Biological Modeling of Neural Networks

Wulfram Gerstner
EPFL, Lausanne, Switzerland

TAs in 2019:
Chiara Gastaldi
Noe Gallice
Martin Barry

COURSE WEBPAGE:
Moodle

Week 1: A first simple neuron model/neurons and mathematics
Week 2: Hodgkin-Huxley models and biophysical modeling
Week 3: Two-dimensional models and phase plane analysis
Week 4: Two-dimensional models, type I and type II models
Week 5,6: Associative Memory, Hebb rule, Hopfield
Week 7-10: Networks, cognition, learning
Week 11,12: Noise models, noisy neurons and coding
Week 13: Estimating neuron models for coding and decoding: GLM
Week x: Online video: Dendrites/Biophysics
LEARNING OUTCOMES

• Solve linear one-dimensional differential equations
• Analyze two-dimensional models in the phase plane
• Develop a simplified model by separation of time scales
• Analyze connected networks in the mean-field limit
• Formulate stochastic models of biological phenomena
• Formalize biological facts into mathematical models
• Prove stability and convergence
• Apply model concepts in simulations
• Predict outcome of dynamics
• Describe neuronal phenomena

Transversal skills

• Plan and carry out activities in a way which makes optimal use of available time and other resources.
• Collect data.
• Write a scientific or technical report.

Look at samples of past exams

Use a textbook, (Use video lectures) don’t use slides (only)

miniproject
Biological Modeling of Neural Networks

Written Exam (70%) + miniproject (30%)

Miniproject consists of 3 extended computer exercises, of which you have to hand in 2

Textbook:
http://neuronaldynamics.epfl.ch/

Video:
https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC1.html
https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC2.html
Welcome back to EPFL!!
Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

1.1 Neurons and Synapses:
   Overview

1.2 The Passive Membrane
   - Linear circuit
   - Dirac delta-function

1.3 Leaky Integrate-and-Fire Model

1.4 Generalized Integrate-and-Fire Model

1.5 Quality of Integrate-and-Fire Models

Reading for week 1:
NEURONAL DYNAMICS
- Ch. 1 (without 1.3.6 and 1.4)
- Ch. 5 (without 5.3.1)

Cambridge Univ. Press
Biological Modeling of Neural Networks

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1.5 Quality of Integrate-and-Fire Models
How do we recognize things?
Models of cognition
Weeks 5-10
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

10,000 neurons
3 km of wire

motor cortex
to motor output
frontal cortex
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

10,000 neurons
3 km of wire

Signal:
action potential (spike)

Ramon y Cajal
Hodgkin-Huxley type models: Biophysics, molecules, ions (week 2)

-70mV

Signal: action potential (spike)

Ions/proteins

Ca^{2+}

Na^{+}

K^{+}
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Signal:
action potential (spike)
Integrate-and-fire models:
Formal/phenomenological
(week 1 and week 7-9)

-spikes are events
-triggered at threshold
-spike/reset/refractoriness
Noise and variability in integrate-and-fire models

Output
- spikes are rare events
- triggered at threshold

Subthreshold regime:
- trajectory of potential shows fluctuations

Spike emission
Random spike arrival
Neuronal Dynamics – membrane potential fluctuations

Spontaneous activity in vivo

What is noise?

What is the neural code?

(week 11-13)

awake mouse, cortex, freely whisking,

Lab of Prof. C. Petersen, EPFL Crochet et al., 2011
A cortical neuron sends out signals which are called:
- [ ] action potentials
- [ ] spikes
- [ ] postsynaptic potential

In an integrate-and-fire model, when the voltage hits the threshold:
- [ ] the neuron fires a spike
- [ ] the neuron can enter a state of refractoriness
- [ ] the voltage is reset
- [ ] the neuron explodes

The dendrite is a part of the neuron where synapses are located
- [ ] which collects signals from other neurons
- [ ] along which spikes are sent to other neurons

In vivo, a typical cortical neuron exhibits:
- [ ] rare output spikes
- [ ] regular firing activity
- [ ] a fluctuating membrane potential

Multiple answers possible!
Biological Modeling of Neural Networks

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Biological modeling of Neural Networks

Course: Monday : 9:15-13:00

A typical Monday:
1st lecture 9:15-9:50
1st exercise 9:50-10:00
2nd lecture 10:15-10:35
2nd exercise 10:35-11:00
3rd lecture 11:15 – 11:40
3rd exercise 11:45-12:40

Course of 4 credits = 6 hours of work per week
4 ‘contact’ + 2 homework

have your laptop with you

paper and pencil

paper and pencil

paper and pencil

OR interactive toy examples on computer

moodle.epfl.ch
Biological Modeling of Neural Networks

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Neuronal Dynamics – 1.2. The passive membrane

Integrate-and-fire model

Spike emission

electrode

synapse

potential

Integrate-and-fire model
Neuronal Dynamics – 1.2. The passive membrane

Spike reception

Subthreshold regime
- linear
- passive membrane
- RC circuit
Neuronal Dynamics – 1.2. The passive membrane

Time-dependent input

Math development: Derive equation (Blackboard)
Passive Membrane Model

\[ I(t) \]

\[ u \]
Passive Membrane Model

Math Development:
Voltage rescaling (blackboard)

\[ \tau \cdot \frac{d}{dt} u = - (u - u_{rest}) + RI(t) \]

\[ \tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest}) \]
Passive Membrane Model

\[ \tau \cdot \frac{d}{dt} u = - (u - u_{\text{rest}}) + RI(t) \]

\[ \tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{\text{rest}}) \]
Passive Membrane Model/Linear differential equation

\[ \tau \cdot \frac{d}{dt} V = -V + RI(t); \]

Free solution: exponential decay
Neuronal Dynamics – Exercises NOW

Start Exerc. at 9:47. Next lecture at 10:15

Step current input:
Pulse current input:
arbitrary current input:

Calculate the voltage, for the 3 input currents

\[ \tau \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

\[ \tau \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{\text{rest}}) \]
Passive Membrane Model – exercise 1 now

Step current input:

Linear equation

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Impulse reception: impulse response function

TA’s: Marco Lehmann

Start Exerc. at 9:47. Next lecture at 10:15
\[
\tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t)
\]
Pulse input – charge – delta-function

\[ I(t) = q \cdot \delta(t - t_0) \]

Pulse current input

\[ \tau \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

\[ u(t) = \]
Dirac delta-function

\[ I(t) = q \cdot \delta(t - t_0) \]

\[ \int_{t_0-a}^{t_0+a} \delta(t - t_0) dt = 1 \]

\[ f(t_0) = \int_{t_0-a}^{t_0+a} f(t) \delta(t - t_0) dt \]
Neuronal Dynamics – Solution of Ex. 1 – arbitrary input

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

Arbitrary input

\[ u(t) = u_{\text{rest}} + \int_{-\infty}^{t} \frac{1}{c} e^{-\frac{(t-t')}{\tau}} I(t')dt' \]

Single pulse

\[ \Delta u(t) = \frac{q}{c} e^{-\frac{(t-t_0)}{\tau}} \]

you need to know the solutions of linear differential equations!
Passive membrane, linear differential equation

$$\tau \cdot \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t)$$
Passive membrane, linear differential equation

If you have difficulties, watch lecture 1.2detour.

Three prerequisites:
- Analysis 1-3
- Probability/Statistics
- Differential Equations or Physics 1-3 or Electrical Circuits

https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC1.html
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Questions?

This is **not** a course on
Deep learning or Artificial neural networks
→ Deep Learning, master EE, *(Fleuret)*
→ Artificial NN, master CS, *(Gerstner)*
Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Week 1 – part 3: Leaky Integrate-and-Fire Model

1.1 Neurons and Synapses:
Overview

1.2 The Passive Membrane
- Linear circuit
- Dirac delta-function
- Detour: solution of 1-dim linear differential equation

1.3 **Leaky Integrate-and-Fire Model**

1.4 Generalized Integrate-and-Fire Model

1.5 Quality of Integrate-and-Fire Models
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]
Neuronal Dynamics – Integrate-and-Fire type Models

Simple Integrate-and-Fire Model:
- passive membrane
- + threshold

Leaky Integrate-and-Fire Model
- output spikes are events
- generated at threshold
- after spike: reset/refractoriness

Input spike causes an EPSP
= excitatory postsynaptic potential

Spike emission
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

\[ u(t) = \vartheta \Rightarrow \text{Fire+reset } u \rightarrow u_r \]

Spike emission
reset

linear
threshold
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

Time-dependent input

Math development: Response to step current

- spikes are events
- triggered at threshold
- spike/reset/refractoriness
Consider the linear differential equation \( \tau \cdot \frac{d}{dt} x = -x + x_c \)
with initial condition \( at \ t = 0 : x = 0 \)

The solution for \( t > 0 \) is

(i) \( x(t) = x_c \exp(t / \tau) \)
(ii) \( x(t) = x_c \exp(-t / \tau) \)
(iii) \( x(t) = x_c [1 - \exp(-t / \tau)] \)
(iv) \( x(t) = 0.5x_c [1 + \exp(-t / \tau)] \)

You will have to use the results: response to constant input/step input again and again
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

CONSTANT input/step input
Leaky Integrate-and-Fire Model (LIF)

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI_0 \]

If \( u(t) = \mathcal{J} \Rightarrow u \to u_r \)

‘Firing’

- Repetitive, current \( I_0 \)
  - Frequency-current relation
  - f-I curve
- Repetitive, current \( I_1 > I_0 \)
Neuronal Dynamics – First week, Exercise 2

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

frequency-current relation

\[ \frac{1}{T} \]

f-I curve
**EXERCISE 2 NOW:** Leaky Integrate-and-fire Model (LIF)

**LIF** \[ \tau \frac{d}{dt} u = - (u - u_{\text{rest}}) + RI_0 \]

If firing: \[ u \rightarrow u_r. \]

**Examine:** \[ u_r = u_{\text{rest}}. \]

**Exercise!**
Calculate the interspike interval \( T \) for constant input \( I \).
Firing rate is \( f = 1/T \).
Write \( f \) as a function of \( I \).
What is the frequency-current curve \( f = g(I) \) of the LIF?

Start Exerc. at 10:53.
Next lecture at 11:15.
Week 1 – part 4: Generalized Integrate-and-Fire Model

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

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EPFL, Lausanne, Switzerland

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1.5 Quality of Integrate-and-Fire Models
Neuronal Dynamics – 1.4. Generalized Integrate-and-Fire

Integrate-and-fire model

LIF: linear + threshold
LIF

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

If firing:

\[ u \rightarrow u_r \]

If \( I = 0 \):

If \( I > 0 \):

repetitive
Neuronal Dynamics – 1.4. Nonlinear Integrate-and-Fire

**LIF**
\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

**NLIF**
\[ \tau \cdot \frac{d}{dt} u = F(u) + RI(t) \]

If firing:
\[ u \rightarrow u_{\text{reset}} \]
Neuronal Dynamics – 1.4. Nonlinear Integrate-and-Fire

Nonlinear Integrate-and-Fire

NLIF

\[ \tau \cdot \frac{d}{dt} u = F(u) + RI(t) \]

firing: \[ u(t) = \theta \Rightarrow u \rightarrow u_r \]
Nonlinear Integrate-and-fire Model

\[ \tau \frac{du}{dt} = F(u) + RI(t) \]

\[ u(t) = \theta_r \Rightarrow \text{Fire+reset threshold} \]
Nonlinear Integrate-and-fire Model

\[
d\frac{u}{dt} = \tau (F(u) + RI(t))
\]

When \( I = 0 \):

\[
u(t) = \mathcal{G}_r \Rightarrow \text{Fire+reset threshold}
\]

Quadratic I&F:

\[
F(u) = c_2(u - c_1)^2 + c_0
\]
Nonlinear Integrate-and-Fire Model

\[ \tau \cdot \frac{d}{dt} u = F(u) + RI(t) \]

\[ u(t) = \mathcal{G}_r \Rightarrow \text{Fire+reset} \]

\[ F(u) = c_2(u - c_1)^2 + c_0 \]

Quadratic I&F:

\[ F(u) = -(u - u_{\text{rest}}) + c_0 \exp(u - \mathcal{G}) \]

exponential I&F:
Nonlinear Integrate-and-fire Model

\[
\frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t)
\]

\[
\tau \frac{d}{dt} u = -\left(u - u_{\text{rest}}\right) + RI(t)
\]

\[F(u) = -(u - u_{\text{rest}}) + c_0 \exp(u - \mathcal{I})\]

\[
u(t) = \mathcal{I}_r \implies \text{Fire+reset threshold}
\]
Nonlinear Integrate-and-fire Model

Where is the firing threshold?

\[
\tau \cdot \frac{du}{dt} = F(u) + RI(t)
\]
Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

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Neuronal Dynamics – 1.5. How good are integrate-and-fire models?

Can we compare neuron models with experimental data?

\[ \tau \cdot \frac{du}{dt} = F(u) + RI(t) \]

if \( u = \vartheta_r \) then \( u \rightarrow u_r \)
Neuronal Dynamics – 1.5. How good are integrate-and-fire models?

What is a good neuron model?

Can we compare neuron models with experimental data?
Neuronal Dynamics – 1.5. How good are integrate-and-fire models?
Nonlinear Integrate-and-fire Model

\[ \tau \cdot \frac{d}{dt} u = F(u) + RI(t) \]

\[ u(t) = \vartheta_r \Rightarrow \text{Fire+reset} \]

Can we measure the function \( F(u) \)?

**Quadratic I&F:**
\[ F(u) = c_2 (u - c_1)^2 + c_0 \]

**Exponential I&F:**
\[ F(u) = -(u - u_{\text{rest}}) + c_0 \exp(u - \vartheta) \]
Neuronal Dynamics – 1.5. How good are integrate-and-fire models?

\[
\frac{du}{dt} - \frac{1}{C} I(t) = F(u) \frac{1}{\tau}
\]

\[
F(u) = -(u - u_{\text{rest}}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right)
\]

Badel et al., J. Neurophysiology 2008
Neuronal Dynamics – 1.5. How good are integrate-and-fire models?
Nonlinear integrate-and-fire models are good.

Mathematical description \rightarrow prediction

Need to add:
- adaptation
- noise
- dendrites/synapses

Computer exercises: Python
Biological Modeling of Neural Networks

http://neuronaldynamics.epfl.ch/

Textbook:

Lecture today:
- Chapter 1
- Chapter 5

Exercises today:
- Install PYTHON for Computer Exercises
- Exercise 3, on sheet

Videos (for today: ‘week 1’):
\[ \tau \cdot \frac{d}{dt} u = F(u) + RI(t) \]
First week – References and Suggested Reading


Selected references to linear and nonlinear integrate-and-fire models

THE END (of main lecture)
MATH DETOUR SLIDES
(for online VIDEO)
Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 1 – neurons and mathematics: a first simple neuron model

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Neuronal Dynamics – 1.2 Detour – Linear Differential Eq.

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]
Neuronal Dynamics – 1.2 Detour – Linear Differential Eq.

Math development: Response to step current

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + R I(t) \]
Neuronal Dynamics – 1.2 Detour – Step current input

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

\( u(t) \)

\( I(t) \)

\( t \)
Neuronal Dynamics – 1.2 Detour – Short pulse input

\[ u(t) = u_{\text{rest}} + RI_0 \left[ 1 - e^{-(t-t_0)/\tau} \right] \]

short pulse: \( (t - t_0) \ll \tau \)

Math development: Response to short current pulse

\[ \tau \cdot \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]
Neuronal Dynamics – 1.2 Detour – Short pulse input

\[ u(t) = u_{\text{rest}} + RI_0 \left[ 1 - e^{-(t-t_0)/\tau} \right] \]

short pulse: \( (t - t_0) \ll \tau \)

\[ \tau \frac{d}{dt} u = -(u - u_{\text{rest}}) + RI(t) \]

\[ u(t) = u_{\text{rest}} + \frac{q}{C} e^{-(t-t_0)/\tau} \]

\( I(t) = q \cdot \delta(t - t_0) \)
Neuronal Dynamics – 1.2 Detour – arbitrary input

Single pulse
\[ u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau} \]

Multiple pulses:
\[ u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt' \]

\[ \tau \frac{du}{dt} = -(u - u_{rest}) + RI(t) \]
Neuronal Dynamics – 1.2 Detour – Greens function

Single pulse
\[ \Delta u(t) = q \frac{1}{C} e^{-(t-t_0)/\tau} \]

Multiple pulses:
\[ u(t) = u_{\text{rest}} + \left[ u(t_0) - u_{\text{rest}} \right] + \int_{t_0}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t') dt' \]

Impulse response function, Green's function

\[ u(t) = u_{\text{rest}} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t') dt' \]
Neuronal Dynamics – 1.2 Detour – arbitrary input

If you don’t feel at ease yet, spend 10 minutes on these mathematical exercises
And quiz 2 in week 1.

Single pulse
\[
\Delta u(t) = \frac{q}{c} e^{-(t-t_0)/\tau}
\]

You need to know the solutions of linear differential equations!