Neural Networks and Biological Modelling Exam
22 June 2016

• Write your name in legible letters on top of this page.

• The exam lasts 160 min.

• Write all your answers in a legible way on the exam (no extra sheets).

• No documentation is allowed apart from 1 sheet A5 of handwritten notes.

• No calculator is allowed.

• Have your student card displayed before you on your desk.

• Check that your exam has 12 pages

Evaluation:

1. ......./6 pts

2. ....... / 18 pts

3. ....... / 8 pts

4. ....... / 14 pts (10 plus 4 bonus points)

Total: ....... / 46 pts (includes N=4 bonus points)
1 Biophysics of ion channels (6 points)

We consider the following model of a ion channel

\[ I_{ion} = g_0 x^p (u - E) \]

where \( u \) is the membrane potential. The parameters \( g_0, p \) and \( E = 0 \) are constants.

(a) What is the name of the variable \( E \)? .........................

Why does it have this name, what does it signify (give answer in one short sentence)

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number of points: 1

(b) The variable \( x \) follows the dynamics

\[ \frac{dx}{dt} = -\frac{x - x_0(u)}{\tau} \]

where \( x_0(u) \) is monotonically increasing and bounded between zero and one. Suppose that, at \( t = 0 \), we make a voltage step from a fixed value \( E \) to a new constant value \( u_0 \). Give the mathematical solution \( x(t) \) for \( t > 0 \).

\[ x(t) = .................. \]

number of points: 2
Experimental colleagues tell you that they are able to apply voltage steps as in (b) and that by measuring the current they want to determine the parameters $g_0$ and $p$ of the ion channel in (a) and (b).

(c) How should they proceed to measure the parameter $p$? What would be different between the case $p = 1$ and $p = 2$? You can sketch a little figure to illustrate your answer.

(d) Under the assumption that $x_0(u)$ is bounded between zero and 1, how can they measure $g_0$?

number of points: 2

number of points: 1
Synaptic plasticity happens at different time scales. The weight \( w \) of a synapse evolves on a time scale \( \tau_w = 1/\gamma \) but it is coupled to a slower variable \( z \) (time scale \( t\tau_z = 1/\epsilon \)). You may want to think of \( z \) as the physical size of the synapse and of \( w \) as the number of AMPA receptors.

Here are the two equations that we will study:

\[
\frac{dw}{dt} = \gamma [f(w) - w + z + H] \quad (1)
\]
\[
\frac{dz}{dt} = \epsilon [f(z) - z + w] \quad (2)
\]

Normally the input \( H \) is zero. But during a pairing protocol the joint activity of pre- and postsynaptic neuron give rise to a positive 'Hebbian' drive \( H > 0 \). The constants \( \gamma \) and \( \epsilon \) have appropriate units (You may assume \( \gamma = 0.25 \) and \( \epsilon = 0.05 \)).

The function \( f \) is a third-order polynomial

\[
f(x) = -2x(1-x)(2-x) \quad (3)
\]

where \( x = w \) or \( x = z \).

(a) Assume \( H = 0 \) and plot the two nullclines. To do so evaluate the \( w \)-nullcline at

\[
\begin{align*}
  w = 0.0 & \rightarrow z = \ldots \\
  w = 0.5 & \rightarrow z = \ldots \\
  w = 1.0 & \rightarrow z = \ldots \\
  w = 1.5 & \rightarrow z = \ldots \\
  w = 2.0 & \rightarrow z = \ldots 
\end{align*}
\]

and the \( z \)-nullcline at

\[
\begin{align*}
  z = 0.0 & \rightarrow w = \ldots \\
  z = 0.5 & \rightarrow w = \ldots \\
  z = 1.0 & \rightarrow w = \ldots \\
  z = 1.5 & \rightarrow w = \ldots \\
  z = 2.0 & \rightarrow w = \ldots 
\end{align*}
\]

Plot the two nullclines in the figure on the next page. Annotate your lines by writing e.g., \( w \)-nullcline or \( z \)-nullcline.

number of points: 3
(b) In the above graph, add an arrow indicating the direction of flow at the point 
\((w = 2, z = 0)\).

number of points: 1

(c) Keep in mind that \(\gamma = 0.25\) and \(\epsilon = 0.05\) and add, in the above graph, 
representative qualitative arrows indicating the flow in six different regions of 
the phase plane and on all segments of the nullclines

number of points: 4

(d) Suppose that, because of a past pairing protocol, the synapse is in a state 
\((w = 2, z = 0)\). Indicate the trajectory of the synapses in the above graph and 
label it with A. (As before \(\gamma = 5\epsilon\).)

number of points: 1

(e) Suppose that, because of a past pairing protocol, the synapse is in a state 
\((w = 2, z = 1)\). Indicate the trajectory of the synapses in the above graph and 
label it with B. (As before \(\gamma = 5\epsilon\).)

number of points: 1
(f) For the cases discussed in (d) and (e) draw qualitatively the trajectory of the synaptic weight $w(t)$ as a function of time. Use the space here:

number of points: 2

(g) At time $t = 100$, a pairing protocol is applied which corresponds to a 'Hebbian' input $H = 1.5$ which is sustained for a long time.

Redraw qualitatively the two nullclines in the graph here:

number of points: 2
(h) Interpret your results: What happens qualitatively to the nullclines and the fixed points? What does it mean for induction of synaptic plasticity?

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number of points: 2

(i) In the above graph, draw qualitatively a trajectory starting at (0,0). Please use $\epsilon \ll \gamma$.

number of points: 2

Free space for your calculations, do not use to write down solutions/answers.
3 Mean-field models (8 points)

Consider a homogeneous network of $N$ neurons. Each neuron has the same parameters and receives $K \leq N$ inputs from other neurons. An input spike causes a postsynaptic potential $w_0 \epsilon(s)$ with two rectangular phases: $\epsilon(s) = +3mV$ for $0 < s < 1ms$ and $\epsilon(s) = -4mV$ for $1 < s < 2ms$. At all other times $\epsilon(s) = 0$. Note that $s = 0$ corresponds to the spike arrival time. The parameter $w_0$ is positive ($w_0 > 0$).

The total input potential of neuron $i$ is

$$u_i(t) = u^\text{ext} + \sum_k \sum_f w_0 \epsilon(t - t^f_k)$$

(4)

where $u^\text{ext} \geq 0$ denotes external input to the network and the sums run over all presynaptic neurons and all firing times, respectively.

Each neuron emits spike trains with a rate of 10Hz if the input potential is below 10mV and spike trains with a rate of 80Hz if the input potential is above 12mV. The firing rate increases linearly in between.

Assume that connections are random in the following sense: in a network of $N$ neurons, **each neuron receives exactly $K = N/2$ inputs.** Take $w_0 = 100 J_0/K$ and assume that $N$ is larger than 10 000 (formally you can assume $N \to \infty$).

(a) Assume stationary asynchronous firing and determine the population activity graphically. Consider four cases: $u^\text{ext} = 5mV$ and $u^\text{ext} = 20mV$ and choose two different values of $J_0 > 0$.

(space for your graphics here)
Complete your graphics on the previous page and write a short comment on your result:

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number of points: 4

(b) Find the solutions for $u^{\text{ext}} = 20mV$ analytically, as a function of the parameter $J_0 > 0$. (Hint: there are three different regimes).

(space for your calculations here)

If $J_0 < \ldots$ then ........................................................................................................................................
........................................................................................................................................

If $J_0 > \ldots$ and $J_0 < \ldots$ then .................................................................
........................................................................................................................................

If $J_0 > \ldots$ then ........................................................................................................................................
........................................................................................................................................

number of points: 4
4 Stochastic Spike Arrivals and Spiking Probability (10 points + 4 bonus points)

Consider a neuron in a network that receives $K$ inputs from other neurons. An input spike at time $s = 0$ causes a postsynaptic potential $\epsilon_0(s)$ with two rectangular phases: $\epsilon(s) = +3mV$ for $0 < s < 1ms$ and $\epsilon(s) = -4mV$ for $1 < s < 2ms$. At all other times $\epsilon(s) = 0$.

The total input potential of neuron $i$ is

$$u(t) = \sum_k \sum_{f} \epsilon_0(t - t_{kf}^i)$$

where the sums run over all presynaptic neurons and all firing times, respectively.

In the following we assume that all presynaptic neurons fire spikes stochastically (i.e., a Poisson process), each neuron with rate $\nu$.

(a) Determine the mean input potential of the postsynaptic neuron. (assuming $K$ presynaptic neurons, each one firing at rate $\nu$)

$$< u > = \sum_k \sum_{f} \epsilon_0(t - t_{kf}^i)$$

number of points: 2

(b) Evaluate the mean input potential for $K = 1000; \nu = 1Hz; \epsilon_0 = 1$. Pay attention to the units

$$< u > = \sum_k \sum_{f} \epsilon_0(t - t_{kf}^i)$$

number of points: 1
(c) Determine the variance of the potential of the postsynaptic neuron (assuming \( K \) presynaptic neurons firing at rate \( \nu \) where \( K \) and \( \nu \) are arbitrary positive parameters)

\[< u^2(t) >= \]

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number of points: 2

(d) Using your results of (a) and (c), determine the standard deviation of the membrane potential

\[< (\Delta u(t))^2 >^{0.5} = \sqrt{< u^2(t) > - < u >^2} = \]

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number of points: 2

(e) Evaluate the standard deviation for \( K = 1000; \nu = 1Hz; w_0 = 1 \). Pay attention to the units

\[< (\Delta u(t))^2 >^{0.5} = \]

................................................................................................................

number of points: 1

(f) The postsynaptic neuron fires a spike as soon as the membrane potential hits the threshold \( \vartheta = 4mV \) from below. Consider initially \( K = 1000, \nu = 1Hz, w_0 = 1 \). Now suppose that we rescale the weights as \( w_0 = 1000/K \) and we increase the number of presynaptic neurons \( K \) from 1000 to 2000 or 4000.

Does the likelihood of the postsynaptic neuron to fire a spike increase or decrease with \( K \)? (Justify your answer on the next page)

Increase or Decrease? ...............
(g) **Bonus:** Suppose the postsynaptic neuron fires a spike as soon as the membrane potential hits the threshold $\vartheta = 4\text{mV}$ from below.

Take $K = 1000, \nu = 1\text{Hz}, w_0 = 1$.

Do you expect the neuron to fire a spike in an observation period of 100 milliseconds? Justify your answer.

My justification is (intuitive, mathematical, OR graphical)

I predict that $\langle \Delta u(t) \Delta u(t + \Delta) \rangle$ is ......................

because, intuitively, ....


(h) **BONUS** Consider the autocorrelation $\langle \Delta u(t) \Delta u(t + \Delta) \rangle$. Is the autocorrelation at $\Delta = 0.9\text{ms}$ positive, zero, or negative? (Give an intuitive, or graphical, or mathematical argument).

I predict that $\langle \Delta u(t) \Delta u(t + \Delta) \rangle$ is ......................

because, intuitively, ....

number of points: 2