8: Introduction to Magnetic Resonance

1. What are the components of an MR scanner?
2. What is the basis of the MR signal?
3. How is nuclear magnetization affected by an external magnetic field?
4. What affects the equilibrium magnetization?
5. How do we best describe the motion of magnetization (in the rotating frame of reference)?

After this week you
1. Are familiar with the prerequisites for nuclear spin
2. Know the factors determining nuclear magnetization
3. Can compare magnetizations for different nuclei and magnetic field
4. Know the equation of motion for magnetization
5. Are able to describe the motion of magnetization in lab and rotating frame
6. Understand that MRI has complex mechanisms

8-1. What are the essential components of an MRI scanner?

It's a complex machine …

This course focuses on the major elements of MRI:
- Nucleus
- Magnet
- RF coil
- Gradient coil

Schematic depiction of all MRI components

Cut-open in real life
What are the risks of the scanner being never off?

Superconducting wires cooled to 4He temperature (4K)
Current stays for 1000 years …
It’s a powerful magnet …

Magnetic field $B_0$ [unit: Tesla, T]
- Earth’s magnetic field $\sim 5 \times 10^{-5}$ T
- Electromagnets $< 1.5$ T
- MRI 1-7 T

8-2. What is the basis of Nuclear Magnetism?
Classical and quantum-mechanical view

Nucleus $\rightarrow$ angular momentum L (here called P)
$\Rightarrow$ Rotation of electrical charge (nucleus)
$\Rightarrow$ Rotating current
$\Rightarrow$ Dipole moment

Magnetic moment $\mu$ of individual spin in induction field $B_0$:
$\mu = \gamma B$

$\gamma$: gyromagnetic ratio (empirical constant)
The angular momentum $P$ of a nucleus is quantized:

$$P_z \text{ has } 2I + 1 \text{ values (m): }$$

$$P = h \frac{\gamma}{2\pi} m, \quad |m| = \frac{1}{2}, \ldots, \frac{I}{2}$$

Spin $\frac{1}{2}$: $P = h\sqrt{3/4\pi}$

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Spin (I)</th>
<th>Gyromagnetic ratio $\gamma/2\pi$ (MHz T$^{-1}$)</th>
<th>Abundance / %</th>
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<tbody>
<tr>
<td>$^1\text{H}$</td>
<td>1/2</td>
<td>42.58</td>
<td>99.98</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>1</td>
<td>6.54</td>
<td>0.015</td>
</tr>
<tr>
<td>$^1\text{P}$</td>
<td>1/2</td>
<td>17.25</td>
<td>100.0</td>
</tr>
<tr>
<td>$^2\text{Na}$</td>
<td>3/2</td>
<td>11.27</td>
<td>100.0</td>
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<tr>
<td>$^1\text{N}$</td>
<td>1/2</td>
<td>4.31</td>
<td>0.37</td>
</tr>
<tr>
<td>$^1\text{C}$</td>
<td>1/2</td>
<td>10.71</td>
<td>1.108</td>
</tr>
<tr>
<td>$^1\text{F}$</td>
<td>1/2</td>
<td>40.08</td>
<td>100.0</td>
</tr>
</tbody>
</table>
What is the basis for nuclear magnetization?

Unequal population of Energy levels

Energy of a magnetic dipole in magnetic field $B_0$ (classical):
$$E = -\mu \cdot B_0 = -\mu \cdot \cos \theta \cdot B_0 = -\mu_z \cdot B_0$$

Energy is minimal, when $\mu || B_0$
(Where is that used?) $\vec{\tau} = \mu \times \vec{B}_0$

Quantum mechanical description:
$$E_i = -\frac{\hbar}{2\pi} m_i \cdot B_0 \quad m_i = -I, \ldots, I$$

Boltzmann statistics/distribution: Unequal population of energy levels
$$\frac{N_1}{N_2} = e^{\frac{\Delta E}{kT}}$$

$k$: Boltzmann's constant ($1.4x10^{-23}$ J/Kelvin)

NB. At 310K: $\sim 1$ in $10^6$ excess protons in low energy state (1Tesla)
$\rightarrow N_1 - N_2 = N/2$ (N = no of spins)

Transitions between $E_1$ and $E_2$ induced by photons
$$h\nu = \Delta E$$

Increasing wavelength [nm]

Increasing frequency [s$^{-1}$]

Increasing energy

8-3. How to classically describe the motion of magnetization?

View each spin as a magnetic dipole $\mu$ (a tiny bar magnet).
Classically: torque $\tau$ of a dipole $\mu$ in $B$

$$\vec{\tau} = \mu \times \vec{B}$$

2nd law of rotations (P: angular momentum)
$$\vec{\tau} = \frac{d\vec{P}}{dt} \quad \vec{\mu} = \gamma \vec{P}$$
$$d\vec{\mu} = \vec{\tau} \times \vec{B}$$

Sum over all $\mu_k$ $\rightarrow$ Magnetization $\vec{M} = \sum \mu_k$

Larmor equation
$$\frac{d\vec{M}}{dt} = -\gamma\vec{B} \times \vec{M}$$

What motion does the Larmor equation describe?

A brief tour back to rotational kinematics
$$\vec{v} = \text{circulates}$$
$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$
$$r_i \rightarrow \text{circulates}$$
$$\vec{r}_i \rightarrow \text{const}$$

Describes a rotation of $r$ about $\omega$
with frequency $f = \omega/2\pi$

$\Rightarrow$ valid for any vector entity $\vec{M}, \vec{L}$
instead of $\vec{r}$

Precession of $\vec{M}$ about $\vec{B}$ with frequency $\gamma B/2\pi$
**What is precession?**

**Observation:** The motion of the axis of the wheel with mass $M_W$ is circular about $O$ with constant angular velocity $\Omega$ dictated by $W_L$.

$$\frac{dL}{dt} = \Omega \times \vec{L}$$

What is the value of $\Omega$?

From Newton's 2nd law (rotations):

$$\frac{dL}{dt} = \vec{r} = -M_W \vec{g} \times \vec{r} = -M_W \frac{\vec{r} g}{r} = -\frac{M_W g}{r} \frac{\vec{L}}{L}$$

$$\frac{dL}{dt} = \frac{M_W g}{r} \frac{\vec{L}}{L} = \vec{L} = \Omega \vec{L}$$

$\Rightarrow$ Precession frequency $\Omega = \frac{r}{L} \frac{M_W g}{L}$

Precession frequency increases with:
1. mass $M_W$ of the wheel $\rightarrow$ gyro magnetic ratio $\gamma$
2. gravitational pull $g$ $\rightarrow$ magnetic field $B_0$

Just like a spinning Gyroscope in gravity

$$\frac{dM}{dt} = -\gamma \vec{B}_0 \times \vec{M}$$

**8-4. What are the essentials of Magnetic Resonance?**

**nucleus & magnetic field**

Nuclear equilibrium magnetization $M_0$

$$M_0 = \left( N_2 - N_1 \right) \mu$$

$$N_2 = N/2$$

$$M_0 = \frac{h \mu}{4\pi kT} \gamma B_0 N$$

Magnetization increases with:
1. No. of spins $N$ (molecules)
2. magnetic field $B_0$
3. gyromagnetic ratio $\gamma$

Nucleus with non-zero spin and high gyromagnetic ratio $\gamma$: $^1H$

Magnet to create magnetic field $B_0 \parallel z$

$\left( N_2 - N_1 \right) \mu_z$ results in equilibrium magnetization $M_0$

$\Delta E$ is small ($\sim \mu eV$)

$\Rightarrow$ Non-ionizing e.m. fields

MRI: $^1H_2O$!

Larmor frequency

$$f_L = \frac{\gamma B_0}{2\pi}$$

$$\omega_L = \gamma B_0$$

Convention in magnetic resonance:

Static magnetic field $B_0 \parallel z$

$\Rightarrow$ thermodynamic equilibrium: $M_0 \parallel z$

MR is safe, but insensitive
How can the sensitivity be increased?

magnetic field strength $B_0$

MRI of the lower abdomen

MRI of the breast (1.5 vs 3 Tesla)

maximum possible MR signal:
determined by
equilibrium nuclear magnetization $M_0$

8-5. Why use a Rotating frame of reference to describe the motion of magnetization?

Rotating frame: A reference frame which rotate about $z$ of the laboratory frame at frequency $\omega_{RF}$

Why use a rotating reference frame?

$$\frac{d}{dt} \vec{M} = \vec{M} \times \gamma \vec{B}$$
What is the equation of motion for magnetization in the rotating reference frame?

Larmor frequency in reference frame rotating with $\omega_{\text{RF}}$: $\Omega = \omega_{\text{L}} - \omega_{\text{RF}}$

$\Rightarrow \Delta B = \Omega / \gamma = B_0 - \omega_{\text{RF}} / \gamma$
[lab frame: $\omega_{\text{RF}} = 0 \Rightarrow \Omega = \omega_{\text{L}} (\Delta B = B_0)$]

For $\omega_{\text{RF}} = \omega_{\text{L}}$, $\Delta B = 0$ (on-resonance):

\[
\begin{align*}
\text{Off-resonance} \\
\frac{dM}{dt} = \Omega \times M \\
\end{align*}
\]

(fictitious) magnetic field $\omega_{\text{RF}} / \gamma$ is progressively subtracted from $B_0$

On-resonance: $\Omega = 0$

\[
\Delta B
\]

Ex. Flipping magnetization over in the rotating reference frame

Start with thermodynamic equilibrium magnetization $M_0$
Reference frame rotating with $\omega_{\text{L}}$ (on-resonance)
Apply additional, constant magnetic field with magnitude $B_1$ (in xy plane) for time $\tau$

What motion can be observed for $M$?

\[
\frac{dM}{dt} = -\gamma B_1 \times M
\]

$M_0$ precesses about $B_1$

Magnetization rotates about $B_1$ with angular velocity $\gamma B_1$

Frequency $\gamma B_1 / 2\pi$

$\Rightarrow$ period $T = 2\pi / \gamma B_1$

Definition Flip angle = angle of rotation $\alpha$ induced by $B_1$ applied for $\tau$ seconds

Special cases of $\alpha$:

90°: Full excitation (all $M_0$ is rotated into transverse plane, xy, i.e. $M_0 \rightarrow M_{xy}$)

180°: Inversion ($M_z \rightarrow -M_z$)

$B_1$ = radiofrequency (RF) field (why?)

Lab frame: $B_1(t) = B_1(\cos \omega_{\text{L}}, \sin \omega_{\text{L}})$

$\gamma \approx 42$MHz/Tesla $\rightarrow \omega_{\text{L}} / 2\pi \approx 100$MHz
Supplement: Why there is only equilibrium magnetization along $B_0$?

Random Phase approximation

Quantized magnetic moment $\mu$

\[
|\vec{\mu}| = \gamma \frac{h}{2\pi} \sqrt{J(J+1)}
\]

\[
\mu_z = \gamma \frac{h}{2\pi} \cdot m_z
\]

$\mu_z < \mu_{\text{tot}}$

Individual spin is never aligned with $B_0$ …

But … Equilibrium $M_0 \parallel B_0$

Phase $\phi$ of $\mu_{xy}$ is random (random phase approximation):

No net $\mu_{xy}$

Bulk Nuclear Magnetization:

\[
\vec{M} = \sum \vec{\mu}
\]

Phase of $\mu_{xy}$