11: Echo formation and spatial encoding

1. What makes the magnetic resonance signal spatially dependent?
2. How is the position of an MR signal identified?
   - Slice selection
3. What is echo formation and how is it achieved?
   - Echo formation
   - Gradient echo sequence
4. How is a two-dimensional MR image encoded?

After this course you
1. Understand the principle of slice selection
2. Are familiar with dephasing and rephasing of transverse magnetization and how it leads to echo formation
3. Understand the principle of spatial encoding in MRI
4. Can describe the basic imaging sequence and the three necessary elements
5. Understand the principle of image formation in MRI and how it impacts spatial resolution

11-1. What do we know about magnetic resonance so far?

Adding a 3rd magnetic field

So far
1) Excite spins using RF field at \( \omega_L \)
2) Record time signal (Known as FID)
3) \( M_{xy} \) decays, \( M_z \) grows (\( T_2 \) and \( T_1 \) relaxation)

RF coils measure signal from entire body (no spatial information)

Precessional Frequency \( \omega_L = \gamma B_0 \) Static Magnetic Field

How to encode spatial position?

\[ B(x) = B_0 + \vec{G} \cdot \vec{x} \]

Magnetic field \( B \) along \( z \) varies spatially with \( x, y, \) and/or \( z \):

\[ \vec{G} = \frac{dB_z}{dr} \]

\[ B_z(\vec{r}) = B_0 + \vec{G} \cdot \vec{r} \]

\( \vec{G} = (G_x, 0, 0) \)
How is the gradient field created?

**One coil for each spatial dimension: \(G_x, G_y, G_z\)**

**G: Gradient Field**

\[
\vec{G} = \frac{dB}{d\vec{r}}
\]

10-50 mT/m in \(~100\mu s\)

Used to determine spatial position of signal (frequency)

Created by a set of 3 additional coils (gradient coil)

**Example: z-gradient coil principle**

(Helmholtz pair)

\[
\vec{B}(\vec{r}) \propto \frac{\mu_0}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}
\]

**NB. Why are MRI scans so loud?**

Lorentz-force of \(B_x\) (3T) on rapidly switched current in gradient coil (wire)

\((\sim 100A \text{ in } \sim 100\mu s)\)

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How is slice-selection achieved?

**Only magnetization on-resonance is excited**

**On-resonance:**

Frequency \(\omega_{RF}\) of RF field \(B_1\) matches the precession frequency of magnetization

\[\omega_{RF} = \gamma B_0 + \gamma G_z z_0\]

**Moving Frequency** \(\omega_{RF}\) alters position of slice:

**NB. Not to confuse:**

\((x,y)\) refers to spatial dimensions

\(M_x, M_y \text{ or } M_z\) refers to transverse magnetization (in magnetization space)

(coordinate systems are different, but share \(z\))
11-2. What is the basic principle of encoding spatial information?

Spatial-varying resonance frequency $\gamma B(x)$ during detection

$B_z(x) = B_0 + G_x x$

Detected signal = sum of all precessing magnetization:

$$S(t) \propto \int_{\text{object}} M_\perp(x,0)e^{ijG_xt} \, dx$$

What does this resemble?

$$S(t) \propto \int_{\text{object}} M_\perp(\omega,0)e^{i\omega t} \, d\omega$$

= Inverse Fourier Transformation!

$$S(k) \propto \int_{\text{object}} M_\perp(x,0)e^{ikx} \, dx$$

For 2D object:

$$M(x) = \int M(x,y) \, dy$$

= Radon Transform

11-3. When is the signal maximal in the presence of G?

Echo formation: Dephasing and rephasing

$S(t) = \{ M(y,0)e^{iG_y t} \} \, dy$

Magnetization in-phase $\rightarrow$ maximal signal (echo formation)

Gradient $G_y(t)$

Magnetization in-phase initially

Phase of magnetization $\phi(t) = \gamma G_y t$

Dephasing

Rephasing

$\tau = TE/2$:

$$S(t) = \int_{\text{object}} M(y,0)e^{-iG_y t} \, dy$$

$$S(t) = \int_{\text{object}} M(y,\tau)e^{iG_y \tau} \, dy$$

$S(t) = \text{maximal (constant } G_y \text{) when } t = \tau$
Is it important when a gradient is applied? gradient applied at different time has the same effect on magnetization phase

Question: Is there a difference in effect on echo?

Application of two orthogonal gradients simultaneously or sequentially generates the same phase for $M_{xy}$

What are the basic elements of the Gradient echo sequence?

NB. Why echo formation?

Gradient switching → Finite rise time

$S \propto \int M(x,y)e^{iG \cdot r} dz \rightarrow 0$

(signal decays like FID in presence of gradient)

⇒ Rephasing (negative) gradient leads to echo formation

$\int \int M(x,y)e^{iG \cdot r} dz = \max_{t \rightarrow 0} \int G_z(t) dt = 0$
11-4. How is the 2nd dimension encoded?

How does the phase encoding gradient encode the 2nd spatial dimension?

Consider two-dimensional object voxel magnetization $M_j$.

Phase encode step 1
Step 2 (twice the gradient strength)

After applying phase encode gradient ($G_y$ for $\tau$ seconds)

Phase of voxel magnetization $e^{i\phi}$:

$$e^{i\gamma G_y y \tau}$$

$$e^{i \gamma k_x x \tau} e^{i \gamma k_y y} = e^{i \gamma (k_x x + k_y y)}$$
How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding?

Phase $\phi$ of a single pixel in $x,y$ plane:

- **Step 1**: Readout ($G_x(t)$)
- **Step 2**: $G_y(t)$
- **Step 3**: $G_x(t)$

Signal of the single voxel:

$$S(n, \Delta G_y, t) \propto M_1(x, y)e^{i n \Delta G_y y} e^{i G_x t}$$

Signal of the entire object:

$$M_2(x, y) \propto \int_{object} S(k_x, k_y) e^{i(k_x x + k_y y)} \, dx \, dy$$

MR image generation:

FT of the signal

11-5. How is the spatial information encoded in MRI?
scanning k-space (Fourier or reciprocal space) sequentially

- **Phase Encode**
- **$G_y(t)$**
- **Sampled Signal**
- **Acq.**

For k-space line every TR=1s: 256$^2$ image matrix > 4 min

One line of k-space acquired per TR

Subject moved head during acquisition

MR scans are long and motion-sensitive

Fourier or reciprocal space ($k_x, k_y$)

Phase Direction (y)

Frequency Direction (x)

Maximum $k_x$ (or $k_y$) $\Leftrightarrow$ Resolution (Nyquist)
Increment $\Delta k$ $\Leftrightarrow$ Field-of-view

Uniform resolution and sensitivity
(Limited by voxel magnetization)

center of k-space ($k_x, k_y=0$)
What are some effects of incomplete sampling of Fourier space (k-space)?

Time of acquisition of center of k-space point \((k_x, k_y=0)\) determines contrast of image:

\[
S(0,0) = \iiint_{\text{object}} M(x,y) \, dx \, dy
\]

Discrete FFT (periodicity + time shift)

Summary: Spatial encoding with gradients

Phase encoding, echo formation + 2DFT

Magnetization at time points specified:

1: \((0,0,M_z)\) rotated by RF pulse by \(\alpha\) about \(x\):
2: \((0,M_y\sin \alpha,M_z\cos \alpha)=\(0,M_{y1},\ldots\) \([\text{now only consider } M_y]\)

Precesses with \(B = -\gamma G_y \gamma G_x\)

3: \(M_y(\sin[\gamma(n\Delta G_y+G_x)x]r), \cos[\gamma(n\Delta G_y+G_x)x]r\)
   \[=M_y(\sin(-\phi), \cos(-\phi))\]
   with \(\phi_i = G_x \tau\) (rotation by angle \(-\phi\))
   inverting gradient, i.e. \(B = +\gamma G_y\gamma G_x\): after another \(\tau\), rotates by angle \(+\phi\)
   \(\Rightarrow\) maximal signal at \(TE=2\tau\) (\(\Delta G_y=0\))

4: \(M_y(0,1,\ldots)=(0,M_y\sin \alpha,M_z\cos \alpha)\)
   \(\Rightarrow\) Echo formation

MRI measures the 2D Fourier transformation of the object (measuring the 2nd dimension requires time!)