Fundamentals of Traffic Operations and Control

Actuated Traffic Signal Design

Nikolas Geroliminis
nikolas.geroliminis@epfl.ch
Outline

• Introduction
• Problem description / Terminology
• Classification of control strategies
  – Fixed-time control
  – Traffic actuated control
  – Traffic responsive control
  – Adaptive control
• Discussion

Acknowledgement: Some material was kindly provided by Dr. T. Kouvelas (EPFL) Prof. M. Papageorgiou (TU Crete) and Prof. A. Skabardonis (UC Berkeley).
Introduction

- Original reason for traffic lights: safe crossing of antagonistic streams of vehicles and pedestrians
- Once they exist, they can be set in different ways: optimization problem
- Difficulties: binary variables, large dimensions, many disturbances, available measurements, real-time constraints
- Many control strategies: both heuristic and systematic
Each vehicle movement has an associated phase number.

Phases have approach detectors and/or stopline detectors.

Stage: set of compatible phases.

Signal cycle: one repetition of all signals.
Example: T–junction

stage 1

stage 2

stage 3

cycle
Isolated intersection control

Fixed-time (pre-timed)
Time-of-Day
Actuated
  • Semi-actuated
  • Fully-actuated
Modelling a plan with rings

- Single ring controller

- Dual ring controller

Permitted/Protected left turns.
Actuated control

Vehicle actuations change the phases durations (and cycle length).
Min, max greens (pedestrians).
Phase skip; gap acceptance; recall.
  • Semi-actuated (coordination)
  • Fully-actuated (change cycles)
Traffic actuated signal control
Operation of an actuated phase

Case 1: Max green not reached (Gap out).
Operation of an actuated phase

Case 2: Max green reached (Max out).
Lead – Lead, Lead – Lead Phasing

Barrier
<table>
<thead>
<tr>
<th>Interval</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2</td>
</tr>
<tr>
<td>Walk</td>
<td>3 4</td>
</tr>
<tr>
<td>Ped Clearance</td>
<td>5 6</td>
</tr>
<tr>
<td>Initial (Min Green)</td>
<td>7 8</td>
</tr>
<tr>
<td>Extension (Passage)</td>
<td></td>
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<tr>
<td>Minimum Gap</td>
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<td>Time Before Reduce</td>
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<td>Time to Reduce</td>
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<tr>
<td>Maximum Green</td>
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<td>Yellow</td>
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<tr>
<td>Red Clearance</td>
<td></td>
</tr>
<tr>
<td>Permit</td>
<td>✓ ✓ ✓</td>
</tr>
<tr>
<td>Max Recall</td>
<td>✓ ✓ ✓</td>
</tr>
<tr>
<td>Min Recall</td>
<td>✓ ✓ ✓</td>
</tr>
<tr>
<td>Ped Recall</td>
<td>✓ ✓ ✓</td>
</tr>
<tr>
<td>Lag Phase</td>
<td>✓ ✓ ✓</td>
</tr>
</tbody>
</table>
Lag – Lag, Lead – Lead Phasing

Barrier

Diagram showing the concept of Lag – Lag, Lead – Lead Phasing with various elements and directions indicated.
<table>
<thead>
<tr>
<th>Interval</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
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<tr>
<td>Walk</td>
<td></td>
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<tr>
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<tr>
<td>Ped Recall</td>
<td></td>
</tr>
<tr>
<td>Lag Phase</td>
<td>✓</td>
</tr>
</tbody>
</table>
Lag – Lag, Lag – Lag Phasing

Barrier
Lead – Lag, Lead – Lead Phasing

Barrier

1 2 3 4
5 6 7 8

1 2 3 4
5 6 7 8

1 2 3 4
5 6 7 8

1 2 3 4
5 6 7 8
Minimum Green or Initial Green:

1. Long enough for at least first vehicle to get to midpoint of intersection
   (When both presence and advance detectors are available or when stop line detector serves for both presence and extension)

"Normal acceleration" \( \approx 5 \text{ ft/sec}^2 \)
Conservative reaction time \( \approx 2 \text{ sec} \)

\[
g_{\text{min}} = \text{Minimum green time} \geq 2 \text{ sec} + \sqrt{\frac{2(d/2)}{5 \text{ ft/sec}^2}} = 2 \text{ sec} + 0.45\sqrt{d}
\]
Minimum Green or Initial Green:

1. Long enough for at least first vehicle to get to midpoint of intersection

\[
g_{\text{min}} \geq 2 \text{ sec} + 0.45\sqrt{100} = 6.5 \text{ sec}
\]

"Normal acceleration" \( \approx 5 \text{ ft/sec}^2 \)
Conservative reaction time \( \approx 2 \text{ sec} \)

\[
g_{\text{min}} = \text{Minimum green time} \geq 2 \text{ sec} + \sqrt{\frac{2(d/2)}{5 \text{ ft/sec}^2}} = 2 \text{ sec} + 0.45\sqrt{d}
\]
Minimum Green or Initial Green:

2. Long enough to clear “undetected” queued vehicles
   (when advance detector serves as both presence and passage detectors)

\[ n = \frac{d_{\text{ext}}}{L} \]

Minimum headway between vehicles \( \approx 2 \text{ sec/veh} \)
Conservative reaction time \( \approx 2 \text{ sec} \)

\[ g_{\text{min}} \geq 2 \text{ sec} + n \cdot 2 \text{ sec/veh} = 2 \text{ sec} + \frac{2d_{\text{ext}}}{L} \text{ sec} \]
Minimum Green or Initial Green:

2. Long enough to clear “undetected” queued vehicles

\[ n = \frac{d_{ext}}{L} \]

\[ g_{\text{min}} \geq 2 \text{ sec} + \frac{2(200 \text{ ft})}{20 \text{ ft}} \text{ sec} = 22 \text{ sec} \]

Minimum headway between vehicles \( \approx 2 \text{ sec/veh} \)

Conservative reaction time \( \approx 2 \text{ sec} \)

\[ g_{\text{min}} \geq 2 \text{ sec} + n \cdot 2 \text{ sec/veh} = 2 \text{ sec} + \frac{2d_{ext}}{L} \text{ sec} \]
Unit Extension or Passage Time:

Long enough for vehicle to make it to stop line
Unit Extension or Passage Time:

Long enough for vehicle to make it to stop line

$50 \text{ mph} = 73 \text{ ft/sec}$

$t_{ext} \geq \frac{200}{73} = 2.7 \text{ sec}$

$t_{ext} \geq \frac{d_{ext}}{\dot{x}}$

200 ft
Allocate Green Time

- Distribute green time such that the v/c ratios are equalized for the critical lane groups

\[ X_i = \frac{v_i}{c_i} = \frac{v_i}{s_i} \ast \frac{g_i}{C} = \frac{v_i}{s_i} \frac{g_i}{C} \]

Equalize \( X_i = X_c \)

\[ g_i = (v_i / s_i) \ast (C / X_c) \]

\( g_i \) : effective green for phase i

\( v_i / s_i \) : flow ratio for phase i from the critical lane group

C : cycle length

\( c_i \) : capacity for phase i

\( X_c \) : degree of saturation for the intersection

\( X_i \) : v/c ratio (degree of saturation) for phase i

\[ \sum g_i = \sum (v_i / s_i) \ast (C / X_c) = C - L \]

\[ X_c = \sum (v_i / s_i) \ast \frac{C}{C - L} \]
For example, suppose the mean flow rate on the approach to NEMA Phase 2 of a particular intersection is 5000 vph, and we have determined that the absolute MAXIMUM GREEN that we can accommodate for this phase is $g_2 = 60$ seconds, with an INITIAL GREEN of 25 seconds.

\[ t_{ext} \geq \frac{200}{73} = 2.7 \text{ sec} \]
Initial or Minimum Green

Unit Extension = 2.7 sec

Initial = 25 sec

Maximum Extension = 35 sec

Maximum Green = 60 sec
Headway Distribution

Recall

\[ P_i(t) = e^{-\lambda t} \cdot \frac{(\lambda t)^i}{i!} \]

Then

\[ P_0(t) = e^{-\lambda t} \cdot \frac{(\lambda t)^0}{0!} = e^{-\lambda t} \]

which is the probability of no arrivals within time period \([0, t]\), which is the same as the probability that the headway between successive vehicles is greater than \(t\), i.e.,

\[ \Pr(h > t) = e^{-\lambda t} \]

Or, conversely, the probability that the headway is less than or equal to \(t\) is

\[ \Pr(h \leq t) = 1 - e^{-\lambda t} \]
\[ \Pr(h \leq t) = 1 - e^{-\lambda t} \]
\[ \Pr(h \leq 2.7) = 1 - e^{-2.7\lambda} \]
1. Suppose that we wanted to ensure that the green would be extended to the MAXIMUM GREEN only if flow conditions were at least equal to those experienced during the peak hour.

2. The minimum number of successive extensions of 2.7 seconds each (the UNIT EXTENSION value) that would have to occur in order for the green to be extended to the MAXIMUM GREEN is $35 \text{ sec} / 2.7 \text{ sec} \approx 13$. (This is the minimum number since it assumes that the extension clock gets reset just in the nick of time.)

3. Since we are assuming independent arrivals, the probability of observing 13 headways $< 2.7$ seconds in a row is simply the product of the individual probabilities, i.e.,

$$ PR(h_1 < 2.7, h_2 < 2.7, \ldots, h_{13} < 2.7) = [\Pr(h \leq 2.7)]^{13} $$
1. Even at a flow rate of 4000 vph, there is approximately a greater than 50% chance that the traffic conditions would produce a MAX OUT, were we to maintain an EXTENSION value of 2.7 sec;

2. There is almost a 25% chance that flows as low as 3000 vph would produce the same result.

3. One way to reduce this “undesirable” result is to employ Gap Reduction.
Instead of keeping the UNIT EXTENSION constant at 2.7 seconds, suppose we employ the following Gap Reduction strategy:

After 15 seconds of the extension period, there would still have to be $(35 - 15) \text{ sec}/1.5 \text{ sec} \approx 13$ successive headways $< 1.5$ seconds in order to extend the green to the MAXIMUM GREEN. The probability of this occurrence is on the order of

$$PR(h_1 < 1.5, h_2 < 1.5, \ldots, h_{13} < 1.5) = \left[Pr(h \leq 1.5)\right]^{13}$$
Under our Gap Reduction policy,

1. At a flow rate of 4000 vph, there is only approximately a 7% chance that the traffic conditions would produce a MAX OUT

2. Only about a 1% chance that flows as low as 3000 vph would produce the same result.
In some cases, in particular on minor side streets, upstream extension detectors are not installed; rather, all of the detection is provided by the stopline presence detectors.

Stopline presence detectors take on the role of “extension” detectors once the INITIAL (or MINIMUM) GREEN has timed out.

Set the INITIAL (or MINIMUM) GREEN to a very small value, say just enough for at least first vehicle to get to midpoint of intersection (approximately 4-6 seconds), and then set the Extension to a value that will hold the green just long enough to dissipate vehicles that have queued up during the Red interval.
Example

Phasing Diagram

<table>
<thead>
<tr>
<th>NEMA Phase</th>
<th>$q_i$ (vph)</th>
<th>$S_i$ (vph)</th>
<th>$q_i/S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1600</td>
<td>4000</td>
<td>0.40</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>2000</td>
<td>0.15</td>
</tr>
<tr>
<td>6</td>
<td>1500</td>
<td>4000</td>
<td>0.375</td>
</tr>
<tr>
<td>8</td>
<td>400</td>
<td>2000</td>
<td>0.20</td>
</tr>
</tbody>
</table>
\[ P_i(t) = e^{-\lambda t} \cdot \frac{(\lambda t)^i}{i!} \]

The mean arrival rate for NEMA Phase 4 is 300 vph
\[ \Rightarrow \lambda = 300 \text{ vph or 0.0833 vehicles per second.} \]

The probability of \( i \) arrivals per 100 second cycle is

\[ P_i(100) = e^{-0.0833(100)} \cdot \frac{(0.0833 \cdot 100)^i}{i!} = e^{-8.33} \cdot \frac{(8.33)^i}{i!} \]
Poision Probability of Exactly \( i \) Vehicles Arriving

Number of Vehicles vs. Probability

- Probability values range from 0.02 to 0.16.
- Number of Vehicles range from 0 to 30.
\[ \Pr(i \leq N) = \sum_{i=0}^{N} e^{-\lambda t} \frac{(\lambda t)^i}{i!} = \sum_{i=0}^{N} e^{-8.33} \frac{(8.33)^i}{i!} \]
In 95% of the cycles we expect that the total number of vehicles arriving on NEMA 4 during any particular cycle of length 100 seconds will be less than or equal to 14 vehicles.

After accounting for startup lost time, queued vehicles can be discharged at the steady state headway $h = \frac{3600}{S} = \frac{3600}{2000} = 1.8$ seconds/vehicle.

Total effective green time that would be needed to just dissipate a queue of 14 vehicles is about 25 seconds.

If we set the MAXIMUM GREEN for NEMA 4 to about 25 seconds, we will be able to accommodate the traffic arriving on this approach for about 95 out of 100 of the cycles.

That is, provided that we set the Extension in a way that would pretty much guarantee extending the green so long as there were vehicles still in the queue.
Recall

\[ h_1 > h_2 > h_3 > h_4 \approx h_5 \approx h_6 \approx \ldots \approx h = \frac{3600}{S} \]

Results of a Study:

Vehicle Headways for NEMA Phase 4

<table>
<thead>
<tr>
<th>Headway</th>
<th>Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>h1</td>
<td>4.2</td>
</tr>
<tr>
<td>h2</td>
<td>3.5</td>
</tr>
<tr>
<td>h3</td>
<td>2.7</td>
</tr>
<tr>
<td>h4</td>
<td>2.3</td>
</tr>
<tr>
<td>h5</td>
<td>2.1</td>
</tr>
<tr>
<td>h6</td>
<td>2.0</td>
</tr>
<tr>
<td>h7</td>
<td>1.9</td>
</tr>
<tr>
<td>h8</td>
<td>1.8</td>
</tr>
<tr>
<td>h9</td>
<td>1.8</td>
</tr>
<tr>
<td>h10</td>
<td>1.8</td>
</tr>
</tbody>
</table>
Vehicle Headways

Time after Onset of Green (Seconds)

Headway (Seconds)
Vehicle Headways

<table>
<thead>
<tr>
<th>Time after Onset of Green (Seconds)</th>
<th>Headway (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>4.2</td>
</tr>
<tr>
<td>1-2</td>
<td>3.5</td>
</tr>
<tr>
<td>2-3</td>
<td>2.7</td>
</tr>
<tr>
<td>3-4</td>
<td>2.3</td>
</tr>
<tr>
<td>4-5</td>
<td>2.1</td>
</tr>
<tr>
<td>5-6</td>
<td>2.0</td>
</tr>
<tr>
<td>6-7</td>
<td>1.9</td>
</tr>
<tr>
<td>7-8</td>
<td>1.8</td>
</tr>
<tr>
<td>8-9</td>
<td>1.8</td>
</tr>
<tr>
<td>9-10</td>
<td>1.8</td>
</tr>
<tr>
<td>10-11</td>
<td>1.8</td>
</tr>
<tr>
<td>11-12</td>
<td>1.8</td>
</tr>
</tbody>
</table>

INITIAL GREEN | EXTENSION
Gap Reduction:

- **4.0 sec** at Time: 5 sec
- **2.0 sec** at Time: 10 sec

Beginning of Extension period:

- **5 sec**
- **10 sec**

Vehicle Headways

<table>
<thead>
<tr>
<th>Time after Onset of Green (Seconds)</th>
<th>Headway (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>0.5</td>
<td>3.5</td>
</tr>
<tr>
<td>1</td>
<td>2.7</td>
</tr>
<tr>
<td>1.5</td>
<td>2.3</td>
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<tr>
<td>2</td>
<td>2.1</td>
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<td>2.5</td>
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<td>3</td>
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<td>3.5</td>
<td>1.6</td>
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<tr>
<td>4</td>
<td>1.8</td>
</tr>
<tr>
<td>4.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

INITIAL GREEN EXTENSION

- Gap Reduction: **4.0 sec**
- Time: 5 sec
- Beginning of Extension period: 10 sec
<table>
<thead>
<tr>
<th>Interval</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
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<tr>
<td>Walk</td>
<td></td>
</tr>
<tr>
<td>Ped Clearance</td>
<td></td>
</tr>
<tr>
<td>Initial (Min Green)</td>
<td>5</td>
</tr>
<tr>
<td>Extension (Passage)</td>
<td>2.9</td>
</tr>
<tr>
<td>Minimum Gap</td>
<td>0.9</td>
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<td>Time Before Reduce</td>
<td>5.0</td>
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<tr>
<td>Time to Reduce</td>
<td>10.0</td>
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<tr>
<td>Maximum Green</td>
<td>25</td>
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<tr>
<td>Yellow</td>
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<tr>
<td>Red Clearance</td>
<td>1</td>
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<tr>
<td>Permit</td>
<td>√</td>
</tr>
<tr>
<td>Max Recall</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>Ped Recall</td>
<td></td>
</tr>
<tr>
<td>Lag Phase</td>
<td></td>
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</tbody>
</table>
Saturation Flow Rates
Thrus: 2000 vphpl
Protected Left Turns: 1850 vphpl
Right Turns: 1600 vphpl
PHASING DIAGRAM

<table>
<thead>
<tr>
<th>NEMA Phase</th>
<th>Yellow (secs.)</th>
<th>All Red (secs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>2</td>
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<tr>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
Set Control Philosophy

1. At most, we are willing to tolerate a 120-second cycle when all of the phases go to their maximum values (i.e., MAX OUT).

2. Design the Maximum Greens in a way that will handle the 85\textsuperscript{th} percentile of arrivals during any cycle

Steps

1. First, let’s compute the flow rates that would be associated with this design philosophy

2. We need to determine which among the combined movements incorporated in any particular NEMA phase will be “critical,”
<table>
<thead>
<tr>
<th>NEMA Phase</th>
<th>Movements</th>
<th>$q_i$ (vph)</th>
<th>$S_i$ (vph)</th>
<th>$Y_i = q_i / S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Through</td>
<td>1200</td>
<td>6000</td>
<td>0.20*</td>
</tr>
<tr>
<td></td>
<td>Right Turn</td>
<td>300</td>
<td>1600</td>
<td>0.188</td>
</tr>
<tr>
<td>6</td>
<td>Through</td>
<td>1200</td>
<td>6000</td>
<td>0.20*</td>
</tr>
<tr>
<td></td>
<td>Right Turn</td>
<td>260</td>
<td>1600</td>
<td>0.163</td>
</tr>
<tr>
<td>4</td>
<td>Left Turn</td>
<td>120</td>
<td>1850</td>
<td>0.065*</td>
</tr>
<tr>
<td></td>
<td>Through</td>
<td>50</td>
<td>2000</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Right Turn</td>
<td>60</td>
<td>1600</td>
<td>0.038</td>
</tr>
<tr>
<td>8</td>
<td>Left Turn</td>
<td>300</td>
<td>1850</td>
<td>0.162</td>
</tr>
<tr>
<td></td>
<td>Through</td>
<td>350</td>
<td>2000</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>Right Turn</td>
<td>290</td>
<td>1600</td>
<td>0.181*</td>
</tr>
</tbody>
</table>

* indicates critical movement
3. Calculate our controlling flow rates for the critical flows identified above and for the flows associated with NEMA phases 1 and 5.

<table>
<thead>
<tr>
<th>NEMA Phase</th>
<th>Critical Movement</th>
<th>$q_i$ (vph)</th>
<th>$\lambda_i = q_i$ (vps)</th>
<th>$S_i$ (vph)</th>
<th>$Y_i = q_i / S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Left Turn</td>
<td>400</td>
<td>0.111</td>
<td>3700</td>
<td>0.108</td>
</tr>
<tr>
<td>2</td>
<td>Through</td>
<td>1200</td>
<td>0.333</td>
<td>6000</td>
<td>0.20*</td>
</tr>
<tr>
<td>5</td>
<td>Left Turn</td>
<td>300</td>
<td>0.083</td>
<td>3700</td>
<td>0.081</td>
</tr>
<tr>
<td>6</td>
<td>Through</td>
<td>1200</td>
<td>0.333</td>
<td>6000</td>
<td>0.20*</td>
</tr>
<tr>
<td>4</td>
<td>Left Turn</td>
<td>120</td>
<td>0.033</td>
<td>1850</td>
<td>0.065*</td>
</tr>
<tr>
<td>8</td>
<td>Right Turn</td>
<td>290</td>
<td>0.081</td>
<td>1600</td>
<td>0.181*</td>
</tr>
</tbody>
</table>
Recall our formula for Poisson arrivals:

\[ P_n(t) = e^{-\lambda t} \cdot \frac{\left(\lambda t\right)^n}{n!} \]

where

\[ P_n(t) = \text{Probability of exactly } n \text{ arrivals during } t \]

The probability of \( n \) arrivals per 120-second cycle is

\[ P_n(120) = e^{-120\lambda_i} \cdot \frac{(120\lambda_i)^n}{n!} \]
Poisson Probability of Exactly \( n \) Vehicles Arriving to be Served by NEMA Phase 1
Poisson Probability of Exactly $n$ Vehicles Arriving to be Served by NEMA Phase 2
Poisson Probability of Exactly $n$ Vehicles Arriving to be Served by NEMA Phase 5
Poisson Probability of Exactly $n$ Vehicles Arriving to be Served by NEMA Phase 6
Poisson Probability of Exactly \( n \) Vehicles Arriving to be Served by NEMA Phase 4
Poisson Probability of Exactly $n$ Vehicles Arriving to be Served by NEMA Phase 8
4. Determine Design Flows

The cumulative probability distribution is

$$\Pr(n \leq N) = \sum_{n=0}^{N} e^{-120\lambda_i} \cdot \frac{(120\lambda_i)^n}{n!}$$

A plot of this cumulative distribution for each of the critical movements, together with the 85th percentile arrivals identified, is shown next:
Poisson Probability of NEMA Phase 1 Vehicles Arriving Being Less Than Number on Abscissa

Number of Vehicles

Probability

0.85

17
Poisson Probability of NEMA Phase 2 Vehicles Arriving Being Less Than Number on Abscissa

Probability

Number of Vehicles

0.85

47
Poisson Probability of NEMA Phase 5 Vehicles Arriving Being Less Than Number on Abscissa

Number of Vehicles

Probability

0.85

13
Poisson Probability of NEMA Phase 6 Vehicles Arriving Being Less Than Number on Abscissa
Poisson Probability of NEMA Phase 4 Vehicles Arriving Being Less Than Number on Abscissa

Number of Vehicles

Probability

0 0.2 0.4 0.6 0.8 1 1.2

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Poisson Probability of NEMA Phase 8 Vehicles Arriving Being Less Than Number on Abscissa

Number of Vehicles

Probability

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

0 0.2 0.4 0.6 0.8 1.0 1.2

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

13
5. Identify the flow rates that we will use in our decisions on the MAXIMUM GREEN values, simply by taking the 85th percentile arrivals during a 120-second cycle and multiplying by 3600/120 = 30 cycles per hour

<table>
<thead>
<tr>
<th>NEMA Phase</th>
<th>Critical Movement</th>
<th>85th % Arrivals</th>
<th>$q_i^*$ (vph)</th>
<th>$S_i$ (vph)</th>
<th>$Y_i = q_i^*/S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Left Turn</td>
<td>17</td>
<td>510</td>
<td>3700</td>
<td>0.138</td>
</tr>
<tr>
<td>2</td>
<td>Through</td>
<td>47</td>
<td>1410</td>
<td>6000</td>
<td>0.235</td>
</tr>
<tr>
<td>5</td>
<td>Left Turn</td>
<td>13</td>
<td>390</td>
<td>3700</td>
<td>0.105</td>
</tr>
<tr>
<td>6</td>
<td>Through</td>
<td>47</td>
<td>1410</td>
<td>6000</td>
<td>0.235</td>
</tr>
<tr>
<td>4</td>
<td>Left Turn</td>
<td>6</td>
<td>180</td>
<td>1850</td>
<td>0.097</td>
</tr>
<tr>
<td>8</td>
<td>Right Turn</td>
<td>13</td>
<td>390</td>
<td>1600</td>
<td>0.244</td>
</tr>
</tbody>
</table>
6. Determine the critical path through the dual ring controller; i.e., which ring on each side of the barrier places the greatest demand for green

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$Y_1 = 0.138$</td>
<td>2</td>
<td>$Y_2 = 0.235$</td>
</tr>
<tr>
<td>5</td>
<td>$Y_5 = 0.105$</td>
<td>6</td>
<td>$Y_6 = 0.235$</td>
</tr>
<tr>
<td>4</td>
<td>$Y_4 = 0.097$</td>
<td>8</td>
<td>$Y_8 = 0.244$</td>
</tr>
</tbody>
</table>

Critical Movement q/S
### Critical Movement \( q/S \)

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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y_1 = 0.138 )</td>
<td>( Y_2 = 0.235 )</td>
<td>( Y_4 = 0.097 )</td>
<td>( Y_8 = 0.244 )</td>
</tr>
<tr>
<td>( Y_5 = 0.105 )</td>
<td>( Y_6 = 0.235 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( Y_5 + Y_6 = 0.340 \)
- \( Y_1 + Y_2 = 0.373 \)
- \( Y_4 + Y_8 = 0.341 \)
7. Distribute the available MAXIMUM GREEN according to Webster’s formulas for optimal greens

\[ g_i = \frac{\theta_i \cdot X_i}{\sum_{j=1}^{n} \theta_j \cdot X_j} \cdot g_t = \frac{q_i/S_i}{\sum_{j=1}^{n} q_j/S_j} \cdot g_t = \frac{Y_i}{\sum_{j=1}^{n} Y_j} \cdot g_t \]

where

\[ n = \text{number of conflicting phases active on the “critical path” through the dual ring diagram} \]

\[ g_t = C - \text{Yellow} \phi_1 - \text{AllRed} \phi_1 - \text{Yellow} \phi_2 - \text{AllRed} \phi_2 - \text{Yellow} \phi_4 - \text{AllRed} \phi_4 - \text{Yellow} \phi_6 - \text{AllRed} \phi_6 = 120 - 3 - 1 - 4 - 2 - 4 - 1 - 4 - 1 = 100 \text{ sec} \]
\[ g_i = \frac{Y_i}{\sum_{j=1}^{n} Y_j} \cdot \sum_{j=1}^{1,2,4,8} Y_j \cdot 100 \, \text{sec} \]

\[ = \frac{Y_i}{0.138 + 0.235 + 0.097 + 0.244} \cdot 100 \, \text{sec} = \frac{Y_i}{0.714} \cdot 100 \, \text{sec} = 140.06 \cdot Y_i \, \text{sec} \]

\[ g_1 = 140.06 \cdot Y_1 = 140.06 \cdot 0.138 \approx 19 \, \text{sec} \]

\[ g_2 = 140.06 \cdot Y_2 = 140.06 \cdot 0.235 \approx 33 \, \text{sec} \]

\[ g_4 = 140.06 \cdot Y_4 = 140.06 \cdot 0.097 \approx 14 \, \text{sec} \]

\[ g_8 = 140.06 \cdot Y_8 = 140.06 \cdot 0.244 \approx 34 \, \text{sec} \]
8. Allocate the MAXIMUM GREEN for NEMA Phases 5 and 6

\[ g_{5+6} = 62\sec - \text{Yellow}_{\phi_5} - \text{AllRed}_{\phi_5} - \text{Yellow}_{\phi_6} - \text{AllRed}_{\phi_6} \]

\[ = 62 - 3 - 1 - 4 - 2 = 52 \sec \]

\[ g_i = \frac{Y_i}{\sum_{j=1}^{n} Y_j} \cdot g_{5+6} = \frac{Y_i}{\sum_{j=5,6}^{n} Y_j} \cdot 52 \sec \; ; \; i = 5, 6 \]

\[ = \frac{Y_i}{0.105 + 0.235} \cdot 52 \sec = \frac{Y_i}{0.34} \cdot 52 \sec = 152.94 \cdot Y_i \sec \]

\[ g_5 = 152.94 \cdot Y_5 = 152.94 \cdot 0.105 \approx 16 \sec \]

\[ g_6 = 152.94 \cdot Y_6 = 152.94 \cdot 0.235 \approx 36 \sec \]
\[ g_5 = 152.94 \cdot Y_5 = 152.94 \cdot 0.105 \approx 16 \text{ sec} \]
\[ g_6 = 152.94 \cdot Y_6 = 152.94 \cdot 0.235 \approx 36 \text{ sec} \]
Adaptive signal control

More sophisticated algorithms:

- include internal models for dynamics and optimization
- compute signal parameters including splits, cycles, offsets in real-time based on measurements
Adaptive strategies

Many strategies (research or commercial):

- SCOOT
- SCATS
- OPAC
- PRODYN
- CRONOS
- RHODES
- UTOPIA
- BALANCE
- TUC
- ACS-Lite

Examples are provided for general knowledge of the students. It will not be taught inside classroom.
**Split Cycle Offset Optimization Technique:**

- **Demand Profiles**
- **Queues**
- **Split & Offset Optimizer**
- **Current SCOOT Timings**
- **Translation Array**
- **Weights & Bias**
- **All Possible Settings**
- **Close Settings**
- **Traffic Engineer**

Flow → Phases

SCOOT – UK
SCOOT – Loop detectors position, demand profile, queue model

- Loop detectors position
- Demand profile
- Queue model

- Red time
- Green time
- Time now
- Flow rate
- Saturation flow rate
- Cruise speed
- Flow adds to back of queue
- Modelled queue at time ‘now’
- Time ‘now’
- ACTUAL QUEUE
- Stop Line
- Cyclic Flow Profile
## Signal optimizers

<table>
<thead>
<tr>
<th>Optimizer</th>
<th>Frequency</th>
<th>Change time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Split</td>
<td>Every stage</td>
<td>-4, 0, +4 (temporary)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1, 0, +1 (permanent)</td>
</tr>
<tr>
<td>Cycle</td>
<td>Once per cycle</td>
<td>-4, 0, +4</td>
</tr>
<tr>
<td>Offset</td>
<td>Every 2.5 or 5 minutes</td>
<td>-4, 0, +4 (32 to 64)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-8, 0, +8 (64 to 128)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-16, 0, +16 (128 to 240)</td>
</tr>
</tbody>
</table>
**SCATS – Australia**

**Sydney Coordinated Adaptive Traffic System:**

**Objectives:**
- Minimize stops (light traffic).
- Minimize delay (heavy traffic).
- Minimize travel time.

- Stopline detection.
- Network divided into regions.
- Each region divided into links and nodes.
- For each region calculate degree of saturation (DS) for all nodes.
SCATS – Stopline detection

Degree of saturation: $DS = \frac{[\text{green}-(\text{unused green})]}{\text{green}}$
SCATS – Optimization

Cycle Length (CL):

• User defined equilibrium DS values are used to determine the relationships between measured DS and CL.

• The relationships are used to select a target CL toward which the actual CL moves.

Offsets:

• Offset plans are selected by comparing traffic flows on the links.

• The weighted three-cycle average volumes are used for each candidate offset.
SCOOT vs. SCATS

- Model
  - Central control
  - Upstream detection
  - Fixed traffic region

- Algorithm
  - Distributed
  - Stop-line detection
  - Adjustable regions

- Closed systems: not all the details are known.
- Involvement of many (implementation specific) heuristics.
Adaptive control: rolling horizon
OPAC/RHODES: no fixed cycle

- Measured and predicted vehicle arrivals
- Optimization: minimize queues
- Rolling horizon

- Upstream detectors can provide history for demand profiles.
- Smoothed volume can be used for uniform profiles.
- Platoon identification and smoothing can be used for cyclic profiles.
TUC strategy

Traffic-responsive Urban Control:

- Based on the store-and-forward modeling paradigm.
- Main control modules:
  - Split control (Linear-Quadratic regulator).
  - Cycle control (feedback P-type regulator).
  - Offset control (decentralized feedback controller).
  - PT Priority (based on mounted GPS devices).
- Field applications in Glasgow, Chania, Munich, Southampton, Tel Aviv, Jerusalem, Brazil (Macae, Maoua and Santos).
Real-time control loop

Disturbances

Demand

Incidents

Total time spent

Measurements

REAL WORLD COMPUTER

Control inputs

Control Devices

Traffic Network

Sensors

Control Strategy

Surveillance

Goals

Human-Machine Interface
Communications requirements

**Central PC**
- Calculation of signal settings without PT priority
- Transmission of control decisions to local controllers (once every cycle)
- Surveillance and administration of control system

**Local controllers**
- Traffic data collection, averaging and transmission to central PC (once every cycle)
- Application of control decisions from central PC
- Local PT data collection
- Local modification of signal settings for PT priority
- Application of stored fixed plans in case of communication problems with central PC

**Traffic detectors**
Traffic data collection (loop occupancy for network links)

**PT detectors**
PT data collection
Implementation: Traffic Control Center