SHAPE FROM X

One image:
• Shading
• Texture

Two images or more:
• Stereo
• Contours
• Motion
SHAPE FROM TEXTURE
SHAPE FROM TEXTURE

Recover surface orientation or surface shape from image texture.

- Assume texture ‘looks the same’ at different points on the surface
- This means that the deformation of the texture is due to the surface curvature
Basic hypothesis: Texture resides on the surface and has no thickness.

--> Computation under:
- Perspective projection
- Paraperspective projection
- Orthographic projection
Pinhole geometry without image reversal
Perspective projection distortion of the texture
- depends on both depth and surface orientation,
- is anisotropic.
Depth vs Orientation:

Infinitesimal vector \([\Delta x, \Delta y, \Delta z]\) at location \([x, y, z]\). The image of this vector is

\[
f \frac{1}{z} [\Delta x - \frac{x}{z} \Delta z, \Delta y - \frac{y}{z} \Delta z]
\]

Two special cases:

\(\Delta z = 0\) : The object is scaled
\(\Delta x = \Delta y = 0\) : The object is foreshortened
ORTHOGRAPHIC PROJECTION

Special case of perspective projection:
- Large $f$
- Objects close to the optical axis
  $\rightarrow$ Parallel lines mapped into parallel lines.

\[ u = sx \]
\[ v = sy \]
ORTHOGRAPHIC PROJECTION
TILT AND SLANT
ORTHOGRAPHIC PROJECTION

**Tilt:** Derived from the image direction in which the surface element undergoes maximum compression.

**Slant:** Derived from the extent of this compression.
CHEETAH

Orthographic projections of squares that are rotated with respect to each other in a plane inclined at $\omega = 60^\circ$ to the image plane.

$$\left| \frac{\mathbf{p}_1}{l_1} \times \frac{\mathbf{p}_2}{l_2} \right| \left( \frac{\mathbf{p}_1}{l_1} \right)^2 + \left( \frac{\mathbf{p}_2}{l_2} \right)^2 = \frac{\cos(W)}{1 + \cos^2(W)}$$
Generalization of the orthographic projection:

- Object dimensions small wrt distance to the center of projection.

→ Parallel projection followed by scaling
For planar texels:

\[ A' = -\frac{f^2}{z_0} \mathbf{n} \cdot \begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix} A \]
Texels:

- Image regions that are brighter or darker than their surroundings.
- Assumed to have the same area in space.

→ Given enough texels, it becomes possible to estimate the normal.
TEXTURE GRADIENT
Mesure texture density as opposed to texel area, that is, the number of textural primitives per unit surface.

Assuming the texture to be homogeneous, we have: \( \psi \mathbf{n} \propto \mathbf{b} \)

\[
\psi = \begin{bmatrix}
u_1 & v_1 & 1 \\
\ldots & \ldots & \ldots \\
u_n & v_n & 1
\end{bmatrix}^t
\]

\[
\mathbf{b} = [b_1, \ldots, b_n]^t
\]

\[
\Rightarrow \mathbf{n} = \frac{\psi \mathbf{n}}{\|\psi \mathbf{n}\|}
\]
STRENGTHS AND LIMITATIONS

Strengths:
• Emulates an important human ability.

Limitations:
• Requires regular texture.
• Involves very strong assumptions.
• Deep learning might weaken them.