3: Interaction of ionizing radiation with tissue

1. What is the basis of contrast for x-ray imaging?
2. By which mechanisms does ionizing radiation interact with matter?
   - Rayleigh scattering
   - Compton scattering
   - Photoelectric effect
   - Pair production
3. How does this interaction depend on the tissue?
   - Energy dependence and effective atomic number \( Z_{\text{eff}} \)
4. How can we protect ourselves against the biological effects of ionizing radiation?
   - A radiation protection primer

After this course you
1. Know the definition of linear and mass attenuation coefficient
2. Understand the major mechanism of x-ray absorption in tissue
3. Understand the dependence of these mechanisms on photon energy and tissue composition
4. Are able to perform contrast-to-noise calculations using effective \( Z \)
5. Understand and are able to apply the basic principles of radiation protection

3-1. How can we describe attenuation of x-rays?

**Linear attenuation coefficient \( \mu \)**

Fates of the photon (other than transmission):
1. Absorption (transfer of \( h\nu \) to lattice)
2. Scattering

Consider situation where \( \Delta x \to 0 \), and \( n = f(x) \)

\[
\frac{\Delta n}{\Delta x} = -\mu n \Delta x
\]

\( \mu \) : **linear attenuation coefficient**

Unit: [cm\(^{-1}\)]

\[ \mu = f(E, Z, \rho) \]

**Definition**

Half value layer (HVL) = The thickness of a material allowing to pass one half of photons:

\[
n(x_{\text{HVL}}) = N_0/2 = N_0 e^{-\mu x_{\text{HVL}}}\]

\[
\text{HVL} = \frac{0.693}{\mu}
\]

Typical HVL values: several cm for tissues, 1-2 cm for aluminum, 0.3 cm for lead
What are typical attenuation coefficients?

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Electrons per Mass (e/g) x 10^13</th>
<th>Electron Density (e/cm³) x 10^13</th>
<th>μ @ 50 keV (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>0.000084</td>
<td>5.97</td>
<td>0.0005</td>
<td>0.000028</td>
</tr>
<tr>
<td>Water vapor</td>
<td>0.000590</td>
<td>3.34</td>
<td>0.0002</td>
<td>0.000128</td>
</tr>
<tr>
<td>Air</td>
<td>0.00120</td>
<td>3.006</td>
<td>0.00038</td>
<td>0.000290</td>
</tr>
<tr>
<td>Fat</td>
<td>0.91</td>
<td>3.34</td>
<td>3.64</td>
<td>0.153</td>
</tr>
<tr>
<td>Ice</td>
<td>0.917</td>
<td>3.34</td>
<td>3.06</td>
<td>0.156</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>3.34</td>
<td>3.24</td>
<td>0.214</td>
</tr>
<tr>
<td>Compact bone</td>
<td>1.85</td>
<td>3.192</td>
<td>5.91</td>
<td>0.573</td>
</tr>
</tbody>
</table>

\[ \mu / \rho \text{ (water)} = \mu / \rho \text{ (ice)} \]

Definition

Mass attenuation coefficient \( \mu / \rho \)

Unit: [cm²/g]

(constant for all forms of the same chemical substance, e.g. water)

Why do we need more sunscreen in the mountains?

3-2. What are the 4 basic interactions of x-rays with biological tissue?

I. Rayleigh scattering & Compton

Elastic scattering of light or other electromagnetic radiation by particles much smaller than the wavelength of the light, \( x = 2 \pi r / \lambda \text{; } x << 1 \).

Elastic: photon energy = constant

No ionization occurs ("classical" scattering)

Scattered X-rays \( \Rightarrow \) deleterious effect on image quality

very low probability (< 5%) in x-ray imaging

\[ E(\text{keV}) = 1.2 / \lambda (\text{nm}) \]

\[ \lambda = 1.2 / E = 0.012 \text{nm} \]

(@100keV)
II. Compton scattering

Electron is initially at rest
\( e \) gains energy
E binding energy is neglected (why?)

Arthur Holly Compton
Physics, 1927

Occurs at the outer shell electrons
\( \Rightarrow \) ionization
Scattered photon: subject to subsequent interactions
(Rayleigh, Compton scattering or photoelectric effect)

Probability increases with
- photon energy
- electron density

Electron/mass density in tissues \( \sim \) constant
(independent of \( Z \))
\( \rightarrow \) proportional to the density of the material

Relativistic linear momentum
a brief tour back to 1st year physics

From the definition \( p=mv \) (which is true at any velocity) it follows
\[
\vec{p} = m(v)\vec{v} = \frac{m_0}{\sqrt{1-v^2/c^2}} \frac{\vec{v}}{1-v^2/c^2}
\]

The value of \( p \) is

**Relativistic kinetic energy** \( E=mc^2 \)
\[
E^2 - m_0^2 c^4 = \frac{m_0^2}{1-v^2/c^2} \left( \frac{c^4}{1-v^2/c^2} - m_0^2 \left( \frac{1-v^2/c^2}{1-v^2/c^2} \right)^2 \right)
\]
\[
m_0^2 \frac{v^2}{1-v^2/c^2} c^2 = (pc)^2
\]

\[
\vec{p} \cdot \vec{p} = \frac{E^2}{c^2} - m_0^2 c^2
\]

NB. Light carries energy, but moves at the speed of light (!) \( c \).

Photon with energy \( E \):
particle with rest mass (\( m_0=0 \))
(otherwise its energy would be infinite, since \( v=c \)):
\[
|\vec{p}| = \frac{E}{c}
\]
Compton scattering
The basic Equations

A simple elastic collision:

**Conservation of energy**

\[ E_a = (E_i - E_f) + m_e c^2 \]

\[ E_f = \frac{E_i^2 - m_e c^2}{c^2 - m_e c^2} \]

**Conservation of linear momentum:**

\[ \vec{p}_i = \vec{p}_f + \vec{p}_e \]

\[ (\vec{p}_i)^2 = (\vec{p}_f - \vec{p}_e)^2 \]

\[ E_f = \frac{E_i}{(1 - \cos \theta)} E_i / m_e c^2 + 1 \]

Conservation of energy:

\[ E_a = (E_i - E_f) + m_e c^2 \]

\[ E_f = \frac{E_i^2 - m_e c^2}{c^2 - m_e c^2} \]

Conservation of linear momentum:

\[ \vec{p}_i = \vec{p}_f + \vec{p}_e \]

\[ (\vec{p}_i)^2 = (\vec{p}_f - \vec{p}_e)^2 \]

\[ E_f = \frac{E_i}{(1 - \cos \theta)} E_i / m_e c^2 + 1 \]

\[ \lambda_f = \frac{h}{m_e c (1 - \cos \theta)} \]

With \( \lambda = \frac{c}{\sqrt{E}} \) it follows:

\[ \frac{E_f}{E_i} \]

\[ \angle \text{(deg)} \]

3-3. Interaction of photons with tissue II
Photoelectric effect, pair production & summary

Photoelectric absorption effect:

Abruptly increases when \( E \) is slightly above \( I_K \) – absorption edges

Absorption edge energy increases with \( Z \)
(very low for H, C, N, O)

Inner shell \( e^- \) is removed
⇒ energy of the incident X-ray quantum \( E_i \)
> ionization energy of an electron \( I_K \)

\[ E_i = E_e^K + I_K \]

Vacancy in K-shell: filled with outer shell \( e^- \)
⇒ cascade of emitting characteristic X-ray quanta
(or Auger electrons, but not so frequent in diagnostic imaging of soft tissues with low \( Z \))

Albert Einstein
Physics, 1921
IV. Pair production

When the photon energy $> 2m_e c^2 = 1.02\text{MeV}$

It interacts with the electric field of the nucleus of an atom

$v \rightarrow e^- + e^+$

The kinetic energy of the produced particles is

$E = E_v - 2m_e c^2$

Not important in Bio-imaging

3-4. What affects the linear attenuation coefficient?

effective $Z$, $Z_{eff}$

Compton:

depends mainly on the $e$ density $\rho_e$

modestly on energy of x-rays $E_v$

Photoelectric effect:

depends strongly on atomic number $Z$

Biology:

empirical $Z_{eff}$

Linear attenuation coefficient of water and the contribution of each interaction to the total attenuation of X-rays as a function of energy.
What is the effective atomic number $Z_{\text{eff}}$ of biological tissue?

Necessary for estimating $\mu$ of the photoelectric effect.

Empirical relationship for compound materials such as biological tissue:

$$Z_{\text{eff}} = \left( \frac{\sum \lambda_i Z_i^{3.4}}{\text{all tissue components}} \right)^{1/3.4}$$

$\lambda_i = \frac{P_i Z_i / A_i}{\sum P_j Z_j / A_j}$

$P$ : percentage weight

$Z$ : atomic number

$A$ : atomic weight

Example: Estimation of $Z_{\text{eff}}$ for water

$^1$H: $Z = A = 1$, $P = 11\%$

$^{16}$O: $Z = 8$, $A = 16$, $P = 89\%$

Denominator of $\lambda$:

$$\sum P_j \frac{Z_j}{A_j} = 1 \frac{1}{1} + 89 \frac{8}{16} = 55.5$$

Protons: $\lambda = 11/55.5 = 0.20$

Oxygen: $\lambda = 44.5/55.5 = 0.80$.

$$Z_{\text{eff}} = (0.2^{3.4} + 0.8^{3.4})^{1/3.4}$$

$$= (0.2 + 1180 \cdot 0.8)^{1/3.4}$$

$$= 944^{1/3.4} = 7.5$$

How good were we?

What is the % mass composition $P_i$ of select biological tissues?

<table>
<thead>
<tr>
<th>% Composition (by Mass)</th>
<th>Adipose Tissue</th>
<th>Muscle (Striated)</th>
<th>Water</th>
<th>Bone (Femur)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>11.2</td>
<td>10.2</td>
<td>11.2</td>
<td>8.4</td>
</tr>
<tr>
<td>Carbon</td>
<td>57.3</td>
<td>12.3</td>
<td></td>
<td>27.6</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.1</td>
<td>3.5</td>
<td>2.7</td>
<td>41.0</td>
</tr>
<tr>
<td>Oxygen</td>
<td>30.3</td>
<td>72.9</td>
<td>88.8</td>
<td>14.0</td>
</tr>
<tr>
<td>Sodium</td>
<td>0.08</td>
<td></td>
<td></td>
<td>7.0</td>
</tr>
<tr>
<td>Magnesium</td>
<td>0.02</td>
<td></td>
<td>7.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.3</td>
<td>0.5</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>Sulfur</td>
<td>0.06</td>
<td></td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Potassium</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calcium</td>
<td>0.007</td>
<td></td>
<td>14.7</td>
<td></td>
</tr>
</tbody>
</table>

$Z_{\text{eff}} \sim 6$

Carbon:

$Z/A = 6/12$

(same for O, 8/16)
3-5. What are the biological effects of ionizing radiation?

Ionization effects:
- instantaneous (10^{-17}-10^{-5}s)
  1. Produce free radicals
  2. Break chemical bonds
  3. Produce new chemical bonds and cross-linkage between macromolecules
  4. Damage molecules that regulate vital cell processes (e.g., DNA, RNA, proteins)

Biological effects are delayed:
- Cataract (months to years)
- Cancer (years-decades)
- Tissue sensitivity to radiation
  - proportional: rate of cell proliferation
  - inversely prop.: degree of cell differentiation
  - Pregnancy vs. old age ...

Radicals (unpaired valence e⁻) and reactive oxygen species, e.g. 2 OH⁻ \rightarrow H₂O₂

Blood-forming organs
Reproductive organs
Skin
Bone and teeth
Muscle
Nervous system

produced by the body (e.g., oxygen consumption)
How does the tissue defend itself against radiation damage?

**DNA repair**

- **Repair**
  - Replace damaged cells with different cell type (fibrosis)
  - Organ not returned to original state
  - Radioresistant tissues (muscle, brain): only repair possible

- **Regeneration**
  - Replace damaged cells by same
  - Organ returned to original state
  - Radiosensitive tissues (skin, digestive system, bone marrow)

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How are the effects of ionizing radiation quantified?

**Three forms of radiation dose**

**Absorbed dose** $D$
- energy deposited by ionizing radiation per unit mass of material:
  \[ D = \frac{\text{Energy}}{\text{mass}} \]
- Units: [Gray [Gy] = 1 J/kg]

**Equivalent dose** $H$
- $H = D \cdot w_R$
- Units: [Sv = J/kg]

**Effective dose** $E$
- $E(Sv) = \sum w_T \cdot H_T (Sv)$

Absorbed dose $D$ depends on:
1. Intensity of incident x-ray
2. Duration of exposure

- $w_R = 1$ (x-rays, $\beta$ particles)
- $20$: for $\alpha$ particles

Bio-imaging:
- Equivalent dose $H = \text{Absorbed dose } D$
3.6 How can we protect ourselves against x-rays?
Exposure time, distance and HVL

Absorbed dose depends on intensity $I_0$ of radiation (activity in Bq for radioactive materials)
$1$ Becquerel (Bq) = $1$ decay/s

NB. $\beta$-emitters ($^{14}$C, $^3$H):
- $\beta$ do not penetrate low $Z$ material (plastic, glass, avoids Bremsstrahlung)
- high energy $\beta$-emitters ($^{32}$P): low $Z$ shielding + lead (attenuate Bremsstrahlung)

Three basic elements of radioprotection:
1. Time
2. Distance
3. Shielding

Occupancy:
- fraction of time of working day (8h), human is in area exposed to radiation

Received radiation = Fraction of solid angle occupied by human surface $A = A/4\pi d^2$
$\sim I_0/d^2$ ($d$ = distance to source)

Depends on energy and type of radiation

<table>
<thead>
<tr>
<th>Material</th>
<th>HVL (cm)</th>
<th>Iron</th>
<th>Brick</th>
<th>Concrete</th>
<th>Dirt</th>
<th>Ice</th>
<th>Wood (soft)</th>
<th>Snow</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.8</td>
<td>5.1</td>
<td>5.8</td>
<td>8.4</td>
<td>17</td>
<td>22</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

30-fold reduction:
- Iron
- Lead
- Concrete
- Water

Depends on energy and type of radiation

Natural radiation exposure:

<table>
<thead>
<tr>
<th>Source of Exposure</th>
<th>mrem/yr</th>
<th>mSv/yr</th>
<th>%total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radon</td>
<td>200</td>
<td>2.0</td>
<td>55%</td>
</tr>
<tr>
<td>Cosmic</td>
<td>27</td>
<td>0.27</td>
<td>8%</td>
</tr>
<tr>
<td>Terrestrial</td>
<td>28</td>
<td>0.28</td>
<td>8%</td>
</tr>
<tr>
<td>Internal</td>
<td>39</td>
<td>0.39</td>
<td>11%</td>
</tr>
<tr>
<td>Total</td>
<td>300</td>
<td>3</td>
<td>82%</td>
</tr>
</tbody>
</table>

Other sources of radiation exposure:

<table>
<thead>
<tr>
<th>Source of Exposure</th>
<th>mrem/yr</th>
<th>mSv/yr</th>
<th>%total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical X-ray</td>
<td>39</td>
<td>0.39</td>
<td>11%</td>
</tr>
<tr>
<td>Nuclear Med.</td>
<td>14</td>
<td>0.14</td>
<td>4%</td>
</tr>
<tr>
<td>Consumer products</td>
<td>10</td>
<td>0.1</td>
<td>3%</td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>0.63</td>
<td>18%</td>
</tr>
</tbody>
</table>

Useful to know:
- $100$ Röntgen equivalent man (REM) = 1 Sievert
Appendix: Derivation of the Compton Relationship

**Situation:** A photon with energy $E_i$ collides with $e$ of mass $m_e$ at rest. One wants to know the energy of the photon after the collision.

Conservation of energy:

\[
E_f = (E_i - E_f) + m_e c^2
\]

\[
E_f^2 = (E_i - E_f)^2 + (m_e c^2)^2 + 2(E_i - E_f) m_e c^2
\]

\[
p_f^2 c^2 = (E_i - E_f)^2 + 2(E_i - E_f) m_e c^2
\]

\[
p_e^2 c^2 = E_e^2 + m_e c^2 + E_e c^2
\]

\[
-2E_i E_f + 2E_i E_f m_e c^2 = -2E_i E_f \cos \theta
\]

\[
E_i (E_i + m_e c^2 - E_i \cos \theta) = E_i m_e c^2
\]

Conservation of momentum:

\[
(p_f)^2 = (p_i - p_f)^2
\]

\[
(p_i) = p_i + p_f - 2p_i p_f \cos \theta
\]

\[
p_i = \frac{E_i}{c}
\]

\[
p_f = \frac{E_f}{c}
\]

\[
(p_f)^2 = \left( \frac{E_f}{c} \right)^2 - 2E_i E_f \cos \theta
\]

\[
p_f^2 c^2 = E_f^2 + E_f^2 - 2E_i E_f \cos \theta
\]

\[
E_f = \frac{E_i m_e c^2}{E_i (1 - \cos \theta) + m_e c^2}
\]