Solution

Problem #1: \( W = 35.6 \text{ eV/ion pair} \) for 5.3 MeV \( \alpha \) particles

- \# ion pairs produced = \( \frac{(5.3 \times 10^6 \text{ eV/\alpha})(100 \text{ \alpha/s})}{35.6 \text{ eV/ion pair}} = 1.489 \times 10^7 \text{ ion pairs/s} \)

- \( I = (1.489 \times 10^7 \text{ electrons/s})(1.6 \times 10^{-19} \text{ Coulomb/electron}) \)
  \[ I = 2.38 \times 10^{-12} \text{ Ampere} \]

This is a very small current, but it can be detected by fairly conventional microamperometers.

- The range of 5.3 MeV \( \alpha \) particle in Al is 5.6 mg/cm².
  \( \rho_m = 2.7 \text{ g/cm}^3 \)

  the necessary thickness will be
  \[ x = \frac{5.6 \times 10^{-3} \text{ g/cm}^2}{2.7 \text{ g/cm}^2} \]
  \[ x = 2.1 \times 10^{-3} \text{ cm} \]

Problem #2: \( I = I_0 e^{-\mu x} \)

- \( 0.05 = e^{-\mu(4.5)} \)
  \[ \mu = 0.666 \text{ cm}^{-1} \]

  the half-thickness is the thickness of lead which will reduce the intensity by one half
  \[ I/I_0 = 0.5 = e^{-\mu x/2} \]
  \[ x_{1/2} = \frac{\ln 0.693}{\mu} = \frac{0.693}{0.666} = 1.041 \text{ cm} \]

  \[ R = \frac{1}{\mu} = \frac{1}{0.666} = 1.5 \text{ cm} \]
Problem #3: The counting error may be expressed by the factor:

\[ \frac{1}{1 - MD} = \frac{1}{1 - (1500 \times 10^6) (200 \times 10^{-6})} = \frac{1}{1 - 5 \times 10^3} = 1.00503 \]

The true count would then be

\[ N = \frac{M}{1 - MD} \]

\[ N = 1500 \times 1.00503 = 1508 \text{ cpm} \]

Problem #4: The standard deviation of the resulting count is the sum of the standard deviations square of the sample counting and the background counting:

\[ \sigma^2 = \sigma_s^2 + \sigma_b^2 \]

\[ \sigma_s = \sqrt{3576 \times \frac{4}{5}} \]

\[ \sigma_b = \sqrt{562/5} \]

\[ \sigma = \sqrt{386.22 + 22.48} \]

\[ \sigma = 20.2 \]

The counting rate due to the source alone, therefore, is

\[ R = (M_s - M_b) \pm \sigma \]

\[ R = ((3476) - 112) \pm 20 \]

\[ R = (3364 \pm 20) \text{ counts per minute} \]
There are 3 situations of interest to be considered when specifying a given standard deviation:

1. Minimum total counting time:
   In this case the differential \( d(t_5 + t_6) = 0 \), and
   \[ t_5 = \frac{R_5 + \sqrt{R_5 R_6}}{\sqrt{2}} \]

2. Equal counts:
   In this case \( M_5 = M_6 \) and
   \[ M_5 = \frac{R_5^2 + R_6^2}{\sqrt{2}} \]

3. Equal counting times:
   In this case \( t_5 = t_6 \) and
   \[ t_5 = \frac{R_5 + R_6}{\sqrt{2}} \]

Problem #5:

- We assume that a typical child has a cross-section of 1.2 m high and 0.3 meter wide, and an average thickness of 0.15 m.

- Cs-137 emits beta electrons, and 661.7 keV gamma photons. This is an open source, so we must also consider the electrons!

- Starting with the electrons, there are two different contributions. First, 94.4% of the electrons populate the excited (661.7 keV) level in Ba-137. Therefore, their energy has a maximum at

  \[ @ E = 662 - 662 = 1176 - 662 = 514 \text{ keV} \]
**Beta Particle Range**

The maximum range, \( R_{\text{max}} \), (material independent) of a beta particle can be computed from an empirical formula given by Katg and Penfold:

\[
R_{\text{max}} = \begin{cases} 
0.412 \times E_b^{1.165} \times 0.0955 \times \ln (E_b) & \text{for } 0.01 \leq E_b \leq 2.5 \text{ MeV} \\
0.530 \times E_b - 0.106 & \text{for } E_b > 2.5 \text{ MeV}
\end{cases}
\]

\( E_b \) is in MeV units.

The ability to stop betas depends primarily on the number of \( \overline{\alpha} \)s in the absorber (i.e. the areal density, which is the number of \( \overline{\alpha} \)/cm\(^2\)). Hence, the range when expressed as a density thickness (g/cm\(^2\)) of the material gives a generic quantifier by which various absorbers can be compared. Within the maximum range known, the actual shielding thickness required can be computed:

\[ t = \frac{R_{\text{max}}}{\text{material density}} \]

- Thus, the Katg-Penfold formula gives us, using \( \text{air} = 1.205 \times 10^{-3} \text{ g/cm}^3 \)

  a range of \( \left( \frac{1}{1.165 \times 10^3} \right) \times 0.412 \times 0.514 \)

  \( \approx 141 \text{ cm} \)

- For the (5.6%) beta-electrons with the max. \( E \) of 1176 keV, we get

  in the same way:

  \( \left( \frac{1}{1.165 \times 10^3} \right) \times 0.412 \times 1.176 \)

  \( \approx 419 \text{ cm} \)
We see that the electrons are indeed stopped by the air, after just a few meters. They should therefore not contribute to the dose!

- For the 662 keV X-rays, we need to consider the air absorption due to the long distance:

\[
\text{fair} = \frac{I}{I_0} = e^{-\mu x} = e^{-8.055 \times 10^{-2} \times 1.205 \times 10^{-3} \times 80 \times 10^2} = 0.46
\]

where we have used the absorption coefficient for 600 keV gamma rays in air. We see that only 46% of the radiation reaches the day-care centre.

- For the absorption of the walls, we have, in the same way:

\[
\text{concrete} = \frac{I}{I_0} = e^{-\mu x} = e^{-8.286 \times 10^{-2} \times 2.3 \times 10} = 0.15
\]

We see that only 15% passes the walls.

- We also need the fraction that interacts with the tissue:

\[
\text{tissue} = 1 - \frac{I}{I_0} = 1 - e^{-\mu x} = 1 - e^{-8.873 \times 10^{-2} \times 1.06 \times 15} = 0.76
\]

- ... and the solid angle fraction:

\[
\text{f.s.a.} = \frac{1.2 \times 0.3}{4\pi \times 80^2} = 4.476 \times 10^{-6}
\]
The energy released per second from the source, in the form of gamma photon (662 keV) is:

\[ P = 662 \times 10^3 \times 100 \times 3.7 \times 10^{10} \times 1.602 \times 10^{-19} \times 0.944 = 3.70 \times 10^7 \text{ J/s} \]

Here, note the 0.944 factor. Only this fraction in the decay of Cs-137 populates the 662 keV state in Ba-137.

Assuming a mass of 25 kg for one child, we have for the absorbed dose:

\[ D = \frac{P \times \text{femtoper centimeter} \times 3600 \times (5 \text{ femtoper centimeter} + 3)}{25} = 3.13 \times 10^{-4} \text{ Gy} \]

\[ \Rightarrow \text{ for } \text{ff}, \text{ and full body dose, we therefore have an effective dose of } 0.21 \text{ mSv}. \]

This is a very small dose, comparable with dose we on average receive from the background radiation over one month. The children should therefore be safe! (No radiation sickness, no long-term effects).