Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 2 – Biophysical modeling: The Hodgkin-Huxley model

2.1 Biophysics of neurons
- Overview

2.2 Reversal potential
- Nernst equation

2.3 Hodgkin-Huxley Model

2.4 Threshold in the Hodgkin-Huxley Model
- Where is the firing threshold?

2.5 Detailed biophysical models
- The zoo of ion channels

Neuronal Dynamics – 2.5 Biophysical models

Neuronal Dynamics – 2.5 Ion channels

Na+ channel from rat heart (Ratlai and Ordi 1985)
A. Traces from a patch containing several channels.
B. Opening times of single channel events
Neuronal Dynamics – 2.5 Biophysical models

There are about 200 identified ion channels
http://channelpedia.epfl.ch/

How can we know which ones are present in a given neuron?

Ion Channels investigated in the study of Toledo-Rodriguez, ..., Markram (2004)

Model of a hypothetical neuron

How many parameters per channel?

C = $g_{Na}E_{Na} + g_{K}E_{K} + g_{leak}E_{leak}$

$\frac{dC}{dt} = -g_{Na}m_{h}(u-E_{Na}) - g_{K}n^4(u-E_{K}) - g_{leak}(u-E_{leak}) + I(t)$

$\frac{dm}{dt} = \frac{1}{\tau_{m}} (n_{i} - n_{m}(u))$

$\frac{dn}{dt} = \frac{1}{\tau_{n}} (n_{i} - n_{n}(u))$

Erisir et al, 1999
Hodgkin and Huxley, 1952

Hodgkin and Huxley, 1952
Model of a hypothetical neuron

Detailed model, based on ion channels

Current pulse

Biophysical model, based on ion channels

Constant current

- Delayed AP initiation
- Smooth f-I curve

type I neuron

Neuronal Dynamics - 2.5 Adaptation

Functional roles of channels?
- Example: adaptation
**Neuronal Dynamics – 2.5 Adaptation: \( I_M \)-current**

- Potassium current
- \( \text{Kv7} \) subunits
- Slow time constant

\[ I_M = g_{\text{K}} m (u - E_K) \]

\( I_M \) current is one of many potential sources of adaptation

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**Neuronal Dynamics – 2.5 Adaptation – \( I_{\text{INaP}} \)-current**

- Persistent sodium current
- Fast activation time constant
- Slow inactivation (~1s)

\[ I_{\text{INaP}} = g_{\text{NaP}} h (u - E_{\text{Na}}) \]

\( I_{\text{INaP}} \) current:
- Increases firing threshold
- Source of adaptation

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**Neuronal Dynamics – 2.5 Biophysical models**

Hodgkin-Huxley model provides flexible framework

Hodgkin & Huxley (1952)

Nobel Prize 1963
Exercise – 2.5. Hodgkin-Huxley model – gating dynamics

A) Often the gating dynamics is formulated as
\[
\frac{dm}{dt} = \alpha_m(u)(1 - m) - \beta_m(u)m
\]
\[
\frac{dm}{dt} = - \frac{m - m_0(u)}{\tau_m(u)}
\]
Calculate \( m_0(u) \) and \( \tau_m(u) \)

B) Assume a form \( \alpha_u(u) = \beta_u(u) = \frac{1}{1 - \exp\left(-(u + \theta)/\beta\right)} \)

How are \( a \) and \( b \) related to \( \gamma \) and \( \theta \) in the equations
\[
\frac{dm}{dt} = - \frac{m - m_0(u)}{\tau_m(u)}
\]
\[
m_0(u) = 0.5(1 + \tanh(\gamma(u - \theta)))
\]

C) What is the time constant \( \tau_m(u) \) ?

Neuronal Dynamics – References and Suggested Reading
