In the class, we looked at the following way to count the elements of $N \times N$

\[(a, b) \mapsto (1+2+3+\ldots+(a+b)) + a =: f(a+b)+a\]
What we Need to Show

$N \times N \rightarrow N$, \((a,b) \mapsto f(a+b)+a\), is injective!

• Suppose that it is not injective.
• Then \(f(a+b)+a = f(x+y)+x\) for two pairs \((a,b) \neq (x,y)\).
• Assume first that \((a+b) \neq (x+y)\).
  • If \((a+b) < (x+y)\)
    • Then \(f(a+b)+a = (1+2+\ldots+(a+b))+a < (1+2+\ldots+(a+b))+a+b+1 = f(a+b+1) \leq f(x+y)+x\), which is a contradiction to \(f(a+b)+a = f(x+y)+x\).
    • Similarly, \((a+b) > (x+y)\) leads to a contradiction.
• Therefore, \((a+b) = (x+y)\).
  • \(f(a+b)+a = f(x+y)+x\) then implies \(a = x\), and since \((a+b) = (x+y)\), also \(b = y\), so \((a,b) = (x,y)\), a contradiction.
• So, the mapping is injective, and we are done!