Exam:
- written exam Tuesday July 3 from 8:15-11:15
- sample exams of previous years online
- miniproject counts 33 percent towards final grade

For written exam:
- bring 1 sheet A5 (double-sided) of own notes/summary
- HANDWRITTEN!
- no calculator, no textbook
LEARNING OUTCOMES
• Solve linear one-dimensional differential equations
• Analyze two-dimensional models in the phase plane
• Develop a simplified model by separation of time scales
• Analyze connected networks in the mean-field limit
• Formulate stochastic models of biological phenomena
• Formalize biological facts into mathematical models
• Prove stability and convergence
• Predict outcome of dynamics
• Describe neuronal phenomena
• Apply model concepts in simulations

Transversal skills
• Plan and carry out activities in a way which makes optimal use of available time and other resources.
• Collect data.
• Write a scientific or technical report.

Look at samples of past exams
Use a textbook, or video lectures. Don’t use slides as only resources.
miniproject
Your Questions for Exam?
LEARNING OUTCOMES (in red: repeated today)
• Solve linear one-dimensional differential equations
• Analyze two-dimensional models in the phase plane
• Develop a simplified model by separation of time scales
• Analyze connected networks in the mean-field limit
• Formulate stochastic models of biological phenomena
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Use a textbook, (Use video lectures) don’t use slides (only)
miniproject
9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM
- Likelihood of a spike train

(9.5 Parameter Estimation)
(- Quadratic and convex optimization)

9.6 Modeling in vitro data
- How long lasts the effect of a spike?

9.7 Helping humans – in vivo data
- What is a good neuron model?
- Estimate parameters of models?
Neuronal Dynamics – 9.1 What is a good neuron model?

A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a ‘black box’)
D) Flexible enough to account for a variety of phenomena
E) Systematic procedure to ‘optimize’ parameters
What is a good choice of $f$?

\[ \tau \frac{du}{dt} = f(u) + RI(t) \]

If $u = \theta_{\text{reset}}$ then reset to $u = u_r$.

See: week 1, lecture 1.5
What is a good choice of $f$?

(i) Extract $f$ from more complex models

(ii) Extract $f$ from data
(i) Extract $f$ from more complex models

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

A. detect spike and reset resting state

Separation of time scales:
Arrows are nearly horizontal

Spike initiation, from rest

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

B. Assume $w = w_{rest}$
(i) Extract $f$ from more complex models

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

Separation of time scales

See week 4:
2dim version of Hodgkin-Huxley

$$\tau_w \frac{dw}{dt} = G(u, w) \quad \Rightarrow \quad w \approx w_{\text{rest}}$$
(ii) Extract $f$ from data \( \text{Badel et al. (2008)} \)

\[
\tau \frac{du}{dt} = f(u) + RI(t) \\
\tau \frac{df}{du} = \frac{f(u)}{\tau} \\
\tau \frac{d\vartheta}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - 9}{\Delta}\right)
\]

**Exp. Integrate-and-Fire, Fourcaud et al. 2003**

A

Pyramidal neuron

\[
f(u) \text{ [mV/ms]} \quad u \text{ [mV]}
\]

linear exponential

B

Inhibitory interneuron

\[
f(u) \text{ [mV/ms]} \quad u \text{ [mV]}
\]

linear exponential

\( \text{Badel et al. (2008)} \)
Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) \[ \tau \frac{du}{dt} = f(u) + RI(t) \]

(2) If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

Best choice of \( f \): linear + exponential

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \mathcal{I}}{\Delta}\right) \]

BUT: Limitations – need to add

- Adaptation on slower time scales
- Possibility for a diversity of firing patterns
- Increased threshold \( \mathcal{I} \) after each spike
- Noise
Biological Modeling of Neural Networks:

- Models and data
- Firing patterns and adaptation
- Integral formulation
- Adding noise to the SRM
- Quadratic and convex optimization
- how long lasts the effect of a spike?

Wulfram Gerstner
EPFL, Lausanne, Switzerland
Neuronal Dynamics – 9.2 Adaptation

Step current input – neurons show adaptation

1-dimensional (nonlinear) integrate-and-fire model cannot do this!

Data: Markram et al. (2004)
Add adaptation variables:

\[
\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) - R \sum_k w_k
\]

\[
\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)
\]

SPIKE AND RESET

after each spike \( w_k \) jumps by an amount \( b_k \)

If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

AdEx model, Brette&Gerstner (2005):

Exponential I&F + 1 adaptation var. = AdEx
Firing patterns:
Response to Step currents,
*Exper. Data, Markram et al. (2004)*

\[ I(t) \]

- **tonic**
- **burst - tonic**
- **delay - tonic**

(high, 35 mV, 350 ms)
Firing patterns: Response to Step currents, AdEx Model, Naud & Gerstner

Neuronal Dynamics – 9.2 Adaptive Exponential I&F

\[ \tau \frac{du}{dt} = - (u - u_{rest}) + \Delta \exp \left( \frac{u - \theta}{\Delta} \right) - Rw + RI(t) \]

\[ \tau_w \frac{dw}{dt} = a \ (u - u_{rest}) - w + b \ \tau_w \sum_f \delta(t - t^f) \]

AdEx model

Phase plane analysis!

Can we understand the different firing patterns?
Neuronal Dynamics – Quiz 9.1. Nullclines of AdEx

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \mathcal{G}}{\Delta}\right) - Rw + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a \left(u - u_{\text{rest}}\right) - w
\]

A - What is the qualitative shape of the w-nullcline?

- [ ] constant
- [ ] linear, slope a
- [ ] linear, slope 1
- [ ] linear + quadratic
- [ ] linear + exponential

B - What is the qualitative shape of the u-nullcline?

- [ ] linear, slope 1
- [ ] linear, slope 1/R
- [ ] linear + quadratic
- [ ] linear w. slope 1/R + exponential

3 minutes
Restart at 9:40
9.1 What is a good neuron model?
   - Models and data

9.2 AdEx model
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9.3 Spike Response Model (SRM)
   - Integral formulation

9.4 Generalized Linear Model
   - Adding noise to the SRM

9.5 Parameter Estimation
   - Quadratic and convex optimization

9.6 Modeling in vitro data
   - How long lasts the effect of a spike?
AdEx model

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - Rw + RI(t) \]

\[ \tau_w \frac{dw}{dt} = a (u - u_{\text{rest}}) - w + b \tau_w \sum_f \delta(t - t^f) \]

after each spike

\[ u \] is reset to \( u_r \)

after each spike

\[ w \] jumps by an amount \( b \)

parameter \( a \) – slope of \( w \)-nullcline

Can we understand the different firing patterns?
AdEx model – phase plane analysis: \textbf{large } b

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - g}{\Delta}\right) + w + RI(t) \]

\[ \tau_w \frac{dw}{dt} = a \ (u - u_{\text{rest}}) - w + b \ \tau_w \sum_f \delta(t - t^f) \]

\text{a}=0

A

B

u-nullcline

\text{b}

\text{u is reset to } u_r
AdEx model – phase plane analysis: small $b$

$$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a(u - u_{\text{rest}}) - w + b \tau_w \sum_f \delta(t - t^f)$$

adaptation

$u$-nullcline

$u$ is reset to $u_r$
 Quiz 9.2: AdEx model – phase plane analysis

\( \tau_w \gg \tau \)

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right) + w + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a \ (u - u_{\text{rest}}) + b \ \tau_w \sum_f \delta(t - t^f)
\]

What firing pattern do you expect?

(i) Adapting  
(ii) Bursting  
(iii) Initial burst  
(iv) Non-adapting

\( u \) is reset to \( u_r \)
AdEx model – phase plane analysis: $a > 0$

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \mathcal{G}}{\Delta}\right) + w + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a \ (u - u_{\text{rest}}) - w + b \ \tau_{w} \sum_f \delta(t - t^f)
\]
Firing patterns arise from different parameters!

Parameter $a$ – slope of $w$ nullcline

$u$ is reset to $u_r$ after each spike

$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)$

$\tau_w \frac{dw}{dt} = a \ (u - u_{\text{rest}}) - w + b \ \tau_w \sum_f \delta(t - t^f)$

$w$ jumps by an amount $b$ after each spike

See Naud et al. (2008), see also Izikhevich (2003)
Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) \[ \tau \frac{du}{dt} = f(u) + RI(t) \]

(2) If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

Best choice of \( f \): linear + exponential

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \mathcal{I}}{\Delta}\right) \]

BUT: Limitations – need to add

- Adaptation on slower time scales
- Possibility for a diversity of firing patterns
- Increased threshold \( \mathcal{I} \) after each spike
- Noise
Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R \sum_k w_k + RI(t)$$

Threshold increases after each spike

$$\vartheta = \theta_0 + \sum_f \theta_1(t - t^f)$$
Neuronal Dynamics – 9.2 Generalized Integrate-and-fire

\[ \tau \frac{du}{dt} = f(u) + R I(t) \]

If \( u = \theta_{\text{reset}} \) then reset to \( u = u_r \)

- Adaptation variables
- Possibility for firing patterns
- Dynamic threshold \( \mathcal{I} \)
- Noise

Use ‘escape noise’ (see earlier lecture)
Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Week 9 – part 3: Spike Response Model (SRM)

- 9.1 What is a good neuron model?
  - Models and data
- 9.2 AdEx model
  - Firing patterns and adaptation
- 9.3 Spike Response Model (SRM)
  - Integral formulation
- 9.4 Generalized Linear Model
  - Adding noise to the SRM
- 9.5 Parameter Estimation
  - Quadratic and convex optimization
- 9.6. Modeling in vitro data
  - how long lasts the effect of a spike?
Exponential versus Leaky Integrate-and-Fire

\[ \tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + RI(t) \]

Reset if \( u = \vartheta \)

\( \Delta \to 0 \)

\[ \tau \frac{du}{dt} = -(u - u_{rest}) + RI(t) \]

Leaky Integrate-and-Fire


\( \Delta = 2mV \)
Neuronal Dynamics – 9.3 Adaptive leaky integrate-and-fire

\[
\tau \frac{du}{dt} = -(u - u_{rest}) - R \sum_k w_k + RI(t)
\]

\[
\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)
\]

SPIKE AND RESET

- \(w_k\) jumps by an amount \(b_k\)
- \(\text{If } u = \delta(t) \text{ then reset to } u = u_r\)

Dynamic threshold
Exercise 1: from adaptive IF to SRM

\[
\begin{align*}
\tau \frac{du}{dt} &= -(u - u_{\text{rest}}) - R \alpha w + RI(t), \quad \alpha = \{0, 1\} \\
\tau_w \frac{dw}{dt} &= -w + b \tau_w \sum_f \delta(t - t^f)
\end{align*}
\]

If \( u = 9 \) then reset to \( u = u_r \)

Integrate the above system of two differential equations so as to rewrite the equations as

\[
\text{potential} \quad u(t) = \int_0^\infty \eta(s) S(t-s) ds + \int_0^\infty \varepsilon(s) I(t-s) ds + u_{\text{rest}}
\]

Hint: voltage reset equivalent to short current pulse

**A** – what is \( \eta(s) \)?

(i) \( x(s) = \frac{R}{\tau} \exp\left(-\frac{s}{\tau}\right) \)  
(ii) \( x(s) = \frac{R}{\tau_w} \exp\left(-\frac{s}{\tau_w}\right) \)

**B** – what is \( \varepsilon(s) \)?

(iii) \( x(s) = C[\exp\left(-\frac{s}{\tau}\right) - \exp\left(-\frac{s}{\tau_w}\right)] \)  
(iv) Combi of (i) + (iii)

Start before break  
Next lecture at 10:20
Linear equation $\rightarrow$ can be integrated!

\[
\begin{align*}
\tau \frac{du}{dt} &= -(u-u_{\text{rest}}) - R \sum_k w_k + RI(t) \\
\tau_k \frac{dw_k}{dt} &= a_k (u-u_{\text{rest}}) - w_k + b_k \tau_k \sum_f \delta(t-t^f)
\end{align*}
\]

Spike Response Model (SRM)

Gerstner et al. (1996)
Neuronal Dynamics – 9.3 Spike Response Model (SRM)

Gerstner et al., 1993, 1996

Input $I(t)$

Arbitrary Linear filters

Spike emission $\mathcal{G}(t)$

Potential

$$u(t) = \sum_{t'} \eta(t-t') + \int_0^\infty \kappa(s) I(t-s) \, ds + u_{\text{rest}}$$

Threshold

$$\mathcal{G}(t) = \theta_0 + \sum_{t'} \theta_1(t-t')$$
SRM with appropriate $\eta$ leads to bursting

$$u(t) = \sum_{f} \eta(t - t^f) + \int_{0}^{\infty} ds \ k(s) I(t - s) + u_{\text{rest}}$$

$$u(t) = \int_{0}^{\infty} ds \ \eta(s) S(t - s) + \int_{0}^{\infty} ds \ k(s) I(t - s) + u_{\text{rest}}$$
**Neuronal Dynamics – 9.3 Spike Response Model (SRM)**

Input $I(t)$

$$u(t) = \sum_{t'} \eta(t - t') + \int_0^\infty \kappa(s) I(t - s) ds + u_{rest}$$

Threshold

$$\mathcal{G}(t) = \theta_0 + \sum_{t'} \theta_1(t - t')$$

Firing if

$$u(t) = \mathcal{G}(t)$$

Gerstner et al., 1993, 1996

Neuronal Dynamics – Spike Response Model (SRM)
Neuronal Dynamics – 9.3 Spike Response Model (SRM)

Potential

\[ u(t) = \sum_{t'} \eta(t-t') + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest} \]

Threshold

\[ \theta(t) = \theta_0 + \sum_{t'} \theta_1(t-t') \]

Linear filters for
- input
- threshold
- refractoriness
Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models
For Coding and Decoding

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Reading for this week:
NEURONAL DYNAMICS
- Ch. 4.6, 6.1, 6.2, 6.4, 9.2
- Ch. 10.2.3, 11.1, 11.3.3
Cambridge Univ. Press

9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM
- Likelihood of a spike train

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- How long lasts the effect of a spike?
Spike Response Model (SRM)
Generalized Linear Model GLM

\[ u(t) = \int \eta(s)S(t-s)ds + \int_0^\infty \kappa(s)I(t-s)ds + u_{rest} \]

potential

\[ \theta(t) = \theta_0 + \int \theta_1(s)S(t-s)ds \]
threshold

firing intensity \[ \rho(t) = f(u(t) - \theta(t)) \]

Gerstner et al., 1992, 2000
Truccolo et al., 2005
Pillow et al. 2008
Neuronal Dynamics – review: Escape noise

**Escape process**

\[ \rho(t) = \rho_0 \exp\left(\frac{u(t) - \vartheta}{\Delta}\right) \]

**Escape rate**

\[ \rho(t) = f(u(t) - \vartheta) \]

**Example: leaky integrate-and-fire model**

\[ \tau \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t) \]

if spike at \( t^f \) \( \Rightarrow u\left(t^f + \delta\right) = u_r \)
Exerc. 2.1: Non-leaky IF with escape rates

\[
\frac{du}{dt} = \frac{RI(t)}{\tau} = \frac{1}{C} I(t) \quad \text{nonleaky}
\]

reset to \( u_r = 0 \)

\[
\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t)
\]

reset to \( u_{\text{rest}} = u_r = 0 \)

Integrate for constant input (repetitive firing)

Calculate
- potential \( u(t - \hat{t}) \)
- hazard \( \rho(t - \hat{t}) = \beta \cdot [u(t - \hat{t}) - \delta]_+ \)
- survivor function \( S(t - \hat{t}) \)
- interval distrib. \( P_0(t - \hat{t}) \)

12 minutes, Next lecture at 10:55
Neuronal Dynamics – review: Escape noise

**Survivor function**

\[
\frac{d}{dt} S_I(t | \hat{t}) = -\rho(t) S_I(t | \hat{t})
\]

\[
S_I(t | \hat{t}) = \exp\left(-\int_{t}^{\hat{t}} \rho(t') dt'\right)
\]

**Interval distribution**

\[
P_I(t | \hat{t}) = \rho(t) \cdot \exp\left(-\int_{t}^{\hat{t}} \rho(t') dt'\right)
\]

**Good choice**

\[
\rho(t) = f(u(t) - \mathcal{G}(t)) = \rho_0 \exp\left[\frac{u(t) - \mathcal{G}(t)}{\Delta u}\right]
\]

**Graphical Representation**

- **Escape process**
  - \( \rho(t) \)
  - \( u(t) \)

- **Escape rate**
  - \( \rho(t) = f(u(t) - \mathcal{G}(t)) \)

- **Survivor function**
  - Graph showing \( S_I(t | \hat{t}) = \exp\left(-\int_{t}^{\hat{t}} \rho(t') dt'\right) \)

- **Interval distribution**
  - Graph showing \( P_I(t | \hat{t}) = \rho(t) \cdot \exp\left(-\int_{t}^{\hat{t}} \rho(t') dt'\right) \)

- **Good choice**
  - Graph showing \( \rho(t) = f(u(t) - \mathcal{G}(t)) = \rho_0 \exp\left[\frac{u(t) - \mathcal{G}(t)}{\Delta u}\right] \)
Neuronal Dynamics – Likelihood of spike train

- linear filters
- escape rate
→ likelihood of observed spike train
Measured spike train with spike times $t^1, t^2, ..., t^N$

Likelihood $L$ that this spike train could have been generated by model?

$$L(t^1, ..., t^N) = \exp(-\int_0^{t^1} \rho(t') dt') \rho(t^1) \cdot \exp(-\int_{t^1}^{t^2} \rho(t') dt') ...$$
Neuronal Dynamics – 9.4 Likelihood of a spike train

\[ S(t) = \sum_{f} \delta(t - t^f) \]

\[
L(t^1, \ldots, t^N) = \exp\left(-\int_{0}^{t^1} \rho(t') \, dt'\right) \rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t') \, dt'\right) \rho(t^2) \cdots \exp\left(-\int_{t^{N-1}}^{T} \rho(t') \, dt'\right) \\
\]

\[
L(t^1, \ldots, t^N) = \exp\left(-\int_{0}^{T} \rho(t') \, dt'\right) \prod_{f} \rho(t^f) \\
\]

\[
\log L(t^1, \ldots, t^N) = -\int_{0}^{T} \rho(t') \, dt' + \sum_{f} \log \rho(t^f) \\
\]
Neuronal Dynamics – 9.4 SRM with escape noise = GLM

- linear filters
- escape rate
→ likelihood of observed spike train
→ parameter optimization of neuron model
### Week 9 – part 5: Parameter Estimation

**Biological Modeling of Neural Networks:**

**Week 9 – Optimizing Neuron Models For Coding and Decoding**

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<td></td>
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<td><strong>9.2</strong></td>
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<td><strong>9.3</strong></td>
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<td><strong>9.5</strong></td>
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<td></td>
<td>- Quadratic and convex optimization</td>
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<td><strong>9.6</strong></td>
<td>Modeling in vitro data</td>
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<td>- how long lasts the effect of a spike?</td>
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Wulfram Gerstner

EPFL, Lausanne, Switzerland
Spike Response Model (SRM)

Generalized Lin. Model (GLM)

\[ u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest} \]

known spike train

known input

Subthreshold potential

Linear filters/linear in parameters
The subthreshold potential is given by:

\[ u(t) = \int_0^\infty \kappa(s) I(t-s) \, ds + u_{\text{rest}} + \int \eta(s) S(t-s) \, ds \]

where \( \kappa(s) \) represents the inhibitory interneuron and \( \eta(s) \) represents the known spike train. The equations show the dynamics of the known input and the known spike train interacting with the neuronal system.

The graphs illustrate the inhibitory and pyramidal effects over time. The inhibitory effects are shown in the left graph, and the pyramidal effects are shown in the right graph. The equations for the inhibitory and pyramidal effects are given in Mensi et al., 2012.
Week 9 – part 5b: Quadratic and Convex Optimization

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner
EPFL, Lausanne, Switzerland

- 9.1 What is a good neuron model?
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  - Quadratic and convex optimization
- 9.6 Modeling in vitro data
  - how long lasts the effect of a spike?
Fitting models to data: so far ‘subthreshold’

A Experimental data set

<table>
<thead>
<tr>
<th>$I(t)$</th>
<th>0.2 nA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(t)$</td>
<td>20 mV</td>
</tr>
<tr>
<td>$y(t)$</td>
<td></td>
</tr>
</tbody>
</table>

C Model

<table>
<thead>
<tr>
<th>40 mV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 nA</td>
</tr>
</tbody>
</table>

Dyn. threshold

Adaptation current
potential \[ u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{\text{rest}} \]

threshold \[ \vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds \]

firing intensity \[ \rho(t) = f(u(t) - \vartheta(t)) \]
Neuronal Dynamics – 9.5 Generalized Linear Model (GLM)

\[
\log L(t^1, \ldots, t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f) = -E
\]

potential \( u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{\text{rest}} \)

threshold \( \mathcal{A}(t) = \theta_0 + \int \theta_1(s) S(t-s) ds \)

firing intensity \( \rho(t) = f(u(t) - \mathcal{A}(t)) \)
Neuronal Dynamics – 9.5 GLM: concave error function

potential \[ u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest} \]

threshold \[ \vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds \]

firing intensity \[ \rho(t) = f(u(t) - \vartheta(t)) \]

\[ \log L(t^1, ..., t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f) \]
Neuronal Dynamics – 9.5 quadratic and convex/concave optimization

Voltage/subthreshold
- linear in parameters
  → quadratic error function

Spike times
- nonlinear, but GLM
  → convex error function
Quiz NOW:

What are the units of $\eta(s)$?

(i) Voltage?
(ii) Resistance?
(iii) Resistance/s?
(iv) Current?

Potential:
$$u(t) = \int \eta(s) S(t-s)ds + \int_{0}^{\infty} \kappa(s) I(t-s)ds + u_{rest}$$

Threshold:
$$\theta(t) = \theta_0 + \int \theta_1(s) S(t-s)ds$$

Firing Intensity:
$$\rho(t) = f(u(t) - \theta(t))$$
Quiz NOW:

What are the units of $\eta(s)$? 

(i) Voltage?
(ii) Resistance?
(iii) Resistance/s?
(iv) Current?

potential 
\[ C \frac{d}{dt} u(t) = - \frac{(u - u_{\text{rest}})}{R} + \int \eta(s) S(t - s) ds + I(t - s) \]

threshold 
\[ \vartheta(t) = \theta_0 + \int \theta_1(s) S(t - s) ds \]

firing intensity 
\[ \rho(t) = f(u(t) - \vartheta(t)) \]
Week 9 – part 6: Modeling in vitro data

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner
EPFL, Lausanne, Switzerland

- 9.1 What is a good neuron model?
  - Models and data
- 9.2 AdEx model
  - Firing patterns and adaptation
- 9.3 Spike Response Model (SRM)
  - Integral formulation
- 9.4 Generalized Linear Model
  - Adding noise to the SRM
- 9.5 Parameter Estimation
  - Quadratic and convex optimization
- 9.6 Modeling in vitro data
  - how long lasts the effect of a spike?
- 9.7 Helping Humans
Neuronal Dynamics – 9.6 Models and Data

comparison model-data

Predict
- Subthreshold voltage
- Spike times
Neuronal Dynamics – 9.6 GLM/SRM with escape noise

potential \[ u(t) = \int \eta(s)S(t-s)ds + \int_0^{\infty} \kappa(s)I(t-s)ds + u_{\text{rest}} \]

threshold \[ \vartheta(t) = \theta_0 + \int \theta_1(s)S(t-s)ds \]

firing intensity \[ \rho(t) = f(u(t) - \vartheta(t)) \]

Jolivet & Gerstner, 2005
Paninski et al., 2004
Pillow et al. 2008
Neuronal Dynamics – 9.6 GLM/SRM predict subthreshold voltage
Role of moving threshold

Mensi et al., 2012
Change in model formulation:

What are the units of \( \ldots \)?

\[ C \frac{d}{dt} u(t) = \int \eta(s) S(t-s) ds + I(t) \]

potential

\[ \mathcal{I}(t) \]

threshold

\[ \mathcal{I}(t) = \mathcal{I}_0 + \int \mathcal{I}_1(s) S(t-s) ds \]

firing intensity

\[ \rho(t) = f(u(t) - \mathcal{I}(t)) \]

‘soft-threshold adaptive IF model’

exponential

adaptation current
How long does the effect of a spike last?

Time scale of filters?

$\eta(s)$

$\theta_1(s)$

Power law

\[ \int \eta(s) S(t-s) \, ds \]

A single spike has a measurable effect more than 10 seconds later!

Pozzorini et al. 2013
Neuronal Dynamics – 9.6 Models and Data

- Predict spike times
- Predict subthreshold voltage
- Easy to interpret (not a ‘black box’)
- Variety of phenomena
- Systematic: ‘optimize’ parameters

BUT so far limited to in vitro
Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

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EPFL, Lausanne, Switzerland

- 9.1 What is a good neuron model?
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  - Integral formulation
- 9.4 Generalized Linear Model
  - Adding noise to the SRM
- 9.5 Parameter Estimation
  - Quadratic and convex optimization
- 9.6. Modeling in vitro data
  - how long lasts the effect of a spike?
- 9.7. Helping Humans: in vivo data
Neuronal Dynamics – 9.7 Model of ENCODING

A) Predict spike times, given stimulus
B) Predict subthreshold voltage
C) Easy to interpret (not a ‘black box’)
D) Flexible enough to account for a variety of phenomena
E) Systematic procedure to ‘optimize’ parameters
Model of ‘Encoding’

Generalized Linear Model (GLM)
- flexible model
- systematic optimization of parameters

Model of ‘Decoding’

The same GLM works!
- flexible model
- systematic optimization of parameters
Model of ‘Decoding’: predict stimulus, given spike times
Model of ‘Decoding’

Predict intended arm movement, given Spike Times

Application: Neuroprosthetics

Neuronal Dynamics – 9.7 Helping Humans

Many groups worldwide work on this problem!
Application: Neuroprosthetics

Decode the intended arm movement

Hand velocity

Fig. 11.12: Decoding had velocity from spiking activity in area MI of cortex. The real hand velocity (thin black line) is compared to the decoded velocity (thick black line) for the $x-$ (top) and the $y-$components (bottom). Modified from Truccolo et al. (2005).
Neuronal Dynamics week 7—Suggested Reading/selected references

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski, *Neuronal Dynamics: from single neurons to networks and models of cognition.* Ch. 6,10,11: Cambridge, 2014

Nonlinear and adaptive IF

Optimization methods for neuron models, max likelihood, and GLM

Encoding and Decoding
Next year a similar class will be taught.
What should be improved?

‘Exercises and Miniprojects take a lot of time,
more than other subjects at 4 ECTS.’

- workload for a 4 credit course (=6 h p. week, for 18 weeks)
  In addition to class 9-12: 2h or less, 3h, 4h or more
  [ 1 credit = 27 hours work total = 1.5 h p. week, for 18 weeks]
Projects: Next year a similar class will be taught. What should be improved?

‘We learn nothing while doing the projects’
agree – not sure - disagree

‘We just connect the dots’
agree – not sure - disagree

‘The instructions of the graded exercises could be more precise’
agree – not sure - disagree

‘we spend more time coding than learning things which is not the goal of the course’
agree – not sure - disagree
Next year a similar class will be taught. What should be improved?

Keep 3 projects?
Reduce to 1 project, but code ‘from scratch’?
More freedom in the projects?
Less freedom (clearer instructions) in the project?
Next year a similar class will be taught. What should be improved?

‘I am more efficient in lectures than in MOOCs’
agree – not sure - disagree

‘2 hours of video needs 4 hours to watch and understand,

‘The inverted classroom is not efficient’
agree – not sure - disagree

- Keep video lectures as an available tool
- Offer at least 10 out of 13
Next year a similar class will be taught. What should be improved?

‘The slides should be redesigned’

‘The slides are too dull (almost without any text)’

agree – not sure - disagree
Quick feedback on course:

“What can I do better and differently next year?”
- support: link to book chapter, video, slides
  not sufficient, sufficient, good, excellent
- integrated exercises?
  repeat next year, do not repeat next year
- workload for a 4 credit course (=6 h p. week, for 18 weeks)
  In addition to class 9-12: 2h or less, 3h, 4h or more
- difficulty?
  easier than other theory classes,
  same, harder than other theory classes
- other points?
The END
What happens if input switches from $I=0$ to $I>0$?

- [] u-nullcline moves horizontally
- [] u-nullcline moves vertically
- [] w-nullcline moves horizontally
- [] w-nullcline moves vertically

Neuronal Dynamics – Quiz 9.2. Nullclines for constant input

\[
\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp \left( \frac{u - \theta}{\Delta} \right) + w + RI(t)
\]

\[
\tau_w \frac{dw}{dt} = a \ (u - u_{rest}) - w + b \ \tau_w \sum_f \delta(t - t')
\]

$\tau = 0$
Neuronal Dynamics – 9.5 Parameter estimation: voltage

Linear in parameters = linear fit = quadratic problem

\[ u(t) = \int_0^\infty \kappa(s) I(t-s) \, ds + u_{\text{rest}} \]

\[ u(t_n) = \sum_k \kappa_{n-k} + u_{\text{rest}} \]

comparison model-data

Blackboard: Riemann-sum

Not done in 2017
Linear in parameters = linear fit

\[ u(t) = \int_0^\infty \kappa(s) I(t - s) ds + u_{\text{rest}} \]

\[ u(t_n) = \sum k_k I_{n-k} + u_{\text{rest}} \]

Blackboard: Error function

\[ E = \sum_n \left[ u_{\text{data}}(t_n) - \sum_{k=1}^K k_k I_{n-k} - u_{\text{rest}} \right]^2 \]

Not done in 2017
Neuronal Dynamics – 9.5 Parameter estimation: voltage

Linear in parameters = linear fit = quadratic optimization

Model

\[ u(t) = \int_0^\infty \kappa(s) I(t-s) \, ds + u_{rest} \]
\[ u(t_n) = \sum_k k_k I_{n-k} + u_{rest} \]

Parameter estimation: voltage

\[ E = \sum_n \left[ u_{data}(t_n) - \sum_{k=1}^K k_k I_{n-k} - u_{rest} \right]^2 \]
Exercise 3 NOW: optimize 1 free parameter

Model
\[ u_n = RI_n \]

Data
\[ u_{\text{data}}(t_n) \]

Optimize parameter R, so as to have a minimal error

\[ E = \sum_n [u_{\text{data}}(t_n) - RI_n]^2 \]

Next lecture At 11h40
Neuronal Dynamics – What is a good neuron model?

A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a ‘black box’)
D) Flexible
E) Systematic: ‘optimize’ parameters

Not done in 2017
Neuronal Dynamics – 9.5 Parameter estimation: voltage

Vector notation

\[ u(t_n) = \sum_k k_k I_{n-k} + u_{rest} \]

\[ u(t_n) = \hat{k} \cdot \chi_n \]

\[ E = \sum_n \left[ u_{data}^n(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest} \right]^2 \]
Neuronal Dynamics – 9.5 Parameter estimation: voltage

Linear in parameters = linear fit = quadratic problem

\[
    u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest} + \int_0^\infty \eta(s) S(t-s) ds
\]

\[
    u(t_n) = \sum k_k I_{n-k} + u_{rest}
\]

\[
    u(t_n) = k \cdot x_n
\]

\[
    E = \sum_n \left[ u_{\text{data}}(t_n) - \sum_{k=1}^K k_k I_{n-k} - u_{\text{rest}} \right]^2
\]

\[
    \begin{bmatrix}
        x_1 \\
        x_2 \\
        \vdots \\
        x_K
    \end{bmatrix}
\]

\[
    \begin{bmatrix}
        I_K \\
        I_{K-1} \\
        \vdots \\
        I_1
    \end{bmatrix}
\]

\[
    \begin{bmatrix}
        I_{K+1} \\
        I_K \\
        \vdots \\
        I_2
    \end{bmatrix}
\]

\[
    \begin{bmatrix}
        I_{K+2} \\
        I_{K+1} \\
        \vdots \\
        I_3
    \end{bmatrix}
\]

\[
    \begin{bmatrix}
        I_T \\
        I_{T-1} \\
        \vdots \\
        I_{T-K+1}
    \end{bmatrix}
\]