Fundamentals of Traffic Operations and Control

Distribution Systems

École Polytechnique Fédérale de Lausanne
Urban Transport Systems Laboratory

Dr. Mor Kaspi

Week 12
December 2017
Outline

➢ The transportation problem
➢ Nestle Challenge
➢ ...

Acknowledgement: Some of the presented material was kindly provided by Prof. Nikolas Geroliminis (EPFL) and Prof. Michal Tzur (Tel-Aviv University).
Total Distribution System

9 Suppliers, 3 Sub-assembly plants, 2 Plants, 5 Warehouses, 52 Retail outlets

- Retail Outlets
- Plants
- Warehouses
- Supply Points
- Sub-assembly plants
Total Distribution System

9 Suppliers, 3 Sub-assembly plants, 2 Plants, 5 Warehouses, 52 Retail outlets

P1

SEA

LA

CHI

ATL

NY

CUSTOMERS
The Transportation Problem

An American company distributes a single product from three production plants in the cities New-York, Chicago and Los-Angeles to five warehouses in the cities: Pittsburgh, Atlanta, Houston, Denver and San-Francisco.

The transportation costs per unit are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Pittsburgh</th>
<th>Atlanta</th>
<th>Houston</th>
<th>Denver</th>
<th>San Francisco</th>
</tr>
</thead>
<tbody>
<tr>
<td>New-York</td>
<td>$2</td>
<td>$3</td>
<td>$4</td>
<td>$5</td>
<td>$8</td>
</tr>
<tr>
<td>Chicago</td>
<td>$2</td>
<td>$4</td>
<td>$3</td>
<td>$3</td>
<td>$7</td>
</tr>
<tr>
<td>Los-Angeles</td>
<td>$8</td>
<td>$6</td>
<td>$4</td>
<td>$3</td>
<td>$2</td>
</tr>
</tbody>
</table>
The Transportation Problem

In addition, the production capacity of each plant is given:

<table>
<thead>
<tr>
<th></th>
<th>New-York</th>
<th>Chicago</th>
<th>Los-Angeles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200</td>
<td>300</td>
<td>500</td>
</tr>
</tbody>
</table>

And so is the demand in each warehouse:

<table>
<thead>
<tr>
<th></th>
<th>Pittsburgh</th>
<th>Atlanta</th>
<th>Houston</th>
<th>Denver</th>
<th>San Francisco</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>150</td>
<td>350</td>
<td>150</td>
<td>250</td>
</tr>
</tbody>
</table>

Formulate the problem as a Linear Programming model!
<table>
<thead>
<tr>
<th>Production Capacity</th>
<th>Transportation Costs</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>300</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td>350</td>
</tr>
</tbody>
</table>

Note: The diagram shows the relationship between production capacity, transportation costs, and demand. Each production capacity is connected to transportation costs, which in turn connect to demand values.
The transportation problem – LP formulation

\(x_{ij}\) - the number of units to be transported from plant \(i\) to warehouse \(j\)

Minimize \(Z =
\begin{align*}
2x_{11} + 3x_{12} + 4x_{13} + 5x_{14} + 8x_{15} + \\
2x_{21} + 4x_{22} + 3x_{23} + 3x_{24} + 7x_{25} + \\
8x_{31} + 6x_{32} + 4x_{33} + 3x_{34} + 2x_{35}
\end{align*}
Subject to

\[ x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 200 \]
\[ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 300 \]
\[ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 500 \]
\[ x_{11} + x_{21} + x_{31} = 100 \]
\[ x_{12} + x_{22} + x_{32} = 150 \]
\[ x_{13} + x_{23} + x_{33} = 350 \]
\[ x_{14} + x_{24} + x_{34} = 150 \]
\[ x_{15} + x_{25} + x_{35} = 250 \]

\[ x_{ij} \geq 0 \quad \forall \ i = 1,2,3 \quad j = 1,2,3,4,5 \]
Model Assumptions

• Each source has a fixed supply which must be distributed to destinations, while each destination has a fixed demand that must be received from the sources
• The cost of distributing commodities from the source to the destination is directly proportional to the number of units distributed

Feasible Solution Property

A transportation problem will have a feasible solution if and only if the sum of the supplies is equal to the sum of the demands.
The General Model of a Transportation Problem

Any problem that attempts to minimize the total cost of distributing units of commodities while meeting the requirement assumption and the cost assumption and has information pertaining to sources, destinations, supplies, demands, and unit costs can be formulated into a transportation model.
Transportation Problem - General LP formulation

**Parameters:**

- $m$ – Number of sources
- $n$ – Number of destinations
- $s_i$ - Supply at source $i$ \[ i = 1, \ldots, m \]
- $d_j$ - Demand at destination $j$ \[ j = 1, \ldots, n \]
- $c_{ij}$ - Unit transportation cost from source $i$ to destination $j$ \[ i = 1, \ldots, m \quad j = 1, \ldots, n \]

**Decision Variables:**

- $x_{ij}$ - the number of units to be transported from source $i$ to destination $j$ \[ i = 1, \ldots, m \quad j = 1, \ldots, n \]
Transportation Problem - General LP formulation

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$\sum_{j=1}^{n} x_{ij} = s_{i} \quad \forall i = 1, \ldots, m$ \hspace{1cm} Supply

$\sum_{i=1}^{m} x_{ij} = d_{j} \quad \forall j = 1, \ldots, n$ \hspace{1cm} Demand

$x_{ij} \geq 0 \quad \forall i, j$
Integer Solutions Property

If all the supplies and demands have integer values, then the transportation problem with feasible solutions is guaranteed to have an optimal solution with integer values for all its decision variables.
Initial Example - Coefficients Matrix

\[
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
Extensions/Adaptations

What if?

• The total supply exceeds the total demand
• The total supply is less than the total demand
• Certain sources may not be able to distribute commodities to certain destinations
• The objective is to maximize profits rather than minimize costs
The Transshipment problem

In this problem, we need to transport a single product from several sources to several destinations (as in the Transportation problem). However, here in addition, there are some intermediate locations that may have their own demand/supply.
Transshipment Problem - Example

<table>
<thead>
<tr>
<th>Production Capacity</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1: 150</td>
<td>Store 1: 130</td>
</tr>
<tr>
<td>Plant 2: 120</td>
<td>Store 2: 120</td>
</tr>
</tbody>
</table>

Transportation costs per unit:

<table>
<thead>
<tr>
<th></th>
<th>Warehouse 1</th>
<th>Warehouse 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1</td>
<td>8$</td>
<td>15$</td>
</tr>
<tr>
<td>Plant 2</td>
<td>15$</td>
<td>25$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Store 1</th>
<th>Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse 1</td>
<td>16$</td>
<td>17$</td>
</tr>
<tr>
<td>Warehouse 2</td>
<td>14$</td>
<td>16$</td>
</tr>
</tbody>
</table>

Can we cast this problem as a Transportation problem?
Route Cost: 99.49
Optimal route cost: 41.3
Traveling Salesman Problem (TSP)

TSP first posed by Irish mathematician W. Hamilton in the 19th century

Given a complete, weighted graph on \( n \) nodes, find the least weight Hamiltonian cycle, a cycle that visits every node once.

In our example problem there were 18 nodes. 177,843,714,048,000 distinct routes are possible!
Increasing the Number of Cities

• One way to find the optimal tour is to consider all possible paths.
• There is a problem with the exhaustive search strategy:
  • the number of possible tours of a map with \( n \) cities is \( (n - 1)! / 2 \)
  • \( n! \) ("\( n \) factorial") is \( n \times (n - 1) \times (n - 2) \ldots \times 2 \times 1 \)
• The number of tours grows incredibly quickly (exponentially) as we add cities to the map.

<table>
<thead>
<tr>
<th>#cities</th>
<th>#tours</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>360</td>
</tr>
<tr>
<td>8</td>
<td>2,520</td>
</tr>
<tr>
<td>9</td>
<td>20,160</td>
</tr>
<tr>
<td>10</td>
<td>181,440</td>
</tr>
</tbody>
</table>

Example: Madeira Island in Atlantic ocean

For 25 cities: 310,224,200,866,619,719,680,000
Real-Life Applications

Several important “real world” problems are the same as finding a tour of a large number of cities.

For many of these problems the number of “cities” can be larger than 500.

- transportation: school bus routes, logistics distribution, airports, ...
- manufacturing: an industrial robot that drills holes in printed circuit boards
- VLSI (microchip) layout
- Genome Sequencing
- communication: planning new telecommunication networks
Example: Building a new tour

Heuristics: “Algorithms that construct feasible solutions, and thus upper bounds for the optimal value.”, Hoffman and Padberg
Nearest Neighbor heuristic

- greedy
- Sensitive to the sequence/order
- Might add neighbors that are not too close
Comparison of methods

How close to the best (optimum) solution

Nearest Neighbor: 26%

Which *local modification* can further improve this tour?
Nearest Neighbor: 45.59
Heuristics (Approximations)

• Definition: if $Z^*$ is the optimal solution, a solution is said to be $\alpha$-optimal if its value can be bounded from above by $\alpha Z^*$ ($\alpha \geq 1$) for any possible instance of the problem (worst case analysis).

• For TSP there is an algorithm that guarantees an $\alpha = 1.5$ approximation ratio (Christofides algorithm).

• Other Heuristics are worse in terms of worst-case analysis but empirically (or probabilistically) provide better results.
Heuristics for TSP

- **Local Search** – a family of heuristic algorithms
- Two solutions are defined as neighbors if one can be reached by a defined change of the other
- Given a solution, we try to find a neighbor that is better than the current solution
- If a better solution is found, it becomes the new current solution
- If no better neighbor is found, the algorithm terminates with the current solution
K-opt Exchange

- Replace k arcs in a given TSP tour by k new arcs, so the result is still a TSP tour.

- Example:
  
  **2-opt** - Replace 4-5 and 3-6 by 4-3 and 5-6.

TSP Tour obtained by a greedy heuristic

Improved TSP tour
The Capacitated Vehicle Routing Problem (VRP/CVRP)

Input:
• Depot (denoted as location 0) and $n$ customers. Customer $i$ has demand $d_i$
• Distances from each customer to the other customers and the depot
• Fleet size, each vehicle has a capacity of $q$ units

Design routes so as to satisfy all customers’ demands

Potential objective function:
• Minimum cost (vehicle fixed costs + vehicle variable costs)
• Minimum distance
• Minimum travel time

Decisions: Assignment of customers to vehicles + Vehicle routing
The Basic VRP

Find a set of routes such that:

• Each route begins and ends at the depot
• The total demand of customers on each route does not exceed $q$
• The demand of all customers must be satisfied

Objective: minimize the total distance

Assumption: Distances are calculate according to the Euclidian metric

Implication: The triangle inequality holds $r_{ij} < r_{ik} + r_{kj}$
VRP – Solution methods

Exact methods –
  • Mathematical programming (various formulations)
  • Advanced enumeration methods

Heuristics –
  • Route first / cluster second
  • Cluster first / route second
  • Route construction methods: saving heuristics, insertion
  • Advanced search methods
  • Math-based heuristics
Route first / cluster second

Tour partitioning

1. Find the optimal TSP
2. Begin at the depot
3. Proceed along the tour until a the capacity is exceeded. Let $j$ denote the first customer that exceeds capacity, and denote by $i$ the customer that precedes it
4. Insert arcs $(i, 0)$ and $(0, j)$. Remove arc $(i, j)$
5. Return to step 3, starting from node $j$, continue until all customers are served.

Extension: start each time from a different customer (Iterated Tour Partitioning) and select the best solution
Cluster first / route second

Sweep Heuristic

Denote each customer by its polar coordinates \((r, \theta)\), where the depot is at the center \((r = 0)\).

Randomly select a customer and set its angle to \(\theta_1 = 0\).

The rest of the customers are sorted in ascending order of their relative angle \(\theta_1 \leq \theta_2 \leq \cdots \leq \theta_n\).
Cluster first / route second

1. Begin with the customer whose angle is $\theta_1 = 0$ and insert customers to a group in ascending order of their angle $\theta$ until the capacity limitation is violated (then, we open a new group). This step is repeated until all customers are assigned to groups.

2. For each group we solve a TSP for all the customers in the group and the depot.
Saving Algorithm
(Clarke and Wright 1964)

1. In the initial solution each customer is assigned to a separate vehicle.
2. For each pair of nodes \((i, j)\) we calculate the saving obtained from joining them together into one route.
   \[
s_{ij} = r_{0i} + r_{0j} - r_{ij}
   \]
3. Sort pairs in descending order of the saving.
4. We search the sorted list for a pair that satisfies the following conditions:
   a)  $i$ and $j$ are assigned to different routes.
   b)  $i$ and $j$ are at the edges of the routes they belong to
   c)  The total demand on the route $i$ belongs to plus the total demand on the route $j$ belongs to does not exceed the capacity.

Arc $(i, j)$ is added to the solution and arcs $(0, i)$ and $(j, 0)$ are deleted.

5. Step 4 is repeated until no pair in the list satisfies all the conditions.
Pure Pickup or Delivery

• **Delivery**: Load vehicle at depot. Design route to deliver to many customers (destinations).

• **Pickup**: Design route to pickup orders from many customers and deliver to depot.

• Examples:
  - UPS, FedEx, etc.
  - Manufacturers & carriers.
  - Carpools, school buses, etc.
Mixed Pickup & Delivery

- Can pickups and deliveries be made on same trip?
- Can they be interspersed?
Mixed Pickup & Delivery

- Pickup
- Delivery

- **One Route Not Interspersed**
  - Depot

- **Separate routes**
  - Depot

- **Interspersed**
  - Depot
Intersperse Pickups and Deliveries?

• Can pickups and deliveries be interspersed?
Complications

- Multiple vehicle types.
- Multiple vehicle capacities.
- Priorities for customers or orders.
- Time windows for pickup and delivery.
- Compatibility
  - Vehicles and customers, Drivers and vehicles etc.
- Driver rules (DOT)
  - Max drive duration = 10 hrs. before 8 hr. break.
- Split Deliveries
- Vehicle Loading/Unloading
Inventory Routing Problem (IRP)

- In the VRP, the size and frequency of deliveries were given.

- Under a VMI strategy, the supplier is responsible for his customers inventories. He decides, which products to deliver and at what frequency. Such decisions are combined with routing decisions.

- Typically, when a vehicle visits a customer, it supplies a quantity that can satisfy the demand of more than one period.

What are the tradeoffs in this problem?
IRP

Decide upon:
• The set of routes that would serve the customers.
• The amount to be delivered to each customer each time a vehicle visits.

Objective:
Minimize distribution costs (fixed costs + traveling costs) and customers’ inventory costs
Discussion