Question Set 7

Exercise 1: Mean field model

Consider a network of $N$ neurons with all-to-all connectivity and scaled synaptic weights $w_{ij} = J_0/N$. The transfer function (rate as a function of input potential) of the neurons is piecewise linear:

$$f = g(h) = \begin{cases} 
0 & , h < h_1 \\
\frac{h-h_1}{h_2-h_1} & , h_1 \leq h \leq h_2 \\
1 & , h_2 < h
\end{cases} \quad (1)$$

The dynamics of the input potential for neuron $i$ is:

$$\tau \frac{dh_i}{dt} = -h_i + RI_i(t) \quad (2)$$

where

$$I_i(t) = I_{\text{ext}}(t) + \sum_j \sum_f w_{ij} \alpha(t-t^{ij}) \quad (3)$$

where $\alpha$ denotes the shape of a the input current caused by a single spike. We assume a constant external current $I_{\text{ext}}$ and are interested in the stationary solutions.

1.1 Find graphically the value of stationary activity $A(t) = A_0$ in the asynchronous state. You may assume that $N$ is large ($N \to \infty$).

1.2 How does the solution change if you change the coupling constant $J_0$? Choose $h_1 = 1$ and $h_2 = 2$ and consider $J_0 = 1$ and $J_0 = 3$. You can assume $R = 1$ and $I_{\text{ext}}(t) = 0$. What happens at $J_0 = 2$? How does the solution change if we additionally have $I_{\text{ext}}(t) \neq 0$?

1.3 Find the solutions analytically (for arbitrary $J_0, h_1, h_2$).
Exercise 2: Randomly connected network: Fixed number of inputs

We consider a homogeneous network of $N$ neurons. Each neuron receives input from $K$ presynaptic neurons (see figure 1). When a spike arrives it generates a postsynaptic current pulse $\alpha(t - t_{fk}^i)$ . The current to neuron $i$ is therefore:

$$I_i = \sum_{k,f} w_{ik} \alpha(t - t_{fk}^i)$$  \hspace{1cm} (4)

Assume the weights are $w_{ik} = \frac{w_0}{K}$ and the network activity is constant: $A(t) = A_0$.

2.1 Give an intuitive or mathematical argument for the following relationship:

$$I_i \approx w_0 A_0 \int_0^\infty \alpha(s) ds$$  \hspace{1cm} (5)

2.2 What happens if $N$ increases? Does the current increase? How about fluctuations?

Figure 1: Two randomly connected networks of different size $N$. The number of inputs per neuron is fixed. (Left: inputs to two representative neurons. Right: inputs to one representative neuron).