Principles of Microeconomics 4

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Chapter 7
(Short Run and Long Run Production)

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Production in the short run

Definitions:

• **Short run**: Period of time during which the quantity of at least one production factor is fixed.  
  => Changes in production can only be obtained by changes in the variable factors.

• **Long run**: Period of time (or forecasting horizon) during which all production factors are variable.
• Initially, we consider one fixed production factor and one variable factor.

Ex.: Fixed factor: “land”, variable factor: “labor”, agricultural production (output). We have:

\[ Q = Q(T, L) \]

where

\( Q \) : harvest (e.g., of wheat) obtained during one period
\( T \) : area of cultivated land, supposed fixed
\( L \) : number of working hours

• Assumption: Labor is perfectly divisible.

• Output (total product): Supposed to change as shown by the illustration below.
2 phases of growth: Initially, the output increases at an accelerated pace. Then, this growth stabilizes (inflection point $I$), before getting slower and slower until point $M$ where the output stops growing.
- The marginal product of the variable factor

**Marginal product** (marginal return) of labor \((MP_L)\):
Defined as the additional output arising from the use of one additional unit of labor in the production process, the quantity of the other factors stays fixed.

\[
MP_L = Q'(L) = \frac{dQ(T,L)}{dL}
\]

- The average product of the variable factor

**Average product** (average return) of labor \((AP_L)\):
Defined as the ratio between the quantity of production (output) and the quantity of labor used for this production.

\[
AP_L = \frac{Q(T,L)}{L}
\]
Note: Returns to factor vs. returns to scale

• By analyzing the marginal and average returns presented by a production function in the short run, we want to know “how the output quantity changes when modifying the variable production factor in the presence of a fixed factor”.

• From now on: We will call these returns “returns to factor” as opposed to returns to scale (change of output due to the change of the quantity of all production factors in the same proportions).

(See long run production.)
Long run production

• Until now, it was assumed that one of the two production factors was fixed, the changes of production being consecutive to the change of the quantity of the other production factor.

• Here: We abandon this hypothesis to allow the analysis of *long run* situations where it is possible to vary simultaneously the quantities of the two production factors.

=> Need to define the concept of the *production function* (PF): The PF establishes a relationship between the quantity produced of a good and the quantities of the different production factors involved in the manufacturing process of this good.
In what follows, we consider the simplified case where there are only **two production factors**, capital and labor.

- \( Q \): quantity produced of the considered good (measured in terms of physical units)
- \( K \): quantity of capital
- \( L \): quantity of labor (measured in terms of services rendered during the period)

**General expression of the production function:**

\[
Q = F(K, L)
\]

**Ex.: Cobb-Douglas production function**

\[
Q = AK^\alpha L^\beta,
\]

where \( A, \alpha \) and \( \beta \) are positive parameters.
Marginal product of each production factor:
Defined as the additional output resulting from the use of an additional unit of this factor, the quantity of the other factor remains fixed.

Assuming the perfect divisibility of each production factor, we have:

\[ MP_L = \frac{\partial F}{\partial L} = F_L' \quad \text{and} \quad MP_K = \frac{\partial F}{\partial K} = F_K' \]
It is supposed that the marginal products are positive and decreasing (decreasing marginal returns to factors), i.e.:

\[ F_L' > 0 \text{ and } F_{LL}'' < 0 \quad F_K' > 0 \text{ and } F_{KK}'' < 0 \]

Ex.: Cobb-Douglas production function

\[ MP_L = A\beta K^\alpha L^{\beta - 1} \]

positive and decreasing for \( 0 < \alpha < 1 \) and \( 0 < \beta < 1 \)

\[ MP_K = A\alpha K^{\alpha - 1}L^\beta \]
**Average product** of each production factor:
Defined as the ratio between the quantity produced of the good and the quantity used of the production factor in question:

\[
AP_L = \frac{Q}{L} \quad \text{and} \quad AP_K = \frac{Q}{K}
\]

Ex.: Cobb-Douglas production function

\[
AP_L = AK^\alpha L^{\beta - 1}
\]

\[
AP_K = AK^{\alpha - 1} L^\beta
\]
• Case of a production function with two factors: Possible to represent the volume of production in the three-dimensional space.

• Thus, we represent:
  – the quantity produced $Q = F(K,L)$ on the vertical axis,
  – the production factors $K$ and $L$ on the two other axis,

so that the level of output is an increasing function of the two factors.
• For reasons of convenience, we cut the output area by horizontal planes, each plane representing a certain level of output.

• The level curves (contour lines) plotted are called **isoquants** (or **iso-product** curves). With a continuous production function, it exists an infinity of level curves, each representing a different level of output.
**Isoquant**

*Definition:*
An isoquant is the set of combinations of production factors which allows to obtain a certain level of output.

- Ex. (chart above): Output $Q_1$ of the good in question can be obtained either by using a lot of capital and little labor (point A), a lot of labor and little capital (point C), or by an intermediate combination (point B).

- Under the assumption of the **perfect divisibility of the production factors**, the isoquants will be continuous.

- => Infinity of combinations of the two factors giving rise to the same output (quantity) produced.
Returns to scale

It is said that there are increasing / constant / decreasing returns to scale when an increase in a certain proportion of the amount of all factors leads to an increase in a larger proportion / in the same proportion / in a smaller proportion of the output level.

• Increasing returns to scale: economies of scale
• Decreasing returns to scale: diseconomies of scale
• A class of production functions, called **homogeneous production functions**, allows to represent the returns to scale.

A production function $Q = F(K,L)$ is called homogeneous of degree $k$ if by multiplying the quantities of the two production factors by a coefficient $\lambda$ (positive), the output is multiplied by the coefficient $\lambda^k$, i.e., $F(\lambda K, \lambda L) = \lambda^k Q$. 
We distinguish between the three following cases:

- \( k < 1 \Leftrightarrow \lambda^k Q < \lambda Q \) : decreasing returns to scale

- \( k = 1 \Leftrightarrow \lambda^k Q = \lambda Q \) : constant returns to scale

- \( k > 1 \Leftrightarrow \lambda^k Q > \lambda Q \) : increasing returns to scale

Ex.: Cobb-Douglas production function

\[
F(\lambda K, \lambda L) = A(\lambda K)^\alpha (\lambda L)^\beta \\
= A \lambda^{\alpha + \beta} K^\alpha L^\beta \\
= \lambda^{\alpha + \beta} F(K, L)
\]

=> The C-D production function is homogeneous of degree \( \alpha + \beta \).
Factors of scale economies

- Division of labor and specialization
- Technological factors
- Financial factors

To these factors of scale economies, we must oppose a factor of **diseconomy of scale** related to the existence of the **limits to an efficient organization**.

Note: The factors of scale economies and diseconomies are all active as **the size of the facility increases**. But while at first the factors of scale economies outweigh those of diseconomies of scale, **from a certain threshold, the relationship is reversed**.

=> The **long run average total cost** curve first decreases, reaches a minimum, and then increases.