Multi-period optimisation problems

- **Type of problems**
  - Periodic
    - same environment, same requirement but independant
    - defined by:
      - conditions(p), requirement(p) for duration(p)
        » ex. typical days, day/night, summer/winter
    - optimal operating conditions for each period
    - storage is not considered
  - Operation horizon
    - defined by:
      - conditions(t), requirement(t) @ time(t)
        » ex. daily profiles, batch processes recipe
    - Scheduling: when to produce what, where
    - Optimal management inc. storage
Targeting the integration of multi-period utility systems for site scale process integration

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Problem definition

- Industrial site

Existing steam network

Import export

Boilers

Turbines

5 process sections
Process Requirements

- 5 processes:

  - **P1**: Heating: 0 kW, Cooling: 3562 kW
  - **P2**: Heating: 25932 kW, Cooling: 0 kW
  - **P3**: Heating: 0 kW, Cooling: 5491 kW
  - **P4**: Heating: 0 kW, Cooling: 24629 kW
  - **SITE reference**: Heating: 22560 kW (38510 kW), Cooling: 17732 kW (33682 kW)

  **P5**: Heating: 12578 kW, Cooling: 0 kW
Total site composite

Integrated SITE
Heating : 9565 kW (22560 kW)
Cooling : 4050 kW (17732 kW)
Refrigeration : 686 kW
Interest of integration

- **Heat pump not useful for P3**
- **Heat pump saving potential for total site**: 2957 kW (30%)

- **Representation with all the hot and cold streams**
  - System sub-divisions
  - No abstraction of pockets potentials
Production levels variations

Market conditions
Productions shifts
Batch
Efficiency variations
Maintenance
Cleaning procedures
Process requirement

Nominal
- Heating: 9565 kW
- Cooling: 4050 kW
- Refrigeration: 686 kW

Average
- Heating: 10270 kW
- Cooling: 386 kW
- Refrigeration: 261 kW

Heat pump?
CHP?
Refrigeration?
Utility system selection

Integrated SITE
Heating: 9565 kW
Cooling: 4050 kW
Refrigeration: 686 kW

Gas turbine

Process Heat + CHP - Heat pump?
Insertion of a gas turbine

Select
Requirements
Variations
Constraints

Expert system

List of technologies

For each tech

Identify parameters

Model the performances

Insertion model parameters

Adapt
Fuels
Ambient cond.

Integrate

Insertion model

Tech data base

Knowledge
Results of the selection

### Table 1
Catalog characteristics of the selected gas turbines

<table>
<thead>
<tr>
<th></th>
<th>El (kWe)</th>
<th>Heat (kW)</th>
<th>$\eta_e$ (%)</th>
<th>$\eta_h$ (%)</th>
<th>IC</th>
<th>$\epsilon$/MWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>GT1</td>
<td>5500</td>
<td>8876</td>
<td>31.5</td>
<td>53.5</td>
<td>4438.4</td>
<td>4</td>
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<td>5300</td>
<td>9024</td>
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<td>5</td>
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<td>15895</td>
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<td>51.3</td>
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<td>4.7</td>
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<tr>
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<td>8018</td>
<td>12904</td>
<td>32.0</td>
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<td>6377</td>
<td>4.7</td>
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</table>
Resolution

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>T</th>
<th>T_{\text{sat}}</th>
<th>type</th>
</tr>
</thead>
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<tr>
<td>S1</td>
<td>145</td>
<td>867</td>
<td>620</td>
<td>production</td>
</tr>
<tr>
<td>S2</td>
<td>54</td>
<td>744</td>
<td>544.7</td>
<td>production/\text{use}</td>
</tr>
<tr>
<td>S3</td>
<td>11</td>
<td>577</td>
<td>456.0</td>
<td>production/\text{use}</td>
</tr>
<tr>
<td>S4</td>
<td>4</td>
<td>477</td>
<td>412</td>
<td>production/\text{use}</td>
</tr>
<tr>
<td>S5</td>
<td>1.5</td>
<td>-</td>
<td>374</td>
<td>\text{use}</td>
</tr>
<tr>
<td>S6</td>
<td>0.1</td>
<td>-</td>
<td>320</td>
<td>\text{use/cond}</td>
</tr>
</tbody>
</table>

Definition

Optimisation

Representation

CHP

10500 kW
Multi-period principle

• Process annual operation defined by a set of operating periods
  – Limited number of sets: \( n_p \)
  – For each set we have to define the operating conditions of utility system

• Definition of operating period \( p \)
  – \( L_{j,p} \): production levels or conditions for process \( j \)
  – \( t_p \): operating time (h/years)

• Assume no heat storage
  – Sequence of operations not constraining
Load scenarios calculation

\[
\min_{L_{j,p}} \sum_{t=1}^{n_t} \left\{ \min_{p=1,\ldots,n_p} \sum_{j=1}^{n_j} \left( \frac{L_{j,p} - q_{j,t}}{\omega_j} \right)^2 \right\}
\]

\( n_p = 5 \)
\( n_t = 8760 \)
\( n_j = 10 \)
Load scenario model

• Problem characteristics
  – Discontinuous
  – Non-linear
  – Multi-modal

• Solved by a Evolutionary Algorithm
  – Easy implementation
  – Robustness
  – Multi-objectives
    • Validity for each process
    • Trade-off
      – Number of representing levels
      – Individual errors
      – Overall error
  – ? Computation time

\[
\min_{L_{j,p}} \sum_{t=1}^{n_t} \left\{ \min_{p=1, \ldots, n_p} \sum_{j=1}^{n_j} \left( \frac{L_{j,p} - q_{j,t}}{\omega_j} \right)^2 \right\}
\]
## Multi-period levels

Minimum energy requirements and levels of operations in the different periods

<table>
<thead>
<tr>
<th></th>
<th>MER hot</th>
<th>MER cold</th>
<th>MER frg</th>
<th>time</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(kW)</td>
<td>(kW)</td>
<td>(kW)</td>
<td></td>
<td></td>
<td>h/year</td>
</tr>
<tr>
<td>L1</td>
<td>22581.</td>
<td>0.</td>
<td>155.</td>
<td>432</td>
<td></td>
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<tr>
<td>L2</td>
<td>0.</td>
<td>18119.7</td>
<td>1027.8</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>L3</td>
<td>19814.</td>
<td>987.</td>
<td>347.</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>L4</td>
<td>10146.</td>
<td>1411.</td>
<td>380.</td>
<td>264</td>
<td></td>
</tr>
<tr>
<td>L5</td>
<td>27771.</td>
<td>0.</td>
<td>0.</td>
<td>720</td>
<td></td>
</tr>
<tr>
<td>L6</td>
<td>13311.</td>
<td>4897.</td>
<td>751.</td>
<td>144</td>
<td></td>
</tr>
<tr>
<td>L7</td>
<td>0.</td>
<td>10526.</td>
<td>677.</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>L8</td>
<td>12252.</td>
<td>173.</td>
<td>311.</td>
<td>1224</td>
<td></td>
</tr>
<tr>
<td>L9</td>
<td>7061.7</td>
<td>831.</td>
<td>310.</td>
<td>2904</td>
<td></td>
</tr>
<tr>
<td>L10</td>
<td>8539.</td>
<td>577.</td>
<td>278.</td>
<td>2592</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10270.</td>
<td>385.6</td>
<td>261</td>
<td>8760</td>
<td></td>
</tr>
</tbody>
</table>

**Endothermic**

**Exothermic**
Multi-period model principle

Objective function = \( \sum_i y_i \ (\text{Inv}_i(f_{\text{size}})) + \sum_i \sum_t f_{ti} \times C_{t1} \times \text{Time}_t \)

\( f_{\text{size}} > f_{ti} \quad \forall t \)

\( y_i > y_{ti} \quad \forall t \)

Utility i

Use in period t : \( y_{it} \)

Flow in period t : \( f_{it} \)

\( y_{1i} \ f_{\text{min}} < f_{ti} < y_{1i} \ F_{\text{max}} \)

\( y_{2i} \ f_{\text{min}} < f_{ti} < y_{2i} \ F_{\text{max}} \)

\( y_{3i} \ f_{\text{min}} < f_{ti} < y_{3i} \ F_{\text{max}} \)

\( y_{4i} \ f_{\text{min}} < f_{ti} < y_{4i} \ F_{\text{max}} \)

List_A
Time = 8000 h/years

List_stopped
Time = 500 h/years

List_min
Time = 1 h

List_max
Time = 1 h
Multi-period problem

Generic formulation

\[
\min_{y_t, x_t, y_s} \sum_{t=1}^{n_t} t_t \cdot c(x_t, s) + I(y, s)
\]

Submitted to:

\[
\begin{align*}
  h_t(x_t, s) &= 0 & \forall t = 1, \ldots, n_t \\
  g_t(x_t, s) &\geq 0 & \forall t = 1, \ldots, n_t \\
  y_t &\leq y & \forall t = 1, \ldots, n_t \\
  y_t, y &\in \{0, 1\} & \forall t = 1, \ldots, n_t
\end{align*}
\]

Only \( s \) (size => investment) and \( y \) (investment decisions) are shared between periods
Multi-period MILP problem formulation

\[
\min_{R^p, y^p, f^p, y^w, f^w} \sum_{p=1}^{n_p} \sum_{w=1}^{n_w} \left( C^{2p} f^p_w + Cel^p Wel^p - Cel^p Wel^p \right) \cdot t_p
\]

Linearised investment

\[
\sum_{w=1}^{n_w} \left( C1_w y^w_{max} \right) + \frac{1}{T} \sum_{w=1}^{n_w} \left( ICF_w y^w_{max} + ICP_w f^w_{max} \right)
\]

For each period

Heat Cascade

\[
\sum_{w=1}^{n_w} f^p_w q_{w,r} + \sum_{i=1}^{n} Q_{i,r} \cdot L_{i,p} + R^p_{r+1} - R^p_r = 0
\]

\[
R^p_1 = 0, R^p_{n_r+1} = 0, R^p_r \geq 0
\]

Electricity Consumption

\[
\sum_{w=1}^{n_w} f^p_w w_w + Wel^p - L_{c,p} \cdot Wc \geq 0
\]

Electricity Production

\[
\sum_{w=1}^{n_w} f^p_w w_w + Wel^p - Wel^p_s - L_{c,p} \cdot Wc = 0
\]

\[
Wel^p \geq 0, Wel^p_s \geq 0
\]

Gas turbines

Combustion

Steam network ...

Part load operation

\[
f_{min} y^p_w \leq f^p_w \leq f_{max} y^p_w
\]

\[
y^p_w \in [0, 1]
\]

Size

\[
f^w_{max} - f^p_w \geq 0
\]

Decision

\[
y^w_{max} - y^p_w \geq 0
\]
Limitations of the model

- **Linearised efficiencies and cost**
  - Mixed Integer Linear programming formulation

- **Targeting approach**
  - Heat exchangers not represented
  - Heat cascade constraints represent the ideal heat exchanger network (feasible)

- **No heat storage considered**

- **DTmin/2 constant**
  - Trade-off DTmin/2 should increase for shorter time feasible exchange
Application

- 5 Processes (60 streams)
- 10 operation levels
- 4 Gas turbines
- Refrigeration system
- Steam network
- Heat pump

- MILP problem characteristics
  - 1900 constraints
  - 1600 variables
  - 50 decision variables
## Results

Table 4
Results with different gas turbines, Natural gas price : 0.135 €/kg, Electricity : 35€/MWh(high)

<table>
<thead>
<tr>
<th></th>
<th>Electricity (MWh)</th>
<th>Fuel (MWh)</th>
<th>Total cost (€/year)</th>
<th>CHP Eff (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GT_{3\text{low}}$</td>
<td>116494</td>
<td>248996</td>
<td>263893</td>
<td>78.6</td>
<td>GT shut down</td>
</tr>
<tr>
<td>$GT_{1\text{int}}$</td>
<td><strong>90303</strong></td>
<td><strong>210622</strong></td>
<td><strong>-226811</strong></td>
<td><strong>82.2</strong></td>
<td>Min post combustion</td>
</tr>
<tr>
<td>$GT_{3\text{int}}$</td>
<td>143949</td>
<td>304960</td>
<td>-78213</td>
<td>70.5</td>
<td>Min post combustion</td>
</tr>
<tr>
<td>$GT_{4\text{int}}$</td>
<td>113220</td>
<td>243344</td>
<td>-220005</td>
<td>79.4</td>
<td>Min post combustion</td>
</tr>
<tr>
<td>$GT_{1\text{high}}$</td>
<td>139690</td>
<td>356721</td>
<td>-377481</td>
<td>54.7</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_{2\text{high}}$</td>
<td>137665</td>
<td>354075</td>
<td>-209181</td>
<td>54.4</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_{3\text{high}}$</td>
<td>192503</td>
<td>451327</td>
<td>-196827</td>
<td>54.9</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_{4\text{high}}$</td>
<td>166219</td>
<td>400155</td>
<td>-381427</td>
<td>55.5</td>
<td>Max post combustion</td>
</tr>
<tr>
<td>$GT_{1\text{hpmp int}}$</td>
<td><strong>88428</strong></td>
<td><strong>204451</strong></td>
<td><strong>-220910</strong></td>
<td><strong>85.3</strong></td>
<td>Min post combustion +hpmp</td>
</tr>
</tbody>
</table>

CHP eff = \( \frac{E_l(MWh)}{Fuel(MWh) - \frac{M_{er}(MWh)}{\eta_{th}}} \) with \( \eta_{th} = 95\% \) the efficiency of a conventional boiler.
### Solution with heat pump

Details of the solution computed with $GT_{int}^{hpmp}$

<table>
<thead>
<tr>
<th>CND</th>
<th>HPMP</th>
<th>GT</th>
<th>CHP</th>
<th>FUEL</th>
<th>EFF</th>
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</thead>
<tbody>
<tr>
<td>kmol/s</td>
<td></td>
<td>kWe</td>
<td>kWe</td>
<td>kWLHV</td>
<td>%</td>
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<tr>
<td>L1</td>
<td>0.013</td>
<td>OFF</td>
<td>5515.0</td>
<td>11455.0</td>
<td>36640.8</td>
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<tr>
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<td>0.349</td>
<td>45.7</td>
<td>5515.0</td>
<td>13679.0</td>
<td>16849.8</td>
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<td>10359.0</td>
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<td>L4</td>
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<td>110.1</td>
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<td>7967.3</td>
<td>19829.7</td>
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<td>5515.0</td>
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<tr>
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<td>OFF</td>
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<td>9875.0</td>
<td>19040.4</td>
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<td>61.5</td>
<td>5515.0</td>
<td>9400.5</td>
<td>20115.5</td>
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</table>
## Part load optimal operation target

Details of the solution computed with $GT_{1_{\text{int}}}$

<table>
<thead>
<tr>
<th></th>
<th>GT (MWh)</th>
<th>CHP power (kW)</th>
<th>CHP total (MWh)</th>
<th>Fuel (MWh)</th>
<th>CHP Efficiency (%)</th>
<th>Condensing kmol/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>2382</td>
<td>11493</td>
<td>4965</td>
<td>15867</td>
<td>88.7</td>
<td>0.013</td>
</tr>
<tr>
<td>L2</td>
<td>265</td>
<td>13660</td>
<td>656</td>
<td>808</td>
<td>81.1</td>
<td>0.34</td>
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<tr>
<td>L3</td>
<td>1059</td>
<td>10879</td>
<td>2089</td>
<td>6457</td>
<td>85.2</td>
<td>0.0</td>
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<tr>
<td>L4</td>
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<td>2219</td>
<td>5618</td>
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<td>3970</td>
<td>12404</td>
<td>8931</td>
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<tr>
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<td>794</td>
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<td>3900</td>
<td>82.4</td>
<td>0.0</td>
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<td>1323</td>
<td>11178</td>
<td>2683</td>
<td>4043</td>
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<td>0.27</td>
</tr>
<tr>
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<td>10379</td>
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<td>30735</td>
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<td>0.0</td>
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<td>57981</td>
<td>81.1</td>
<td>0.0</td>
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<tr>
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<td>25006</td>
<td>54240</td>
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<td>0.0</td>
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<tr>
<td>Total</td>
<td>48311</td>
<td>90303</td>
<td>210622</td>
<td>82.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Influence of electricity cost

Total cost as a function of the electricity cost

- Max post combustion
- Gas turbine shut down
- Low post combustion

Solution:
- Gas turbine shut down
- Condensation turbine

Electricity price in Euro/MWh

Total cost (Euro/year)
Comments on the formulation

- **Consistent data set needed**
  - Linear programming goes to the constraint
- **CHP solutions highly sensitive to electricity and fuels costs**
  - Replace “yes/no” investment decisions with “what if?”
    => sequential resolution (for each period) instead of simultaneous resolution
- **Sensitivity analysis is important**
- **First step**
  - To combine with multi-period HEN design?
  - System models and NLP optimisation
  - Reliability …
Conclusions

• MILP formulation
  – Target multi-period utility system integration
    • CHP and gas turbines
    • Total site integration
  – Part load optimal targeted operation for utility system

• Integrated into a www based tool
  – Process integration
  – Technology selection
  – Technology integration

• Helps understanding process integration
• Supports engineers creativity
• Decision support