Networks Out of Control: Dynamics 2 (Cascades)
Cascades

- There is a network of people, and a behavior that spreads through the network from person to person across edges.
  - E.g., an idea, product, illness or habit spreads across the network.
  - (Already discussed this informally in the context of homophily)

- How does it spread?
  - Last week: probabilistic effects
  - This week:
    - Direct-benefit effects
    - Rational effects

- What do we study?
  - Will it spread to the entire network?
  - Are there threshold properties that determine if/when it does?
Dynamics 2: Shelling Threshold Model
Schelling Threshold Model

- There are \( n \) people (labeled \( i = 0, 1, 2, \ldots, n-1 \)). Each has a “willingness to riot” coefficient \( r_i \) which is *how many others* decide to riot before they join in.

- Can think of this process as occurring on the complete graph:
  - Is there a complete riot in this situation?
Schelling Threshold Model

- There are 100 people (labeled $i = 0, 1, 2, \ldots, 99$). Each has a “willingness to riot” coefficient $r_i$ which is *how many others* decide to riot before they join in.

- Can think of this process as occurring on the complete graph:
  - Is there a complete riot in this situation?

- I.e., the existence of a cascade depends on the parameters.
Schelling Threshold Model

- There are 100 people (labeled $i = 0, 1, 2, \ldots, 99$). Each has a “willingness to riot” coefficient $r_i$ which is how many others decide to riot before they join in.

- Can think of this process as occurring on arbitrary graphs:
  - Is there a complete riot in this situation?

- I.e., the existence of a cascade depends on the network.
Cascades:
Direct-Benefit Effects
Crash Course in Game Theory

- **Agents**: two or more participants.
- **Actions**: options available to the agent.
- **Outcome**: global end result (function of all agent’s actions).
- **Utility**: real-valued function of outcome for a given agent.

In game theory:
- Agents want to maximize their own utility (selfish behavior)
- We study Nash equilibria, i.e., outcomes such that no agent can unilaterally increase their utility by changing their action.
Diffusion on Networks

- There are two products, A and B.
- There is a social network G, and each node can select to use a single product A or B.
- Let $n_A(v)$ be the number of v’s neighbors using A.
- Let $a, b > 0$. The utility for agent v is
  - If v uses A: $a n_A(v)$
  - If v uses B: $b n_B(v)$
- When does v select A instead of B?
  - $p > b / (a+b)$
- When does the whole network converge (cascade) to one option?
Example

- Let \( a = 3 \) and \( b = 2 \)
  - So \( p = \frac{2}{5} \) is the threshold for selecting \( A \)
Example

- Let $a = 3$ and $b = 2$ (so $p = 2/5$).
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- Has applications to viral marketing – which nodes should one target (and then hope they influence their neighbors)?

- Will a cascade always occur?
Cascades

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- Will a cascade always occur?
  - No – “clusters” can get in the way.

- A p-dense cluster is a set of nodes such that all of nodes in the cluster have at least a p fraction of their neighbors inside the cluster.
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Theorem: Consider an initial set of adopters $V'$ and a threshold $p$ for adoption. The nodes in $V \setminus V'$ contain a cluster of density greater than $1-p$ if and only if a complete cascade does not occur.
A p-dense cluster is a set of nodes such that every node in the set has a $p$ of its neighbors inside the set.

Theorem: Consider an initial set of adopters $V'$ and a threshold $q$ for adoption. The nodes in $V \setminus V'$ contain a cluster of density greater than $1-q$ if and only if a complete cascade does not occur.
**Small Exercise**

- For which range of $p = \frac{b}{(a+b)}$ does a complete cascade occur from white to pink?

- If you wanted to initiate a cascade, which vertex would you want to as an initial adopter?
Cascades: Rational Effects
Information Cascades

- Two options A and B
  - You have some (private) information as to which option is better.
  - You have some public information as to which option other people think is better.
**Cascades**

- **Example:**
  - A = guess red urn,  B = guess blue urn
  - Noisy signal = sample from urn
  - Public decision = guess of previous players
Cascades

- Two options A and B
  - Each player receives a private *noisy signal* that indicates which option is better.
  - Each player observes sequential public *decisions*.
Cascade Example

- How should players make their decisions?

\[
\Pr \left[ \text{majority-blue} \mid \text{what she has seen and heard} \right] > \frac{1}{2}
\]

- In order to analyze this, we’ll make heavy use of Bayes’ rule:

\[
\Pr \left[ A \mid B \right] = \frac{\Pr \left[ A \cap B \right]}{\Pr \left[ B \right]}
\]

- Note:

\[
\Pr \left[ \text{majority-blue} \right] = \Pr \left[ \text{majority-red} \right] = \frac{1}{2}, \\
\Pr \left[ \text{blue} \mid \text{majority-blue} \right] = \Pr \left[ \text{red} \mid \text{majority-red} \right] = \frac{2}{3}.
\]
Cascade Example

Consider the first player:

- Assume they saw blue (the argument for red is symmetric).

\[
\Pr[\text{majority-blue} | \text{blue}] = \frac{1/3}{1/2} = \frac{2}{3}.
\]
Consider the second player:

- If the first player saw blue and they also see blue:
  - Clearly $P[\text{majority-blue}] > \frac{1}{2}$ so they say blue
- If the first player saw blue and they see red:
  - Won’t do this formally, but with Bayes’ rule can prove that $P[\text{majority-blue}] = P[\text{majority-red}] = \frac{1}{2}$
  - Tiebreak – (let’s assume they stick with color they see).
Cascade Example

- Consider the third player:
  - If the first two see one red and one blue
    - Reduces to the first-player setting because first two samples were not informative.
  - If all three see blue
    - Clearly $P[\text{majority-blue}] > \frac{1}{2}$, and they say blue.
  - If the first two see blue and they see red...
Consider the third player:

Assume the first two see blue, but they see red.

\[ \Pr[\text{majority-blue} | \text{blue, blue, red}] \]

\[ \Pr[\text{blue, blue, red} | \text{majority-blue}] \]

\[ \Pr[\text{blue, blue, red}] \]

\[ \Pr[\text{majority-blue} | \text{blue, blue, red}] = \frac{4}{27} \cdot \frac{1}{2} = \frac{2}{3}. \]
Cascade Example

Consider the fourth player:
- Assume the first two see blue, and they see red.

$$\Pr[majority-blue | blue, blue, red]$$

- Reduces to the three-player setting!

$$\Pr[A | B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$ 

$$\Pr[majority-blue] = \Pr[majority-red] = \frac{1}{2}.$$ 

$$\Pr[blue | majority-blue] = \Pr[red | majority-red] = \frac{2}{3}.$$
Cascade Example

cascade starts above this line

cascade starts below this line
Information Cascades

- Generally, cascades
  - Can lead to sub-optimal outcomes (the crowd may not be wise)
  - Can be based on very little information
  - Are fragile

- Main bottleneck to “wisdom” -- decisions NOT independent.
  - Also known as “herding”

- Can think of this process as occurring on a graph.