Solution: Exercise for non-smokers

- The potassium we have in our bodies consists of (checking the abundances e.g. on the nuclide chart) 93% K-39, 7% K-41, and 0.0117% K-40.

- We note from the nuclide chart, and the table of isotopes, the K-40 is radioactive, so this is the nuclide of interest here.

- K-40 has an atomic mass \( M_u \) of 39.36 u, so the number \( N \) of K-40 nuclei in the body with mass 70 kg is (with 2 grams of potassium per kilogram body weight) calculated by:

\[
N = \frac{0.0117 \times 10^{-2} \times 140}{N_A} \cdot \frac{M_u}{1.6 \times 10^{-25}}
\]

The activity \( A \) for a 70 kg body is then:

\[
A = 2N = \frac{1.17 \times 10^{-2} \cdot \ln(2) \times 140 M_u}{1.6 \times 10^{-25}} = \frac{\ln(2) \times 1.17 \times 10^{-4} \times 140 \times 6.023 \times 10^{23}}{1.28 \times 10^9 \times 365.25 \times 24 \times 3600} = 4237 \text{ Bq}
\]
• We see that we have a natural activity of above 4 kBq in our body originating from potassium-40. K-40 has the possibility to decay both by $\beta^-$ decay (in 89% of the decays), and by electron capture decay to Ar-40, followed by the emission of a 1461 keV gamma (in 11% of the decay).

• We see that the $\beta^-$-minus decay go directly to the ground state of Ca-40, with $Q_{\beta} = 1311$ keV. For the $\beta^-$-decay we make the simple assumption that, on average, only half of the energy is left for the electron (the rest goes to the escaping anti-neutrino).

• With Katz and Penfold formula we see that the range of beta electrons below 1 MeV is just a few mm, so that almost all the beta electrons are stopped inside the body.

\[
\text{Katz and Penfold formula:}
\]

\[
R_{\max} \left[ \text{cm} \right] = \begin{cases} 
0.912 \times E_\beta^{0.265} - 0.0959 \ln(E_\beta) & \text{for } 0.01 \leq E_\beta \leq 2.5 \text{MeV} \\
0.530E_\beta - 0.106 & \text{for } E_\beta > 2.5 \text{MeV}
\end{cases}
\]

\[
[ E_\beta \text{ in MeV} ]
\]

shield thickness \( t = \frac{R_{\max}}{\rho \cdot \text{vola. density}} \)
• We therefore assume that approximately 650 keV (Q\textsubscript{3} - \textfrac{1}{2}) of the beta-minus decay energy is deposited in the body.

• For the 1461 keV gamma, we have to assume the absorption in the body tissue. A rough estimate would be that an escaping gamma photon has to pass through about 10 cm of body tissue in order to escape the body.

• The ratio of gamma absorption then becomes:

\[
\frac{I}{I_0} = 1 - e^{-\mu t} = 1 - e^{-5.701 \times 10^{-2} \times 1.06 \times 10} = 1 - 0.556 = 0.454
\]

where the mu-value for tissue at 1.5 keV was used, together with a tissue density of 1.06 g cm\(^{-3}\).

• For photons and betas over the whole body, the weighting factors are both 1.

• Thus, for the total effective dose D, we get:

\[
D = \frac{A \times E_{\text{rel}} \times t}{\mu} = \frac{4237 \times (650 \times 10^3 + 0.270 \times 1461 \times 10^3) \times 6.02 \times 10^{-15} \times 365 \times 24 \times 3600}{70} = 4.034 \times 10^{-5} \text{ Sv} = 0.4 \text{ mSv}
\]
We see that we get (from our own body) an effective dose of about 0.4 mSv over one year.