Problem 1

(a) From Fig. 7-9 the maximum range of 1.71 MeV $\beta$-particles is 810 $\frac{\text{mg}}{\text{cm}^2}$ or 0.81 $\frac{\text{g}}{\text{cm}^2}$.

Hence, the thickness of polyethylene required to absorb the 1.71 MeV $\beta$-particles is:

$$\frac{0.81 \frac{\text{g}}{\text{cm}^2}}{0.93 \frac{\text{g}}{\text{cm}^2}} = 0.87 \text{ cm}$$

(b) Essentially all of the Bremsstrahlung can be assumed to be produced in the water solution since the thin polyethylene walls are of similar density. From Table 8-1 about 0.6% of 1.7 MeV $\beta$-energy is converted to photons due to absorption in water. As the average $\beta$-energy for $^{32}$P was given as

$$E_\beta = 0.695 \text{ MeV}$$

the photon emission rate is:

$$E_{\text{rad}} = 3.7 \times 10^{10} \text{s}^{-1} \times 0.695 \text{ MeV} \times 6 \times 10^{-3} = 1.55 \times 10^8 \text{ MeV/s}$$

Due to Bremsstrahlung production.
This energy rate is assumed to produce photons of energy equal to the maximum $\beta$-energy 1.71 MeV (conservative estimate).

The photon emission rate is then:

$$\frac{1.55 \times 10^8 \text{ MeV/s}}{1.71 \text{ MeV/photons}} = \frac{9.1 \times 10^7 \text{ photons}}{\text{s}}$$

Assuming isotropy, the photon flux @ 1 m is:

$$\frac{9.1 \times 10^7 \text{ photons}}{\text{s}} \times \frac{1}{4\pi \times 100^2 \text{ cm}^2} = \frac{7.24 \times 10^2 \text{ photons}}{\text{cm}^2 \cdot \text{s}}$$

\[\text{NB: What should be the thickness of lead required to ensure that the dose equivalent rate due to Bremsstrahlung photons is less than 1 mrem/s at 1 m?}\]

- Dose equivalent rate: slide 4/week 2a
  
  Dose equivalent \( H = \frac{dE}{dM} \cdot D \) where \( D = \frac{dE}{dM} \cdot Q 

- Bremsstrahlung photons $\gamma$-rays (continuous) \( Q = \alpha_k = 1 \)

- 1 rem = 1 Röntgen equivalent in man (or mammal)

  - unit to measure the biological effect of ionizing radiation
  - the dosage in rads that will cause the same amount of biological injury as 1 rad $\gamma$-rays or X-rays

\[1 \text{ rad} = 0.01 \text{ Gy} = 0.01 \frac{\text{J}}{\text{kg}} = 100 \frac{\text{rads}}{\text{g}}\]

For $\gamma$-rays \( Q = 1 = \alpha_k \) \( \Rightarrow \) 100 $\frac{\text{rads}}{\text{g}} = 1 \text{ rad} = 1 \text{ rem}$
Thus, from Table 8-2 the mass energy absorption coefficient for 1.71 MeV photons for tissue is (by interpolation): \(0.027 \text{ cm}^2 \text{ g}^{-1}\). Therefore, the photon flux produces an energy absorption rate in tissue of:

\[
E_{ab} = 7.24 \times 10^2 \left( \frac{\text{cm}^2}{\text{g}} \right) \times 1.71 \text{ MeV} \times 0.027 \left( \frac{\text{cm}^2}{\text{g}} \right) \times 1.6022 \times 10^6 \left( \frac{\text{ergs}}{\text{MeV}} \right) \times 3600 \left( \frac{\text{sec}}{\text{hr}} \right) = \\
= 0.193 \left( \frac{\text{ergs}}{\text{g} \cdot \text{sec}} \right) = \text{thus corresponds to an absorbed dose rate of 1.93 mrad/hr, or a dose equivalent rate of 1.93 mrem/hr}
\]

For \(\gamma\) rays \((Q = k_\gamma = 1)\) \(\Rightarrow 100 \frac{\text{ergs}}{\text{g}} = 1 \text{ rad} = 1 \text{ reu}\)!

---

Now: To reduce the dose below 1 mrem/h it is necessary to add a Pb shield. From Table 8-2 the attenuation coefficient \(\mu\) for lead and 1.71 MeV photons is \(\mu = 0.565 \text{ cm}^{-1}\). Assuming that there are no scattered photons one obtains from:

\[1.0 \text{ mrem} / \text{h} = 1.93 \text{ mrem} / \text{h} \times e^{-0.565x}\]

the required thickness \[x = 1.16 \text{ cm}\]

\[\text{NB}: \ 1 \text{ mrem} = 10^{-5} \text{ Sv}\]
Problem 2: (a) From Table 8-3 we get $HVL = 1.47 \text{cm}$ for 1 MeV photons.

As $200 \text{ mR/hr} = \frac{800 \text{ mR/hr}}{2 \text{ m}}$ with $\mu = 2$

$\Rightarrow$ two HVLs or $2 \times 1.47 \text{ cm} = 2.94 \text{ cm}$ of iron shield are needed.

(b) Applying the exponential attenuation law: $I = I_0 e^{-\mu x}$

We find with $\mu = \frac{\ln 2}{HVL} = \frac{\ln 2}{1.47 \text{ cm}} = 0.47 \text{ cm}^{-1}$

$\ln \frac{I}{I_0} = -\mu x$

$\ln \frac{800}{150} = -\mu x$

$-1.67398 = -\mu x$

$x = \frac{-1.67398}{0.47} = 3.56 \text{ cm}$
Problem 3: From Table 8-2 the linear attenuation coefficient \( \mu \) for 1.5 MeV photons in lead is 0.5927 cm\(^{-1} \), and for good geometry:

\[
I(x) = 10^5 \frac{\Phi}{cm^2} \times e^{-(0.5927x)} 
\]

\[
= 3.06 \times 10^4 \frac{\Phi}{cm^2}
\]

From Table 8-4 the buildup factor for 1.5 MeV photons in lead for \( \mu x = 0.5927x = 1.1854 \) can be extracted (by interpolation) to \( B = 1.45 \), and the fluence including buildup is:

\[
I_0(x) = B \times I_0 \times e^{-\mu x} = 4.43 \times 10^4 \frac{\Phi}{cm^2}
\]

which contains primary unscattered photons and scattered ones with lower energy.

Conservatively the best estimate for the energy fluence is:

\[
I_e(x)_E = 4.43 \times 10^4 \frac{\Phi}{cm^2} \times 1.5 MeV = 6.65 \times 10^5 \frac{MeV}{cm^2}
\]
Problem 4: This kind of design problem has to be solved iteratively. First, we need to compute the thickness for the unscattered beam \( \beta = 1 \).

As given, the exposure rate in air is related to the photon flux by:

\[
\text{Exposure (mR/h)} = 0.0658 \phi E \left( \frac{T_{\text{air}}}{T} \right)_{\text{air}}
\]

For 1 MeV photons, Table 8-2 gives

\[
\left( \frac{T_{\text{air}}}{T} \right)_{\text{air}} = 0.0279 \text{ cm}^2 / \text{g}.
\]

Then, the flux that produces 1 mR/h is:

\[
\phi = \frac{1 \text{ mR/h}}{0.0658 \times 1 \text{ MeV} \times 0.0279 \text{ cm}^2 / \text{g}} = 544.72 \frac{\phi}{\text{cm}^2 \cdot \text{s}}
\]

The flux at 60 cm from a point source that isotropically emits \( 10^8 \) photons/s is:

\[
\phi_1 = \frac{10^8 \text{ p/s}}{4\pi R^2 \text{ cm}^2} = \frac{10^8 \text{ p/s}}{4\pi (60)^2 \text{ cm}^2} = 2.21 \times 10^3 \frac{\phi}{\text{cm}^2 \cdot \text{s}}
\]

\[
= 2210 \frac{\phi}{\text{cm}^2 \cdot \text{s}}
\]
To reduce this flux $\phi_1$ to $\phi$ we need, as the linear attenuation coefficient for 1 MeV photons in iron is 0.472 cm$^{-1}$ (from Table 8-2) iron shielding with a thickness $x$ of:

$$\phi = \phi_1 e^{-\mu x} \Rightarrow \ln \left( \frac{\phi_1}{\phi} \right) = -\mu x$$

$$x = \frac{\ln \left( \frac{544.72}{0.21 \times 10^3} \right)}{-0.472} = 2.96711 \text{ cm}$$

This is the thickness without a buildup factor.

Now: In order to take the scattering of photons into account the buildup factor $B(1 \text{ MeV}, (\mu x)_\text{Fe})$ is needed. From Table 8-4 we get for $\mu x = 0.472 \text{ cm}^{-1} \times 2.967 \text{ cm} = 1.40$ and 1 MeV photon energy, the value (by interpolation) $B = 2.25$.

With this information we obtain a flux:

$$\phi_2 = 2.21 \times 10^3 \times 2.25 e^{-0.472 \times 2.967} \approx 1225.7 \frac{\text{cm}^2}{\text{s}}$$

which still yields an exposure $\gg 1 \text{ mR/h}$.

The process: $\phi_i \rightarrow \text{new } x \rightarrow \mu x \rightarrow B \rightarrow \phi_{i+1}$

must be repeated until $\phi_i = \phi$. :}
Problem 5: (a) **Point sources** that emit \( S \) neutrons/s can be specified at a distance \( R \) (cm) in terms of a neutron flux \( \phi \):

\[
\phi \left[ \frac{1}{cm^2 s} \right] = \frac{S \left[ \frac{n}{s} \right]}{4\pi R^2 \left[ cm^2 \right]}
\]

Table 14-4 indicates that the flux corresponding to a dose equivalent rate of 1.0 mrem/h due to 4.5 MeV neutrons is about 6.4 neutrons/cm²·s or 0.156 mrem/h per 1.0 neutron/cm²·s.

The dose equivalent rate @ 100 cm from the unshielded neutron source is:

\[
H_0 = \frac{3 \times 10^4 \times 0.156}{4\pi \times 100^2} = 37.2 \text{ mrem/h}
\]

**NB:**

\[
\text{units} \left[ \frac{\text{mrem}}{h} \right] \times \left[ \frac{1}{S} \times \frac{\text{mrem}}{h} \right] = \left[ \frac{\text{mrem}}{h} \right]
\]
(b) If the source is shielded by water, scattering of neutrons will take place and a buildup factor $B$ is needed.

With $B = 5$ (see slide 25/WEEK 3) and $Z_{m}$ extracted from Table 14-5 as 0.103 cm$^{-1}$, one obtains:

\[
\dot{H}(25\text{cm H}_2\text{O}) = \dot{H}_0 \times B \times e^{-\frac{Z_{m}}{25} \times 0.103 \times 25}
\]

\[
\dot{H}(25\text{cm H}_2\text{O}) = \frac{142}{\text{Mrad/cm}^2 / \text{h}}
\]