Renewable Energy: Exercise 10 solution

In this exercise, you will learn about energy storage solutions.

1. Application of Flywheels in Cars

(a) Kinetic Energy: \( E_{\text{kin}} = \frac{1}{2} M \cdot \nu^2 \approx 320 \text{ kJ} \approx 0.089 \text{ kWh} \)

(b) Losses due to air drag: \( P_{\text{air}} = F_{\text{air}} \cdot \nu = \frac{1}{2} \rho_{\text{air}} \cdot c_d \cdot A_{\text{front}} \cdot \nu^3 \approx 4.5 \text{ kW} \)

(c) Necessary \( E_{\text{flywheel}} = \frac{1}{\eta} (E_{\text{kin}} + P_{\text{air}} \cdot \frac{d_{\text{range}}}{\nu}) \approx 35 \text{ MJ} \approx 9.8 \text{ kWh} \)

(d) In a car, there is only space for wheels with a radius \( R \) of up to 70 cm. Therefore \( R \) is set to 70 cm.

The maximal angular frequency is \( \omega = \frac{2}{R} \sqrt{\frac{\sigma_{\text{CFP}}}{\rho_{\text{CFP}}} K} \approx 2500 \text{ rad/s} \approx 24000 \text{ U/min} \)

Comment: This is a rather high value, which probably causes additional losses due to aerodynamic and bearing drag.

The rotational energy of a disc with radius \( R \) and constant thickness \( D \) is

\[
E_{\text{flywheel}} = \frac{1}{2} \Theta \cdot \omega^2 = \frac{1}{2} \omega^2 \int_V r^2 \cdot \rho_{\text{CFP}} \cdot dV = \frac{1}{2} \omega^2 \cdot 2\pi \cdot D \cdot \rho_{\text{CFP}} \int_0^R r^3 \cdot dr = \frac{\pi}{4} \rho_{\text{CFP}} \cdot \omega^2 \cdot D \cdot R^4
\]

According to (c), each flywheel has to store \( E_{\text{flywheel}} = 18 \text{ MJ} \). So now, the thickness \( D \) of one flywheel can be calculated:

\[
D = \frac{4E_{\text{flywheel}}}{\pi \cdot \rho_{\text{CFP}} \cdot \omega^2 \cdot R^4} \approx 9.6 \text{ mm}
\]

The mass of both flywheels is accordingly \( m = 2\rho_{\text{CFP}} \cdot \pi \cdot R^2 \cdot D \approx 44 \text{ kg} \)

(e) The pair of flywheels should store the kinetic energy of a car moving at a speed of 120 km/h:

\[
2E_{\text{flywheel}} = E_{\text{kin}} = \frac{1}{2} M \cdot \nu^2 \approx 720 \text{ kJ} \approx 0.20 \text{ kWh}
\]

Losses due to air resistance are neglected in this case, because the conventional engine can compensate them. There is less space for a supplementary device. As a consequence, the radius of the flywheels \( R \) is set to 30 cm.

The maximal angular frequency is \( \omega = \frac{2}{R} \sqrt{\frac{\sigma_{\text{CFP}}}{\rho_{\text{CFP}}} K} \approx 6000 \text{ rad/s} \approx 57000 \text{ U/min} \)
2. Pumped air storage:

(a) Uncompressed air: \( p_0 \approx 1 \text{ bar} \approx 100 \text{ kPa}, T_0 \approx 25 \degree C \)
Compressed air (gas tank): \( p_1 \approx 300 \text{ bar} \approx 30 \text{ MPa}, T_1 = T_0 \approx 25 \degree C \)
Released air: \( p_2 = p_0 \approx 1 \text{ bar} \approx 100 \text{ kPa}, T_2 < T_0 \)

Isothermal process: \( p \cdot V = n \cdot R \cdot T \) = const. or \( V(p) = \frac{n \cdot R \cdot T}{p} \)

Adiabatic process: \( p \cdot V^\kappa \) = const. or \( V(p) = V_1 \cdot \left(\frac{p_1}{p}\right)^{1/\kappa} = \frac{n \cdot R \cdot T_1}{p_1} \cdot \left(\frac{p_1}{p}\right)^{1/\kappa} \)

(b) Isothermal compression work:
\[
W_{comp} = - \int_0^1 p \cdot dV = - \int_{p_0}^{p_1} p \frac{dV}{dp}
\bigg|_{\text{isothermal}}
\]
\[
dp = nRT_0 \int_{p_0}^{p_1} \frac{dp}{p} = nRT \cdot \ln\left(\frac{p_1}{p_0}\right) \approx 14.1 \text{ kJ/mol}
\]
Adiabatic expansion work:
\[ W_{exp1} = - \int_1^2 p \cdot dV = - \int_{p_0}^{p_1} \frac{dV}{dP}_{adiabatic} dp \]

\[ = \frac{p_1^{1/\kappa} \cdot V_1}{\kappa} \int_{p_1}^{p_0} \frac{p^{1/\kappa} \cdot V_1}{ \kappa} \left( \frac{p_{1/\kappa}}{p_0} - \frac{p_{1/\kappa}}{p_1} \right) = nRT \frac{p_1^{\kappa-1}}{\kappa - 1} \left( \frac{p_0}{p_1} - 1 \right) \approx -5.0 \text{ kJ/mol} \]

Isobaric expansion work:

\[ W_{exp2} = - \int_0^2 p \cdot dV = -p_0(V_0 - V_2) = nRT \left( \frac{p_0}{p_1} \right)^{\frac{\kappa-1}{\kappa}} \approx -2.0 \text{ kJ/mol} \]

Losses:

\[ W_{losses} = W_{comp} - W_{exp1} - W_{exp2} \approx 7.1 \text{ kJ/mol} \]

Efficiency:

\[ \eta = \frac{W_{exp1} + W_{exp2}}{W_{comp}} \approx 50\% \]

(c) From Problem 1c:

Energy needed for 120 km: \[ E_{drive} = P_{air} \cdot d_{range} \approx 24.3 \text{ MJ} \]

Released work from pumped air storage:

\[ W_{released} = W_{exp1} + W_{exp2} \approx 7.0 \text{ kJ/mol} \]

\[ \rightarrow \text{ Minimal amount of air: } n = \frac{E_{drive}}{W_{released}} \approx 3470 \text{ mol}, V_{air} = \frac{R \cdot T_0}{p_1} \frac{E_{drive}}{W_{released}} \approx 0.287 \text{ m}^3 \]

There should be enough space in a car for a 300 litre tank.


3. Pumped water storage:

(a) Potential energy of 1 m\(^3\) water: \[ E_{pot} = m \cdot g \cdot \Delta h = 1000 \cdot 9.81 \cdot 1000 \approx 9.81 \text{ MJ} \]

Annual production of 100 MW\(_p\) PV plant:

\[ E_{prod} = \eta \cdot P_p \cdot t = 0.15 \cdot 10^8 \cdot 365 \cdot 24 \cdot 3600 \approx 4.7 \cdot 10^{14} \text{ J} \]

Amount of water: \[ V_{water} = \eta_{pump} \cdot \frac{E_{prod}}{E_{pot}} = 0.85 \cdot \frac{4.7 \cdot 10^{14}}{9.8 \cdot 10^6} \text{ m}^3 \approx 4.1 \cdot 10^7 \text{ m}^3 \]

(b) Annual production of 100 MW\(_{av}\) PV plant:

\[ E_{prod} = P_{av} \cdot t = 10^8 \cdot 365 \cdot 24 \cdot 3600 \approx 3.2 \cdot 10^{15} \text{ J} \]

Amount of water: \[ V_{water} = \eta_{pump} \cdot \frac{E_{prod}}{E_{pot}} = 0.85 \cdot \frac{3.2 \cdot 10^{15}}{9.8 \cdot 10^6} \text{ m}^3 \approx 2.7 \cdot 10^8 \text{ m}^3 \]

4. Batteries:

(a) for the discharge:

Anode: \( \text{Pb}^{2+} + \text{SO}_4^{2-} \rightarrow \text{PbSO}_4 + 2e^- \)

Cathode: \( \text{PbO}_2 + 4\text{H}^+ + \text{SO}_4^{2-} + 2e^- \rightarrow \text{PbSO}_4 + 2\text{H}_2\text{O} \)
(b) Equation for the electrochemical equilibrium: \( U^0 = \Delta E^0 = -\frac{\Delta G^0}{z \cdot F} \)

\( \Delta G^0 \) for Pb-Acid and F are given, it is possible to see from point a) that \( z = 2 \).

\[ \rightarrow U^0 = 2.04 \text{ V} \]

If a 24 V battery is required, a series of at least 12 Pb-Acid cells is needed \( \rightarrow = \text{c.a 24.5 V} \)

(c) How many moles of Pb got converted? (= moles of PbSO\(_4\) formed on the anode only)

\[ n = \frac{m_{C_d}}{M_{C_d}} = \frac{11.6g}{207.2g/mol} = 0.056 \text{ mol} \]

With the help of the Faraday constant (which defines the mol-specific charge of matter), we can now calculate the overall charge in A.s (=C) we get, when the 56 mmol are converted. Note from the half-cell reaction, that there are 2 electrons involved when 1 Pb is converted.

\[ F = \frac{Q_0}{z \cdot n} \]

\( Q_0 = F \cdot z \cdot n = 96485 \text{ A.s/mol} \cdot 2 \cdot 0.056 \text{ mol} = 10815.9 \text{ C} \)

To determine the time it will take to recharge the battery, we divide the charge by the given current:
\[ 10815.9 \text{ A.s/ 1.5 A} = 7210.6 \text{ s} = 2.0 \text{ h} \]

(d) For obtaining the mass specific charge \( Q \) in Ah/kg we use the Faraday law again. Note, that all the charge-carrying species (educts, left side of the overall reaction equation) are involved in the calculation by their molar masses:

\[ Q = \frac{z \cdot F}{\sum_i M_i} \]

\( \sum_i M_i = 1 \cdot \text{M(Pb)} + 1 \cdot \text{M(PbO}_2\text{)} + 2 \cdot \text{M(H}_2\text{SO}_4\text{)} \)

From the given molar masses for Pb,O,S,H to be 207.2, 16, 32, 1 g/mol respectivey, it is possible to obtain: \( \sum_i M_i = 642.4 \text{ g/mol} \)

Having in mind that \( z \) is still 2 and one hour is made up of 3600 seconds, the specific charge now calculates to \( Q = 83.44 \text{ Ah/kg} \).

The energy density can be obtained from the charge density (= mass specific charge) by multiplying by the reversible cell voltage, since voltage \( U[V].\text{current[A]} = \text{Power P[W]} \) and Power \( \text{P[W].time [s]} = \text{Energy E[W]} \):

\[ E = Q \cdot U^0; \text{ using } U^0 \text{ from above } = 2.04 \text{ V, it follows: } E=170.22 \text{ Wh/kg}. \]

(e) i. Equation for the electrochemical equilibrium: \( U^0 = \Delta E^0 = -\frac{\Delta G^0}{z \cdot F} \)

\[ \rightarrow U^0 = 4.20 \text{ V}. \]

ii. \( Q = \frac{z \cdot F}{\sum_i M_i} \)

\( \sum_i M_i = 1 \cdot \text{M(LiC}_6\text{)} + 1 \cdot \text{M(CoO}_2\text{)}=169.8 \text{ g/mol}; z=1 \)

\[ \rightarrow Q_{\text{Li-ion}} = 157.84 \text{ Ah/kg } \rightarrow U^0_{\text{Li-ion}} \rightarrow E_{\text{Li-ion}} = 662.93 \text{ Wh/kg} \]

compare:
\[ \rightarrow Q_{Pb-Acid} = 83.44 \text{ Ah/kg} \rightarrow U_{Pb-Acid}^0 \rightarrow E_{Pb-Acid} = 170.22 \text{ Wh/kg} \]

iii. reason 1): reversible cell voltage has doubled
reason 2): less weight of the charged electrode
→ both parameters bring big advantage in salebility of a battery system