Bonus Tests

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If the data is in the critical region we...

A. Accept $H_0$
B. Reject $H_0$
C. It depends on the test
D. I don’t know
Solution

Answer B, by definition of the critical region
Saying that a test is of size 5% means that...

A. The probability to accept $H_0$ when it does not hold is $\leq 0.05$
B. The probability to reject $H_0$ when it holds is $\leq 0.05$
C. Both
D. I don’t know
Solution

Answer B, by definition of the size of a test.
If the $p$-value of a test is small we...
Solution

Answer B, by definition of the p-value
We have a collection of random variables $X_i, Y_i$ which correspond to non paired simulation results with configuration 1 or 2. How can you test whether the configuration plays a role or not?

A. With a Wilcoxon Rank Sum test
B. With an ANOVA test
C. With either
D. With none
E. I don’t know
Solution

Answer C

A is robust and can be used if we can ensure that the simulation runs are independent and that the two distributions differ by a location shift, i.e. have same variance

B is applicable if, in addition, $X_i$ and $Y_i$ can be assumed gaussian with same variance
We test whether a distribution is gaussian using a Kolmogorov-Smirnov test against the fitted distribution. We obtain a \( p \) –value

A. The true p-value is smaller
B. We have the true p-value
C. The true p-value is larger
D. It depends on the data
E. I don’t know
Solution

Answer A

The KS test applies if we are testing against a fixed, non fitted distribution F. By using a fitted distribution, we are biasing the test, we are making it more likely than should be to accept the distribution F, i.e. to accept $H_0$. The $p$-value is higher when we accept $H_0$, i.e. we are overestimating the $p$-value.
We have two data sets $X_i$ and $Y_j$ believed to be iid and from one exponential distribution each. We test whether they come from the same distribution and make a likelihood ratio test. The log likelihood ratio statistic is $lrs$. The p-value is...

A. $p \approx 1 - \chi^2_1(2lrs)$
B. $p \approx 1 - \chi^2_2(2lrs)$
C. $p \approx 1 - F(2lrs)$ where $F$ is the CDF of the standard exponential distribution
D. $p \approx 1 - F(2lrs)$ where $F$ is the CDF of the standard Laplace distribution
E. I don’t know
Solution

1. The log-likelihood is \( m \log \lambda - \lambda \sum_i x_i + n \log \mu - \mu \sum_j y_j \) where \( i = 1 \ldots m, j = 1 \ldots n \), \( \lambda \) [resp. \( \mu \)] is the parameter of the expo distrib of \( X_i \) [resp. \( Y_j \)]

2. Under \( H_0: \lambda = \mu \), max is for \( \lambda^{-1} = \mu^{-1} = \frac{\sum_i x_i + \sum_j y_j}{m+n} \)

\[
\ell_0 = (m + n) \log \left( \frac{m + n}{\sum_i x_i + \sum_j y_j} \right) - m - n
\]

3. Under \( H_1 \), max is for \( \lambda^{-1} = \frac{\sum_i x_i}{m} \), \( \mu^{-1} = \frac{\sum_j y_j}{n} \)

\[
\ell_1 = m \log \left( \frac{m}{\sum_i x_i} \right) + n \log \left( \frac{n}{\sum_j y_j} \right) - m - n
\]

\[
l_{rs} = \ell_1 - \ell_0
\]

4. The order is \( p = 2 - 1 = 1 \), \( p \approx 1 - \chi^2_1 (2l_{rs}) \)

Answer A