4: From x-ray to image – Computed Tomography

1. What factors influence contrast in x-ray imaging?
   - Beam hardening
   - Sensitivity and resolution considerations
2. What influences CNR of x-ray imaging?
3. What is the fundamental basis for image reconstruction using x-ray absorption?
   - Radon Transform
4. How can x-ray images be reconstructed?
   - Sinogram
   - Backprojection vs. filtered backprojection
   - Central Slice Theorem
5. Examples & Summary

After this course you
1. Understand the consequences of the Bremsstrahlung continuum on image contrast
2. Understand how Compton scattering reduces image contrast and how its influence can be reduced
3. Are familiar with the Radon transform
4. Understand the principle of matrix reconstruction and backprojection
5. Understand the major mechanisms leading to CT contrast

4-1. What does absorption in the real world imply?

Linear attenuation coefficient $\mu$

\[
\begin{align*}
\mu & : \text{linear attenuation coefficient} \\
\text{Unit: [cm}^{-1}\text{]} \\
\end{align*}
\]

But, $\mu = f(E, Z, \rho)$

Two consequences:
- Beam hardening
- Depth dependent contrast

Contrast is “well-defined” for monochromatic x-rays

\[
\ln \left( \frac{n(x)}{N_0} \right) = -\mu x
\]

($\mu$ for a homogeneous object of thickness $x$)
**What does the Energy Spectrum of an x-ray tube really look like?**

Filtered Bremsstrahlung and characteristic emission

\[ I_0 = \int_{0}^{\infty} i(E) \, dE \]

\( i(E) \): complex function

- **Minimal energy is zero, but:**
  - Soft x-rays (low energy) are filtered by instrument
  - **E\(_{\text{eff}}\)** is increased by instrument (filtering of soft x-rays)
  - **Maximal x-ray energy is =** kinetic energy of e\(^{-}\) (eV\(_{\text{cathode}}\))

---

**What is the consequence of energy-dependent absorption?**

Beam Hardening - Effective energy depends on depth

**A similar consequence arises in tissue:**

**Ideal:**
- Monochromatic x-rays
- \( E(x) = \delta(x) \)

**Reality:**
- Polychromatic, multienergetic \( i(E) \)

\[ \text{Absorption is not uniform with } E \]
- Contrast changes with large objects and depth
- Excessive radiation dose to superficial tissue

**“Solution”:** Reduce \( i(E) \) for soft x-rays
- (e.g. 3mm Al eliminates 90% of 20keV photons)
4-2. How does x-ray scattering impact CNR?

Scattering increases with FOV (field-of-view) of irradiation
⇒ Further reduction of image contrast

Solution: Anti-scatter grid (collimator)

Compton scattering

increased masking of object.

Photoelectric effect:
complete absorption of photons → Object (lesion) easily detected

With \( E > \) Compton scattering → increased masking of object.

Collimation principle:
establishes directionality of x-rays

High Z, \( \rho \): stop x-rays

Detector

Slice selection

Collimation principle: establishes directionality of x-rays

Fund BioImag 2017

How is CNR quantified?

Signal:
\[ \propto I(d) \text{ (no. photons detected)} \]
\[ \propto I_0 \text{ (no. photons irradiated)} \]

Contrast: \( \Delta I(d) \) due to \( \mu(d) \) differences

- \( i(E) \), \( \mu_c \) produces reduced contrast
- Compton scattering: Antiscatter grid

CT intensity can be measured in absolute terms (CT-number)

Soft tissue: Typically has weak contrast (small Hounsfield units)

CT-numbers of tissue in Hounsfield units (HU)

HU : attenuation normalized to water (\( =0 \))
range from -1000 (air) to +3000 (bone and contrast agents)
soft tissues: -300 to +100

\[ CT \text{ – number} = \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \cdot 1000 \]
4-3. What is the basis of image reconstruction?

The Radon transform

Given a certain beam intensity (no. of photons) \( I \) at a given position \( y_0 \), \( I(y_0) \), the beam intensity at \( y_0 + \Delta y \) is

\[
I(y_0 + \Delta y) = I(y_0) e^{-\mu(y_0) \Delta y}
\]

Considering a two-dimensional object:

\[
I_{\text{detected}}(x) = I_0 e^{-\int_{\text{arc}} \mu(x,y) \, dy'}
\]

\( I_0 \): intensity of incident x-ray beam

Recursive application to derive \( I(y_0 + 2\Delta y) \)

\[
I(y_0 + 2\Delta y) = I(y_0 + \Delta y) e^{-\mu(y_0 + \Delta y) \Delta y}
\]

\[
I(y_0 + 2\Delta y) = I(y_0) e^{-\mu(y_0 + \Delta y) \Delta y} e^{-\mu(y_0) \Delta y}
\]

\[
I(y_0 + 2\Delta y) = I(y_0) e^{-\mu(y_0 + \Delta y) \Delta y + \mu(y_0) \Delta y}
\]

\[
\lim_{\Delta y \to 0} I_{\text{detected}} = I(y_0) e^{-\int_{\text{arc}} \mu(x,y) \, dy'}
\]

Radon transform of a point-like homogeneous object

Radon transform of a rectangular object

Radon transform of a circular object

Each point in space is uniquely represented by Amplitude R and phase \( \phi_0 \) of sinusoidal trajectory in Sinogram (sic!): \((x,y) \to (R, \phi_0)\)
Can a CT image be constructed by Matrix inversion?

Decomposing an object into a 2x2 matrix requires a minimum of 4 measurements:

\[ I_1 = I_0 e^{-(\mu_1 \Delta x + \mu_2 \Delta y)} \]

\[ \ln(I_1/I_0) = -\mu_1 \Delta x - \mu_2 \Delta y \]

\[ \ln(I_2/I_0) = -\mu_3 \Delta x - \mu_4 \Delta y \]

\[ \ln(I_3/I_0) = -\mu_1 \Delta y - \mu_3 \Delta y \]

\[ \ln(I_4/I_0) = -\mu_2 \Delta y - \mu_4 \Delta y \]

Setting \( \Delta x = \Delta y \) yields a linear 2x2 inversion problem linking \( \mu_k \) to \( I_k \).

\[ \begin{bmatrix} \mu_1, \mu_2 \\ \mu_3, \mu_4 \end{bmatrix} = \begin{bmatrix} \ln(I_1/I_0) \\ \ln(I_2/I_0) \\ \ln(I_3/I_0) \\ \ln(I_4/I_0) \end{bmatrix} \]

In principle such an \( n^2 \) matrix can be inverted.

\[ \Rightarrow \text{Too complex, computationally intensive and unstable} \]

CT was introduced in 1970 \( \Rightarrow \) simple reconstruction algorithm!

4-4. What algorithm is adapted to 1970’s computing power?

Backprojection reconstruction

Basic reconstruction principle: Along the measured projection direction fill in each pixel constant numbers corresponding to the Radon transform (projection intensity).

Repeat for next orientation of the projection, sum the values in overlapping pixels.

Illustration with gray shades (point-like object):

2 Projections

4 Projections

8 Projections

16 Projections
Why does simple Backprojection have poor spatial resolution?

Backprojection has poor spatial resolution:

Reconstruction of a point-object falls off with $\frac{1}{r}$

The reconstruction falls off with $\frac{1}{r}$ (in analogy to the decrease of light intensity in 2D)

$$dx\,dy=dA=\rho \sin(d\phi)\,d\rho \equiv \rho \, d\phi \, d\rho$$

Number of rays (projections): constant with $d\phi$

But:

$$dA \propto \rho$$

pixel size = $dx\,dy \propto dA = \text{const}$

$\Rightarrow$ No. of rays $\propto \frac{1}{\rho}$

How can good image resolution be maintained?

Filtered Backprojection

Filtered Backprojection Example
How is Backprojection linked to Fourier transform?

Central Slice Theorem

**Image space**

- **x**
- **y**

**Final Image**
- **CT: acquired data**

**k-space (Fourier space)**

- **k_x**
- **k_y**

**FT**

**IFT**

**MRI: Acquired Data**

\[ M(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) e^{-i k_x x} e^{-i k_y y} \, dx \, dy \]

FT of projection \( \rightarrow \) 1 line in k-space

\[ M(0, k_y) = \int_{-\infty}^{\infty} \mu(x, y) \, dx \quad e^{-i k_y y} \, dy \]

\[ M(k_x, 0) = \int_{-\infty}^{\infty} \mu(x, y) \, dx \quad e^{-i k_x x} \, dy \]

Reconstruction using FT:

**See also: Signals and Systems (SV)**

4-15

---

4-5. X-ray CT: Examples (Human)

**Bone (calcification): bright (high absorption)**

**Air is dark**

**Different densities of tissue give intermediate results**

**Imaging of mummified bodies**

**Dislodged arrow head**

Fund BioImag 2017

4-16
CT: Examples (mouse)

3D CT scan of rodent spine treated with human mesenchymal stem cells (transduced with the human BMP-9 gene via an adenoviral vector) significant bone formation at the treatment sites (arrows)

13μm micro CT of mouse placenta vasculature

Micro-CT of mouse femur bone

CT: Summary

Main contrast is bone vs. soft tissue (or air) (calcium content i.e. e' density p)
Contrast agents (increase Z_eff) allow depiction of vessel architecture and lesions

SNR and CNR:
1. Intensity can be increased by cathode current
2. High spatial resolution possible (limited only by radiation dose in humans)

Insect with iodine contrast agent
How have CT scanners evolved?

Generations of CT scanners

First Generation
- Parallel beam design
- One/two detectors
- Translation/rotation

2nd Generation
- Small fan beam
- Translation/rotation
- Larger no. of detectors

3rd Generation
- Multiple detectors
- Large fan beam

4th Generation
- Detector ring
- Large fan beam