Problem 1. Until recently, all books were identified by a 10-digit International Standard Book Number (ISBN-10), a 10-digit code $x_1x_2 \ldots x_{10}$, assigned by the publisher. An ISBN-10 consists of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally, a check digit that is either a digit or the letter $X$ (used to represent 10). This check digit is selected so that
\[ \sum_{i=1}^{10} ix_i \equiv 0 \mod 11. \]

1. Check if 084930149 is a valid ISBN-10 number.
2. Compute the last digit of the ISBN-10 number 335128442__.
3. In the ISBN-10 number 0387027333 two consecutive digits were swapped. Find which digits these were and what the correct ISBN-10 number is.
4. Find an example of a ISBN-10 number where two swapped digits cannot uniquely be corrected.

Problem 2. Some airline tickets have a 15-digit identification number $a_1a_2 \ldots a_{15}$ where $a_{15}$ is a check digit that equals $a_1a_2 \ldots a_{14}$ mod 7 (where $a_1a_2 \ldots a_{14}$ is a 14-digit number, not a product).

1. Find the check digit $a_{15}$ for the following airline ticket identification number: 00193222543435__.
2. Determine whether 101333341789013 is a valid airline ticket identification number.
3. Can the accidental transposition of two consecutive digits in the first 14 positions of an airline ticket identification number be detected?

Problem 3. Compute $3^{21}$ mod 11 with the binary expansion of 21

1. from left to right.
2. from right to left (Horner’s method).

Problem 4. Use mathematical induction to prove that

1. $3^n < n!$ if $n$ is an integer greater than 6.
2. $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer $n$. 