Problem 1.

(a) \[ \frac{dn}{dt} = q_1 + q_2 - O(n) \] (n: number of vehicles in the center)

(b) \[ n_0 = 500 \text{ [veh]} \text{ at 7am}, O(n): \text{Piece-wise linear function} \]

1) \[ \frac{dn}{dt} = q_1 + q_2 - O_1(n) \] \( \Rightarrow \) This is valid till we reach \( n_{cr1} \)

\[ \frac{dn}{dt} = 150 - 0.1n \Rightarrow \frac{dn}{150 - 0.1n} = dt \Rightarrow \int_{n_0}^{n_{cr1}} \frac{dn}{150 - 0.1n} = \int_0^{t_1} dt \Rightarrow \]

\[ -10 \ln(n + 1500) \Bigg|_{n_0}^{n_{cr1}} = t_1 \Rightarrow -10 \ln(500) + 10 \ln(1000) = t_1 \Rightarrow t_1 = 10 \ln(2) \approx 0.7 \text{ min (time that } n = n_{cr1} \text{)} \ (7:07 \text{ am}) \]

2) \[ \frac{dn}{dt} = q_1 + q_2 - O_2(n) \Rightarrow \frac{dn}{dt} = 50 \Rightarrow n = n_{cr1} + \frac{(t - 7) \times 50}{50} \Rightarrow \]

\[ t_2 = 7 + \frac{n_{cr2} - n_{cr1}}{50} = 7 + \frac{500}{50} = 17 \text{ (time that } n = n_{cr2} \text{)} \ (7:17 \text{ am}) \]

3) \[ \frac{dn}{dt} = q_1 + q_2 - O_3(n) \Rightarrow \frac{dn}{dt} = 150 - \frac{(4000 - n)}{25} \Rightarrow \]

\[ \frac{dn}{150 - \frac{4000 - n}{25}} = dt \Rightarrow \int_{n_{cr2} \frac{150 - \frac{4000 - n}{25}}{t_2}}^n \frac{dn}{150 - \frac{4000 - n}{25}} = \int_{t_2}^{30} dt \Rightarrow \ldots \Rightarrow n \approx 2352 \text{ [veh]} \text{ at time 7:30 am} \]
c. $\frac{dn}{dt} = 60 + \alpha \cdot 90 - \frac{(4000 - n)}{25} \cdot \frac{25}{2352}$ ratio of vehicles that could enter the center

\[ \int \frac{dn}{60 + \alpha \cdot 90 - \frac{(4000 - n)}{25}} = \int \frac{dn}{730}. \]

\[ t = 30 \rightarrow \alpha = \frac{10.25}{0.25} \]

\[ n = 2443.5 \text{ [veh]} \]

number of vehicles (queue) in the periphery (outside the city center)

\[ \frac{dn_2}{dt} = q_1 - \alpha q_1 \Rightarrow \frac{dn_2}{dt} = 81.45 \Rightarrow n_2 = 81.45 \times 30 \text{ [veh]} \]

e. $n(k+1) = n(k) + \frac{\Delta t}{5\text{min}} \cdot (q_1(k) + q_2(k))$ - mass conservation equation (Discrete form)

or

\[ n(t + \Delta t) = n(t) + \Delta t \cdot (q_1(t) + q_2(t)) - O(n(t)) \], $\Delta t = 5\text{min}$

* The same approach with discrete equation presented above. For small number of steps, it could be done by hand. In order to solve in a more precise way, you need to choose smaller time unit (e.g., 1 min) and solve it in Excel. The results would be similar to the one obtained in previous sections with dynamic equations.