Solution of Exercise 4, question 1: Ramp metering

\[ c = 6000 \text{ veh/h} \]
\[ d = 5400 \text{ veh/h} \]
\[ q_0 = 300 \text{ veh/h} \]
\[ q_p = 1000 \text{ veh/h} \]

1) During off-peak:
\[ 5400 \text{ veh/h} + 300 \text{ veh/h} = 5700 \text{ veh/h} < c \]
During peak:
\[ \# hov = 1000 \text{ veh/h} \times \frac{20}{100} = 200 \text{ veh/h} \]
\[ \# sup = 1000 \text{ veh/h} \times \frac{80}{100} = 800 \text{ veh/h} \]
\[ \Rightarrow 800 : x = 400 \Rightarrow x = 50\% \]

2)

\[ \begin{array}{c}
\text{Area of the triangles}
\end{array} \]

3)

Sup

\[ \begin{array}{c}
\text{Now treated the same}
\end{array} \]

4)

Max queue length is at \( t = 1 \text{ h} \)
\[ \Rightarrow \text{in the first case it is 400 vehicles (2 the second case)} \]
Solution of Exercise 4, question 2: Coordinated ramp metering

During the off-peak the total demand is lower than the mainline capacity \( 4000 + 800 + 1000 < 6000 \)

a) At the beginning of the peak hour we have a bottleneck downstream of \( r_2 \) (demand > capacity)

b) Ramp \( r_2 \) must start ramp metering and the metering rate is:

\[
4600 + 800 + a \cdot 1000 = 6000 \quad \rightarrow \quad a = 60\%
\]

c) Vehicles queue up with rate 400 veh/h, so the queue will reach capacity at:

\[
400 \cdot t_1 = 100 \quad \rightarrow \quad t_1 = 15 \text{ min}
\]

At this point ramp \( r_2 \) runs out of space so ramp \( r_1 \) starts ramp metering (slave).
The metering rate of the slave is the following:

\[ 4600 + B \cdot 800 + 1000 = 6000 \rightarrow B = \frac{50}{\%} \]

d) Vehicles now queue up at ramp \( r_1 \) with a rate of 400 veh/h. This ramp will reach queue capacity at:

\[ 400 \cdot t_2 = 100 \rightarrow t_2 = 15 \text{min} \]

plus 15 min of \( t_1 \) = 30 min after peak hour

At this point demand exceeds capacity and there's no more available space on the ramps! Strategy should move further upstream (if there are available other slaves; otherwise congestion is unavoidable!)