For Problems 1 and 2 we need the following definitions:

- A cycle $C_n$, $n \geq 3$, consists of $n$ vertices $v_1, v_2, \ldots, v_n$ and edges \{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$ and \{v_n, v_1\}.
- A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of $m$ and $n$ vertices, respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.
- A simple graph is called regular if every vertex of this graph has the same degree.
- The complementary graph $\overline{G}$ of a simple graph $G$ has the same vertices as $G$. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in $G$.

**Problem 1.** Draw each of these graphs.

1. The complementary complete graphs $\overline{K_2}, \overline{K_3}, \overline{K_4}$.
2. The complementary complete bipartite graphs $\overline{K_{2,2}}, \overline{K_{2,3}}, \overline{K_{2,4}}, \overline{K_{3,4}}$.
3. The complementary cycles $\overline{C_3}, \overline{C_4}, \overline{C_5}$.

Find a general description of $\overline{K_n}, \overline{K_{m,n}}, \overline{C_n}$.

**Problem 2.**

1. How many vertices does a regular graph of degree four with ten edges have?
2. For which values of $n$ is the cycle $C_n$ regular?
3. For which values of $m$ and $n$ is the complete bipartite graph $K_{m,n}$ regular?
4. For which values of $m$ and $n$ is $\overline{K_{m,n}}$ regular?
Problem 3. Find an adjacency matrix for the graph and determine how many paths of length 3 there are from $A$ to $D$. How many paths of length 3 are there from $E$ to itself?

Problem 4. A directed graph is called strongly connected if there is a path from $A$ to $B$ and from $B$ to $A$, for any pair of vertices $A, B$ in the graph. A directed graph is called weakly connected if there is a path between every two vertices in the underlying undirected graph. Determine whether each of these directed graphs is strongly connected and if not, whether it is weakly connected.

1.  

2.  

3.
Problem 5. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path, if one exists.

1.

2.

3.
Problem 6. A simple path in a graph $G$ that passes through every vertex exactly once is called a *Hamilton path*, and a simple circuit in a graph $G$ that passes through every vertex exactly once is called a *Hamilton circuit*.

For each of these graphs, determine whether the graph has a Hamilton circuit and construct such a circuit, if it exists. If no Hamilton circuit exists, determine whether the graph has a Hamilton path and construct such a path, if one exists.

1.

![Graph 1](image1)

2.

![Graph 2](image2)
Problem 7. (This is taken from Rosen's book, Exercise 17 of Section 10.6.)
The weighted graph shows some major roads in New Jersey and the distances between cities on these roads.

Use Dijkstra’s algorithm to find the shortest path (and its length) between

1. Newark and Camden,
2. Newark and Cape May.