Exam:
- written exam Wedn. 3.07.2019 from 8:15-11:00
- sample exams of previous years online
- miniproject counts 30 percent towards final grade

For written exam:
- bring 1 sheet A5 of own notes/summary
- HANDWRITTEN!
- no calculator, no textbook
LEARNING OUTCOMES

• Solve linear one-dimensional differential equations
• Analyze two-dimensional models in the phase plane
• Develop a simplified model by separation of time scales
• Analyze connected networks in the mean-field limit
• Formulate stochastic models of biological phenomena
• Formalize biological facts into mathematical models
• Prove stability and convergence
• Apply model concepts in simulations
• Predict outcome of dynamics
• Describe neuronal phenomena

Transversal skills

• Plan and carry out activities in a way which makes optimal use of available time and other resources.
• Collect data.
• Write a scientific or technical report.

Look at samples of past exams

Use a textbook, (Use video lectures) don’t use slides (only)

miniproject
Biological Modeling of Neural Networks:

Week 9 – Decision models:

Competitive dynamics

Wulfram Gerstner
EPFL, Lausanne, Switzerland

9.1 Introduction
- decision making

9.2 Perceptual decision making
- V5/MT
- Decision dynamics: Area LIP

9.3 Theory of decision dynamics
- competition via shared inhibition
- effective 2-dim model

9.4. Solutions
- symmetric case
- biased case

9.5. Simulations and Experiments
- simulations and theory
- simulations and experiments

9.6. Decisions, actions, volition
- the problem of free will

Reading for week 9: NEURONAL DYNAMICS
Ch. 16 (except 16.4.2)

Cambridge Univ. Press
9.1. How do I decide?

We take decisions all the time
- Coffee before class or not?

- Vote for candidate A or B?

- Turn left or right at the crossing?
9.1. How do I decide?
9.1. Decision making

[Diagram showing decision-making process with options 'Left?' and 'Right?']
9.1. Review of week 8: High-noise activity equation

Population activity

\[ A(t) = F(h(t)) \]

Membrane potential caused by input

\[ \tau \frac{d}{dt} h(t) = -h(t) + R I(t) \]

\[ \tau \frac{d}{dt} h(t) = -h(t) + R I^{\text{ext}}(t) + w_{ee} F(h(t)) \]

Attention:
- valid for high noise only, else transients might be wrong
- valid for high noise only, else spontaneous oscillations may arise
9.1. Review: microscopic vs. macroscopic
9.1. Competition between two populations

Input indicating ‘left’

\[ A_{e,1}(t) \]

\[ w_{ee} \]

\[ w_{ei} \]

\[ w_{ie} \]

\[ A_{inh}(t) \]

Input indicating ‘right’

\[ A_{e,2}(t) \]
9.1. How do YOU decide?

As selected EPFL student, pick your money at EPFL:

30CHF tomorrow / 100 CHF May first next year

90CHF tomorrow / 100 CHF May first next year

‘Neuro-economics’
9.1. Perceptual decision making?

Bisection task:

‘Is the middle bar shifted to the left or to the right?’
9.1. decision making - aims

Decisions are everywhere

Model: populations of neurons

Model feature Competition

Experimental data

Perceptual Decision task
Biological Modeling of Neural Networks:

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Bisection task:

‘Is the middle bar shifted to the left or to the right?’

e.g., Herzog lab, EPFL
1) Cells in visual cortex MT/V5 respond to motion stimuli

2) Neighboring cells in visual cortex MT/V5 respond to motion in similar direction cortical columns

Albright, Desimone, Gross, J. Neurophysiol, 1985
9.2. Detour: receptive fields in V5/MT

Recordings from a single neuron in V5/MT

Receptive Fields depend on direction of motion

Random moving dot stimuli:
e.g. Salzmann, Britten, Newsome, 1990
Roitman and Shadlen, 2002
Gold and Shadlen 2007
Receptive Fields depend on direction of motion: $\beta = \text{preferred direction} = P$
9.2. Experiment of Salzmann et al. 1990

monkey indicates decision by eye movement

coherence 0.8 = 80%

coherence 0.5 = 50%

coherence 0.0

coherence -1.0

Eye movement

Image: Salzman, Britten, Newsome, 1990
Monkey behavior w. or w/o Stimulation of neurons in V5/MT

X = coherent motion to bottom right

Salzman, Britten, Newsome, 1990

Blackboard: Motion detection/stimulation
9.2: Experiment of Salzman et al. 1990

A diagram showing the experiment setup:

- Pref LED
- FP
- Receptive field
- Stimulus aperture
- Null LED

- Fixation point
- Vis. stimulus
- Elect. stimulus
- Target LEDs
- Eye position
- Time

A timeline with steps:

- T1
- T2
- T3

The diagram explains that the experiment excites this group of neurons and shows coherence values of:

- 0.8 = 80%
- 0.5 = 50%
- 0.0
- -1.0

The text credits the source:

NATURE • VOL 346 • 12 JULY 1990
© 1990 Nature
9.2. Experiment of Salzman et al. 1990

Behavior: psychophysics

With stimulation

Image: Gerstner et al. (2014), Neuronal Dynamics; Redrawn after Salzman et al, 1990
9.2. Perceptual Decision Making

9.1 Review: Population dynamics
- competition

9.2 Perceptual decision making
- V5/MT
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9.3 Theory of decision dynamics
- shared inhibition
- effective 2-dim model

9.4. Decisions in connected pops.
- unbiased case
- biased input

9.5. Decisions, actions, volition
- the problem of free will
RF of Neuron in LIP:
- selective to target of saccade
- response increases faster if signal is stronger
- activity is noisy

LIP is somewhere between MT (movement detection) and Frontal Eye Field (saccade control)

Roitman and Shadlen 2002

Neurons in LIP:
- selective to target of saccade
- increases faster if signal is stronger
- activity is noisy

LIP is somewhere between MT (movement detection) and Frontal Eye Field (saccade control)

Figure 4. Response of an LIP neuron during the RT-direction-discrimination task. Data were obtained from the block of RT trials.
Neurons in LIP:
- Selective to target of saccade
- Activity increases faster if signal is stronger
- Activity is noisy
- Located in the signal processing stream between sensory areas and saccade control
- I do not claim that these neurons ‘take the decision’
- Interesting correlations with decision outcome
Receptive field in LIP
[ ] related to the target of a saccade
[ ] depends on movement of random dots
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Reading for week 9:
NEURONAL DYNAMICS
Ch. 16 (except 16.4.2)
9.3. Theory of decision dynamics

$$A_n(t) = F(h_n(t))$$

Activity equations

Membrane potential caused by input

$$\tau \frac{d}{dt} h_1(t) = -h_1(t) + R I_1^{ext}(t) + w_{ee} F(h_1(t)) + w_{ei} F(h_{inh}(t))$$

$$\tau \frac{d}{dt} h_2(t) = -h_2(t) + R I_2^{ext}(t) + w_{ee} F(h_2(t)) + w_{ei} F(h_{inh}(t))$$

Input indicating left movement

$$A_{e,1}(t)$$

population activity

Input indicating right movement

$$A_{e,2}(t)$$

Blackboard: reduction from 3 to 2 equations
9.3. **Theory of decision dynamics**

**Population activity**

\[ A_n(t) = F(h_n(t)) \]

**Inhibitory Population**

\[ A_{inh}(t) = F(h_{inh}(t)) = h_{inh}(t) = w_{ie}(A_{e,1}(t) + A_{e,2}(t)) \]

Activity equations

\[ F(h) = h \text{ for } 0.2 < h < 0.8 \]
\[ F(0) = 0.1 \]
\[ F(1) = 0.9 \]

Blackboard: Linearized inhibition
9.3. Effective 2-dim. model

Activity equations:

Membrane potential caused by input

\[ \tau \frac{d}{dt} h_1(t) = -h_1(t) + h_1^{ext}(t) + (w_{ee} - \alpha) F(h_1(t)) - \alpha F(h_2(t)) \]

\[ \tau \frac{d}{dt} h_2(t) = -h_2(t) + h_2^{ext}(t) + (w_{ee} - \alpha) F(h_2(t)) - \alpha F(h_1(t)) \]

Input indicating left movement

Input indicating right movement

Population activity
Exercise 1 now: draw nullclines and flow arrows

\[ \tau \frac{d}{dt} h_1(t) = -h_1(t) + h_1^{ext}(t) + (w_{ee} - \alpha) g(h_1(t)) - \alpha g(h_2(t)) \]

\[ g(h) = h \text{ for } 0.2 < h < 0.8 \]

\[ g(0) = 0.1 \]

\[ g(0.9) = 0.85 \]

\[ g(1) = 0.9 \]

\[ h_1^{ext} = h_2^{ext} = 0.8; \ w_{ee} = 1.5; \ \alpha = 1 \]

\[
\begin{array}{c|c|c|c}
\frac{d}{dt} h_1 = 0 & h_1 & g(h_2) & h_2 \\
1.0 & 1.0 & 1.0 & 1.0 \\
0.8 & 0.8 & 0.8 & 0.8 \\
0.2 & 0.2 & 0.2 & 0.2 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\end{array}
\]

Next Lecture at 10:45
Week 9 – Decision models: Competitive dynamics

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Reading for week 9:
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Ch. 16 (except 16.4.2)
Cambridge Univ. Press
Phase plane, strong external input

\[ \frac{d}{dt} h_1 = 0 \]

\[ h_1^{\text{ext}} = 0.8 = h_2^{\text{ext}} \]

\[ g(h) = h \quad \text{for} \quad 0.2 < h < 0.8 \]

\[ g(0) = 0.1 \]

\[ g(1) = 0.9 \]
Continue with Exercise 1, but change the external inputs.

A  Keep the input to population 1, but reduce the input to population 2 from 0.8 to 0.2
[ ] The nullcline for $dh_2/dt$ shifts vertically downward
[ ] The nullcline for $dh_2/dt$ shifts horizontally leftward
[ ] The nullcline for $dh_1/dt$ shifts vertically downward
[ ] The number of fixed points changes

B In addition, you now also reduce the input to population 1, 0.8 to 0.2
[ ] The nullcline for $dh_2/dt$ shifts vertically downward
[ ] The nullcline for $dh_2/dt$ shifts horizontally leftward
[ ] The nullcline for $dh_1/dt$ shifts vertically downward
9.4. Theory of decision dynamics: biased input

Population activity

\[ \frac{d}{dt} h_1 = 0 \quad \frac{d}{dt} h_1 = 0 \]

\[ h_1^{ext} = 0.2 \]

\[ \frac{d}{dt} h_2 = 0 \]

\[ h_2^{ext} = 0.2 \]

Phase plane – biased input:

\[ h_2^{ext} < h_1^{ext} \]

\[ h_2^{ext} = 0.2 \]
9.4. Theory of decision dynamics: unbiased weak

Phase plane – symmetric but small input

\[ \frac{d}{dt} h_1 = 0 \]

\[ h_1^{ext} = 0.2 = h_2^{ext} \]

Weak external input:
Stable fixed point

\[ \frac{d}{dt} h_2 = 0 \]
9.4. decision dynamics: unbiased strong to biased

Symmetric, but strong input

Phase plane

unbiased strong input = 2 stable fixed points
9.4. Theory of decision dynamics: biased strong

Population activity

\[ \frac{d}{dt} h_1 = 0 \]

\[ \frac{d}{dt} h_2 = 0 \]

Phase plane

\[ h_1^{ext} = 0.8; \]
\[ h_2^{ext} = 0.2 \]

Biased input = stable fixed point  → decision reflects bias
9.4. Theory of decision dynamics: unbiased strong

Phase plane

\[ \frac{d}{dt} h_1 = 0 \]

\[ h_1^{ext} = 0.8 = h_2^{ext} \]

Homogeneous solution
= saddle point
\rightarrow decision must be taken

\[ \frac{d}{dt} h_2 = 0 \]
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Competitive dynamics

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Cambridge Univ. Press
9.5. Decisions in populations of neurons: simulation

Simulation of 3 populations of spiking neurons, unbiased strong input

X.J. Wang, 2002

NEURON
1) Before stimulus is given: symmetric but small input

\[ \frac{d}{dt} h_1 = 0 \]

\[ h_1^{\text{ext}} = 0.2 = h_2^{\text{ext}} \]

Weak unbiased input: Stable fixed point \( \rightarrow \) no decision

Exercise 2 at home: stability of symmetric solution
2) When stimulus is given: symmetric but strong input

\[
\frac{d}{dt} h_1 = 0
\]

\[
\frac{d}{dt} h_2 = 0
\]

Phase plane

Homogeneous solution
= saddle point
\rightarrow decision must be taken

\[
h_{1}^{\text{ext}} = 0.8 = h_{2}^{\text{ext}}
\]
Simulation of 3 populations of spiking neurons, unbiased strong input

9.5: Decisions in populations of neurons: simulation

X.J. Wang, 2002

NEURON
9.5. Comparison with experiment: biased strong input

Prediction by theory - for input potential $h_1(t)$ and population activity $A(t) = F(h(t))$

$\frac{d}{dt} h_1 = 0$

$\frac{d}{dt} h_2 = 0$

$h_1^{ext} = 0.8;$

$\color{red}{h_2^{ext} = 0.2}$

Biased input = stable fixed point \( \rightarrow \) decision reflects bias
9.5. Decisions in populations of neurons: LIP data

Figure 7. Time course of LIP activity in the RT-direction-discrimination task. A, Average response from 54 LIP neurons. Responses are grouped by motion strength and choice as indicated by color and line type. The responses are aligned to two events in the trial. On the left, responses are aligned to the onset of stimulus motion. Response averages in this portion of the graph are drawn to the median RT for each motion strength and exclude any activity within 100 msec of eye movement initiation. On the right, responses are aligned to initiation of the eye movement response. Response averages in this portion of the graph show the buildup and decline in activity at the end of the decision process. They exclude any activity within 200 msec of motion onset. The average firing rate was smoothed using a 60 msec running mean. Arrows indicate the epochs used to compare spike rate as a function...
9.5. Decisions in populations of neurons: simulations and data

Simulation of competing populations shares properties with LIP data:
- faster increase for strong bias
- suppression for opposite saccade

BUT: there is no claim that decision is taken in LIP

LIP is somewhere in the processing stream from input to saccades
Biological Modeling of Neural Networks:

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Cambridge Univ. Press
9.6. Decision: risky vs. safe

How would you decide?
How would you decide?
9.6. fMRI variant of Libet experiment: volition and free will

- Subject decides spontaneously to move left or right hand
- Report when they made their decision

Libet, Behav. Brain Sci., 1985
Soon et al., Nat. Neurosci., 2008
What decides? Who decides?

‘Your brain decides what you want or what you prefer … ’

‘ … but your brain – this is you!!!’

- Your experiences are memorized in your brain
- Your values are memorized in your brain
- Your decisions are reflected in brain activities

‘We don’t do what we want, but we want what we do’ (W. Prinz)

The problem of Free Will
(see e.g. Wikipedia article)
9.6. Decision: risky vs. safe

- decisions are taken in the brain
- competition between populations is a transparent model
- relevant decisions involve personal values and experiences
9.6. Selected References: Decision Making

Suggested Reading:  - Salzman et al. Nature 1990
    - Roitman and Shadlen, J. Neurosci. 2002
    - Abbott, Fusi, Miller: Theoretical Approaches to Neurosci.
    - X.-J. Wang, Neuron 2002
    - Soon et al., Nat. Neurosci., 2008
    - free will, Wikipedia

Chapter 16, Neuronal Dynamics, Gerstner et al. Cambridge 2014
Exercise 2.1 now: stability of homogeneous solution

\[ A_n(t) = g(h_n(t)) \]

Membrane potential caused by input

\[ \tau \frac{d}{dt} h_1(t) = -h_1(t) + b + (w_{ee} - \alpha)g(h_1(t)) - \alpha g(h_2(t)) \]

\[ \tau \frac{d}{dt} h_2(t) = -h_2(t) + b + (w_{ee} - \alpha)g(h_2(t)) - \alpha g(h_1(t)) \]

Assume: \( h_1^{ext} = h_2^{ext} = b \)

a) Calculate homogeneous fixed point \( h_1 = h_2 = h^*(b) \)

b) Analyze stability of the fixed point \( h(b) \) as a function of \( b \)
Quick feedback on course:

“What can I do better and differently next year?”

- support: link to book chapter, video, slides
  not sufficient, sufficient, good, excellent
- integrated exercises?
  repeat next year, do not repeat next year
- workload for a 4 credit course (=6 h p. week, for 18 weeks)
  In addition to class 9-12: 2h or less, 3h, 4h or more
- difficulty?
  easier than other theory classes,
  same, harder than other theory classes
- other points?
Biological Modeling of Neural Networks

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EPFL, Lausanne, Switzerland

Week 1: A first simple neuron model/neurons and mathematics
Week 2: Hodgkin-Huxley models and biophysical modeling
Week 3: Two-dimensional models and phase plane analysis
Week 4: Two-dimensional models, type I and type II models
Week 5,6: Associative Memory, Hebb rule, Hopfield
Week 7-9: Perception, Cognition, Decision
Week 10-12: Noise models, noisy neurons and coding
Week 13: Learning in Networks
Week x: Online video: Dendrites/Biophysics
Week xx: Online video: GLM/ estimating neuron models
LEARNING OUTCOMES
• Solve linear one-dimensional differential equations
• Analyze two-dimensional models in the phase plane
• Develop a simplified model by separation of time scales
• Analyze connected networks in the mean-field limit
• Formulate stochastic models of biological phenomena
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Transversal skills
• Plan and carry out activities in a way which makes optimal use of available time and other resources.
• Collect data.
• Write a scientific or technical report.

Look at samples of past exams
Use a textbook, (Use video lectures) don’t use slides (only)
miniproject
Biological Modeling of Neural Networks

Written Exam (70%) + miniproject (30%)

Written exam:
- checks ‘Learning Outcomes’
- no calculator, no textbook
- bring A5 sheet of *handwritten* notes

Textbook:

http://neuronaldynamics.epfl.ch/

Videos:

https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOCall.html
The end:
good luck with your exams!