Fundamentals of Traffic Operations and Control

Distribution Systems

École Polytechnique Fédérale de Lausanne
Urban Transport Systems Laboratory

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Outline

- The transportation problem
- Nestle Challenge
- ...

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Total Distribution System

9 Suppliers, 3 Sub-assembly plants, 2 Plants, 5 Warehouses, 52 Retail outlets
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9 Suppliers, 3 Sub-assembly plants, 2 Plants, 5 Warehouses, 52 Retail outlets
The Transportation Problem

An American company distributes a single product from three production plants in the cities New-York, Chicago and Los-Angeles to five warehouses in the cities: Pittsburgh, Atlanta, Houston, Denver and San-Francisco.

The transportation costs per unit are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Pittsburgh</th>
<th>Atlanta</th>
<th>Houston</th>
<th>Denver</th>
<th>San Francisco</th>
</tr>
</thead>
<tbody>
<tr>
<td>New-York</td>
<td>$2</td>
<td>$3</td>
<td>$4</td>
<td>$5</td>
<td>$8</td>
</tr>
<tr>
<td>Chicago</td>
<td>$2</td>
<td>$4</td>
<td>$3</td>
<td>$3</td>
<td>$7</td>
</tr>
<tr>
<td>Los-Angeles</td>
<td>$8</td>
<td>$6</td>
<td>$4</td>
<td>$3</td>
<td>$2</td>
</tr>
</tbody>
</table>
The Transportation Problem

In addition, the production capacity of each plant is given:

<table>
<thead>
<tr>
<th>New-York</th>
<th>Chicago</th>
<th>Los-Angeles</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>300</td>
<td>500</td>
</tr>
</tbody>
</table>

And so is the demand in each warehouse:

<table>
<thead>
<tr>
<th>Pittsburgh</th>
<th>Atlanta</th>
<th>Houston</th>
<th>Denver</th>
<th>San Francisco</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>150</td>
<td>350</td>
<td>150</td>
<td>250</td>
</tr>
</tbody>
</table>

Formulate the problem as a Linear Programming model!
The transportation problem – LP formulation

\( x_{ij} \) - the number of units to be transported from plant \( i \) to warehouse \( j \)

Minimize \( Z = \)

\[
2x_{11} + 3x_{12} + 4x_{13} + 5x_{14} + 8x_{15} + \\
2x_{21} + 4x_{22} + 3x_{23} + 3x_{24} + 7x_{25} + \\
8x_{31} + 6x_{32} + 4x_{33} + 3x_{34} + 2x_{35}
\]
Subject to
\[ x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 200 \]
\[ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 300 \]
\[ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 500 \]
\[ x_{11} + x_{21} + x_{31} = 100 \]
\[ x_{12} + x_{22} + x_{32} = 150 \]
\[ x_{13} + x_{23} + x_{33} = 350 \]
\[ x_{14} + x_{24} + x_{34} = 150 \]
\[ x_{15} + x_{25} + x_{35} = 250 \]
\[ x_{ij} \geq 0 \quad \forall \ i = 1,2,3 \ j = 1,2,3,4,5 \]
Model Assumptions

• Each source has a fixed supply which must be distributed to destinations, while each destination has a fixed demand that must be received from the sources.

• The cost of distributing commodities from the source to the destination is directly proportional to the number of units distributed.

Feasible Solution Property

A transportation problem will have a feasible solution if and only if the sum of the supplies is equal to the sum of the demands.
The General Model of a Transportation Problem

Any problem that attempts to minimize the total cost of distributing units of commodities while meeting the requirement assumption and the cost assumption and has information pertaining to sources, destinations, supplies, demands, and unit costs can be formulated into a transportation model.
Transportation Problem - General LP formulation

**Parameters:**
- $m$ – Number of sources
- $n$ – Number of destinations
- $s_i$ - Supply at source $i$  
  $i = 1, \ldots, m$
- $d_j$ - Demand at destination $j$  
  $j = 1, \ldots, n$
- $c_{ij}$ - Unit transportation cost from source $i$ to destination $j$  
  $i = 1, \ldots, m \quad j = 1, \ldots, n$

**Decision Variables:**
- $x_{ij}$ - the number of units to be transported from source $i$ to destination $j$  
  $i = 1, \ldots, m \quad j = 1, \ldots, n$
Transportation Problem - General LP formulation

\[ \text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \]

subject to

\[ \sum_{j=1}^{n} x_{ij} = s_i \quad \forall i = 1, \ldots, m \] \hspace{1cm} \text{Supply} \\
\[ \sum_{i=1}^{m} x_{ij} = d_j \quad \forall j = 1, \ldots, n \] \hspace{1cm} \text{Demand} \\
\[ x_{ij} \geq 0 \quad \forall i, j \]
Integer Solutions Property

If all the supplies and demands have integer values, then the transportation problem with feasible solutions is guaranteed to have an optimal solution with integer values for all its decision variables.
Initial Example - Coefficients Matrix

\[
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Extensions/Adaptations

What if?

- The sum of supply exceeds the sums of demand
- The sum of the supplies is less than the sum of demands
- Certain sources may not be able to distribute commodities to certain destinations
- The objective is to maximize profits rather than minimize costs
The Transshipment problem

In this problem, we need to transport a single product from several sources to several destinations (as in the Transportation problem). However, here in addition, there are some intermediate locations that may have their own demand/supply.
Transshipment Problem - Example

Production Capacity

<table>
<thead>
<tr>
<th>Plant 1</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 2</td>
<td>120</td>
</tr>
</tbody>
</table>

Demand

<table>
<thead>
<tr>
<th>Store 1</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Store 2</td>
<td>120</td>
</tr>
</tbody>
</table>

Transportation costs per unit:

<table>
<thead>
<tr>
<th></th>
<th>Warehouse 1</th>
<th>Warehouse 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1</td>
<td>8$</td>
<td>15$</td>
</tr>
<tr>
<td>Plant 2</td>
<td>15$</td>
<td>25$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Store 1</th>
<th>Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse 1</td>
<td>16$</td>
<td>17$</td>
</tr>
<tr>
<td>Warehouse 2</td>
<td>14$</td>
<td>16$</td>
</tr>
</tbody>
</table>

Can we cast this problem as a Transportation problem?