.9</td>
<td>5515.0</td>
<td>9403.0</td>
<td>22696.2</td>
<td>108.3</td>
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<tr>
<td>L7</td>
<td>0.309</td>
<td>172.1</td>
<td>5515.0</td>
<td>11246.0</td>
<td>16849.8</td>
<td>66.7</td>
</tr>
<tr>
<td>L8</td>
<td>OFF</td>
<td>OFF</td>
<td>5515.0</td>
<td>10379.0</td>
<td>25110.5</td>
<td>85.0</td>
</tr>
<tr>
<td>L9</td>
<td>OFF</td>
<td>70.2</td>
<td>5515.0</td>
<td>9875.0</td>
<td>19040.4</td>
<td>85.1</td>
</tr>
<tr>
<td>L10</td>
<td>OFF</td>
<td>61.5</td>
<td>5515.0</td>
<td>9400.5</td>
<td>20115.5</td>
<td>84.5</td>
</tr>
</tbody>
</table>
### Part load optimal operation target

Details of the solution computed with $GT_{1_{int}}$

<table>
<thead>
<tr>
<th></th>
<th>GT (MWh)</th>
<th>CHP power (kW)</th>
<th>CHP total (MWh)</th>
<th>Fuel (MWh)</th>
<th>CHP Efficiency (%)</th>
<th>Condensing kmol/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>2382</td>
<td>11493</td>
<td>4965</td>
<td>15867</td>
<td>88.7</td>
<td>0.013</td>
</tr>
<tr>
<td>L2</td>
<td>265</td>
<td>13660</td>
<td>656</td>
<td>808</td>
<td>81.1</td>
<td>0.34</td>
</tr>
<tr>
<td>L3</td>
<td>1059</td>
<td>10879</td>
<td>2089</td>
<td>6457</td>
<td>85.2</td>
<td>0.0</td>
</tr>
<tr>
<td>L4</td>
<td>1456</td>
<td>8407</td>
<td>2219</td>
<td>5618</td>
<td>79.3</td>
<td>0.0</td>
</tr>
<tr>
<td>L5</td>
<td>3970</td>
<td>12404</td>
<td>8931</td>
<td>30971</td>
<td>90.0</td>
<td>0.02</td>
</tr>
<tr>
<td>L6</td>
<td>794</td>
<td>10775</td>
<td>1552</td>
<td>3900</td>
<td>82.4</td>
<td>0.0</td>
</tr>
<tr>
<td>L7</td>
<td>1323</td>
<td>11178</td>
<td>2683</td>
<td>4043</td>
<td>66.3</td>
<td>0.27</td>
</tr>
<tr>
<td>L8</td>
<td>6750</td>
<td>10379</td>
<td>12704</td>
<td>30735</td>
<td>85.0</td>
<td>0.0</td>
</tr>
<tr>
<td>L9</td>
<td>16015</td>
<td>10158</td>
<td>29499</td>
<td>57981</td>
<td>81.1</td>
<td>0.0</td>
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<tr>
<td>L10</td>
<td>14295</td>
<td>9647,6</td>
<td>25006</td>
<td>54240</td>
<td>80.8</td>
<td>0.0</td>
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<tr>
<td>Total</td>
<td>48311</td>
<td></td>
<td>90303</td>
<td>210622</td>
<td>82.2</td>
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</table>