Forecasting Bonus Exercise

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Exercise: Compute the prediction

We have a times series $Y_t$. We computed the differenced time series $X_t = Y_t - Y_{t-1}$ and found that $X_t$ can be modelled as an AR process: $X_t = \epsilon_t + 0.5 X_{t-1}$ where $\epsilon_t \sim \text{iid } N(0, \sigma^2)$

1. Is this a valid ARIMA model?
2. Compute a point forecast $\hat{X}_t(2)$
3. Compute a point forecast $\hat{Y}_t(2)$
4. Compute the first 3 terms of the impulse response of the filter $\epsilon \rightarrow Y$
5. Compute a prediction interval for $Y_{t+2}$ done at time $t$
6. How would you compute a prediction interval using the bootstrap?
1. Is this a valid ARIMA model?

A. Yes because $X = F \epsilon$ and $F$ is an ARMA filter
B. Yes because $X = F \epsilon$ and $F$ is a stable ARMA filter with stable inverse
C. No because differencing is not a stable filter
D. I don’t know
2. The point predictions for $X$ are...

A. $\hat{X}_t(2) = X_t + 0.5\hat{X}_t(1), \quad \hat{X}_t(1) = 0.5X_t$
B. $\hat{X}_t(2) = 0.25X_t + 0.5\hat{X}_t(1), \quad \hat{X}_t(1) = 0.5X_t$
C. $\hat{X}_t(2) = 0.5X_t + 0.5\hat{X}_t(1), \quad \hat{X}_t(1) = -0.5X_t$
D. $\hat{X}_t(2) = 0.5\hat{X}_t(1), \quad \hat{X}_t(1) = 0.5X_t$
E. I don’t know
3. The point predictions for $Y$ are ...

A. $\hat{Y}_t(2) = \hat{X}_t(2) + 0.5Y_t(1), \hat{Y}_t(1) = \hat{X}_t(1) + 0.5Y_t$

B. $\hat{Y}_t(2) = \hat{X}_t(2) - 0.5Y_t(1), \hat{Y}_t(1) = \hat{X}_t(1) - 0.5Y_t$

C. $\hat{Y}_t(2) = \hat{X}_t(2) + \hat{Y}_t(1), \hat{Y}_t(1) = \hat{X}_t(1) + Y_t$

D. $\hat{Y}_t(2) = \hat{X}_t(2) + Y_{t+1}, \hat{Y}_t(1) = \hat{X}_t(1) + Y_t$

E. I don’t know
4. What is the impulse response of the filter $\epsilon \rightarrow Y$?

A. 1.000 -1.000 -1.500 ...
B. 1.000 -1.500 -2.250 ...
C. 1.000 1.500 1.750 ...
D. None of the above
E. I don’t know
5. A prediction interval for $Y_{t+2}$ done at $t$ is ...

A. $\hat{Y}_t(2) \pm 1.96 \times \sigma$
B. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{1.25}\sigma$
C. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{2.25}\sigma$
D. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{3.25}\sigma$
E. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{4.25}\sigma$
F. $\hat{Y}_t(2) \pm 1.96 \times \sqrt{5.25}\sigma$
G. I don’t know
6. Which is a correct implementation of the bootstrap for computing 95%-prediction intervals at time $t$ and lag 2?

A. A
Compute the time series $\epsilon_s = X_s - 0.5X_{s-1}$, $s = 3: t$
do $r = 1: 999$

[draw $e^r_s$, $s = 3: (t + 2)$ with replacements from $\epsilon_s$, $s = 3: t$]
[compute $X^r_{1:t}$, $Y^r_{1:t}$ and $\hat{\gamma}^r_t (2)$ using $X^r_s = e^r_s + 0.5X_{s-1}^r$, $Y^r_s = X^r_s + Y^r_{s-1}$ and the formula for $\hat{\gamma}^r_t (2)$]

$$Y^r_{t+2} = e^r_{t+2} + 1.5e^r_{t+1} + \hat{\gamma}^r_t (2)$$

Prediction interval is $[Y^{(25)}_{t+\ell}, Y^{(975)}_{t+\ell}]$

B. B
Compute the time series $\epsilon_s = X_s - 0.5X_{s-1}$, $s = 3: t$
do $r = 1: 999$

[draw $e^r_1$, $e^r_2$ with replacements from $\epsilon_s$, $s = 3: t$]

$$Y^r_{t+2} = e^r_1 + 1.5e^r_2 + \hat{\gamma}^r_t (2)$$

Prediction interval is $[Y^{(25)}_{t+\ell}, Y^{(975)}_{t+\ell}]$