a) We can visualize the dynamics of the city center coupled with
the parking search mechanism as three reservoirs in series, with the
levels representing the three accumulations, as depicted in fig. 1.

We write the continuous mass conservation equations as:

\[ \dot{n}_m(t) = q - o_m(n_m(t), n(t)) \]
\[ \dot{n}_s(t) = o_m(n_m(t), n(t)) - o_s(n_s(t), n(t)) \]
\[ \dot{n}_p(t) = o_s(n_s(t), n(t)), \]

where the transfer flows (i.e., trip completion rates) are:

\[ o_m(n_m(t), n(t)) = \frac{n_m(t)}{n(t)} \frac{P(n(t))}{l_m} \]
\[ o_s(n_s(t), n_p(t), n(t)) = \frac{n_s(t)}{n(t)} \frac{P(n(t))}{l_s(n_p(t))} = \frac{N_p - n_p(t)}{n(t)} \frac{n_s(t)}{n(t)} P(n(t)). \]

Discretizing eq. (1) using the forward Euler method, we obtain:

\[ n_m(k + 1) = n_m(k) + T \cdot (q - o_m(n_m(k), n(k))) \]
\[ n_s(k + 1) = n_s(k) + T \cdot (o_m(n_m(k), n(k)) - o_s(n_s(k), n_p(k), n(k))) \]
\[ n_p(k + 1) = n_p(k) + T \cdot o_s(n_s(k), n_p(k), n(k)), \]

where \( T \) is the step size.

b) Simulating using eq. (2) we obtain the trajectory of \( n_p(k) \) given
in fig. 2, which shows that all spots will be filled in 27 minutes.

c) When \( N_p \) is infinite we can assume that the dynamics of \( n_s(k) \)
disappears (as drivers would not spend any time cruising for parking
spots), resulting in the following model (where \( n(k) = n_m(k) \) since
\( n_s(k) \) is always 0):

\[ n(k + 1) = n(k) + T \cdot (q - o_m(n(k))) \]
\[ n_p(k + 1) = n_p(k) + T \cdot o_m(n(k)). \]

Simulating the system using eq. (3), we obtain the trajectory of
\( n(k) \) and compare it with that of part b) as shown in fig. 3. The gray
shaded area between the two curves (5645 veh.min) is the extra delay
due to parking space limitation. Average trip lengths are compared
in fig. 4, showing that limitation results in increased trip lengths,
whereas for infinite availability the value is fixed at \( l_m \) by definition.

d) Perimeter control can be used (changing the term \( q \) in eq. (1) to
\( u(t) \cdot q \), where \( u(t) \) is the perimeter control input, with \( 0 \leq u(t) \leq 1 \))
to operate the city center at maximum production \( P_{max} \).