Biological Modelling of Neural Networks Exam
23 June 2015

- Confirm that your exam copy has 8 pages total.
- Write your name in legible letters on top of the first page.
- The exam lasts 160 min.
- Write all your answers in a legible way on the exam (no extra sheets).
- No documentation is allowed apart from 1 sheet A5 of handwritten notes.
- No calculator is allowed.
- Have your student card displayed before you on your desk.

Evaluation:

1. ....... / 9 pts
2. ....... / 22 pts
3. ....... / 11 pts

Total: ....... / 42 pts
1 Synaptic Plasticity (9pts)

Throughout this section, we consider a synaptic plasticity rule of the form

\[
\frac{dw_{ij}}{dt} = -a_0 \nu_{j}^{\text{pre}} + a_2 \nu_{j}^{\text{pre}} \nu_{i}^{\text{post}} - a_4 (\nu_{i}^{\text{post}})^4
\]

where \(i\) denotes the postsynaptic neuron and \(w_{ij}\) the synaptic weight from neuron \(j\) to neuron \(i\), and \(a_0, a_2, a_4\) are positive constants.

(a) Is this a Hebbian rule? Justify your answer

yes, no ... because .................................................................................................................................

number of points: /1

(b) Is this a local rule? Justify your answer

yes, no ... because .................................................................................................................................

number of points: /1

(c) Does this learning rule allow for both potentiation and depression? [Hint: assume \(0 < a_4 \ll a_2\) and consider presynaptic neuron ON/OFF; postsynaptic neuron ON/OFF where ON is suitably defined]

yes, no ... because .................................................................................................................................

number of points: /1
(d) Suppose we have a network of 4 neurons. Each neuron is described by a firing rate $\nu = g(h) > 0$. The network is in a homogeneous state, so that all neurons always have the same firing rate: $\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu$. The initial state of all weights is $w = 0.0001$.

Write down the differential equation for the evolution of the weight from neuron 2 to neuron 4.

\[
\text{number of points: } /1
\]

Assume $a_0 = a_2$ and $0 < a_4 \ll a_2$. Will the weight grow, or decay? Always? Approach a fixed point or not? [Hint: Consider different initial states of the weight and of the rates!]

Please justify your answers, by mathematical calculation or suitable graphics.

\[
\text{number of points: } /3
\]

(e) Would your answers change if $a_0 \ll a_2$?

Yes/No because ................................................................................................................

Would your answers change if $a_0 = 0$?

Yes/No because ................................................................................................................

\[
\text{number of points: } /2
\]

(space for calculations)
We now combine synaptic plasticity with neuronal dynamics. We consider four rate neurons with gain function

\[ \nu = g(h) = [1 - (1 - h)^2] \nu_0 \quad \text{for} \quad 0.1 < h < 1 \]

and \( g(h) = 0.19 \nu_0 \) for \( h \leq 0.1 \) and \( g(h) = \nu_0 \) for \( h \geq 1 \);

and dynamics (for neuron \( i \))

\[ \tau \frac{dh_i}{dt} = -h_i + \sum_{j \neq i} w_{ij} g(h_j) \]

Each neuron receives input from the three other neurons in the network via weights \( w_{ij} \) that follow a dynamics

\[ \tau_w \frac{dw_{ij}}{dt} = -\tilde{a}_0 g_{pre}^j + g_{pre}^j g_{post}^i - \tilde{a}_4 (g_{post}^i)^4 \]

(a) How many **differential equations** are necessary to describe the dynamics in our plastic network of 4 neurons?

number of points: /1

(b) Assume that all weights are identical (but time dependent), and that all neuronal firing rates are identical (but time dependent). Assume unit-free variables and set \( \tilde{a}_0 = 0.00 \) and \( \tilde{a}_4 = 1/4 \) and \( \nu_0 = 8/3 \). Write down the two resulting equations (you may keep \( g(h) \) without inserting the expression).

\[ \tau \frac{dh}{dt} = \]

\[ \tau_w \frac{dw}{dt} = \]

number of points: /2

(c) Calculate the nullclines [Hint: you may keep \( g(h) \) without inserting the expression and note that \( g(h) > 0 \) for all \( h \)]

number of points: /3

Space for calculations
(d1) Evaluate the $h$-nullcline for

$h = 0.0 \rightarrow w =$................................................................................................................
$h = 0.1 \rightarrow w =$................................................................................................................
$h = 0.5 \rightarrow w =$................................................................................................................
$h = 1.0 \rightarrow w =$................................................................................................................

number of points: /2

[Hints: insert the definitions of $g$ in the regimes $h \leq 0.1$, $0.1 < h < 1$ and $h \geq 1$; note that for $h = 0.5$ we have $g(h) = 2$, and for $h = 0.1$ we have $g(h) = 0.19 \cdot 8/3$ as you can easily verify by inserting the value for $\nu_0$].

(d2) sketch the nullclines in the range $0 < w < 0.4$ and $-0.1 < h < 1$ in the space here:

number of points: /4

(e) in the graph above, draw arrows on all relevant parts of the nullclines

number of points: /2

(f) in the above graph indicate qualitatively the direction of arrows in all relevant regions of the phase plan. Consider in particular the points $(h=1, w=0.2); (h=0, w=0.2); (h=1, w=0); (h=0.2, w=0)$

(You may assume $\tau_w = 3\tau$.)

number of points: /2
(g) in the above graph sketch the evolution of a network where the four neurons
start with an input potential of $h = 1.0$ and weights $w_{ij} = 0.2$. You may assume
that $\tau_w \gg \tau$. number of points: $/2$

(h) in the above graph sketch the evolution of a network where the four neurons
start with an input potential of $h = 0.2$ and weights $w_{ij} = 0.01$. You may assume
that $\tau_w \gg \tau$. number of points: $/2$

(i) Is the network useful as a memory unit? If yes, why? Would your answer
change if we set $a_0 > 0$? Would your answer change if we set $a_0 < 0$? Justify
your answers.

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number of points: $/2$
3 Stochastic Neuron Model (11pts)

Throughout this exercise, we consider stochastically spiking neurons with a stochastic intensity (or 'hazard')

\[ \rho(t) = g(h) = [1 - (1 - h)^2] \nu_0 \quad \text{for} \quad 0 < h < 1 \]

and \( g(h) = 0 \) for \( h \leq 0 \) and \( g(h) = \nu_0 \) for \( h \geq 1 \). After each spike, the neuron has an absolute refractory time of \( \Delta \) during which it does not fire (i.e., the hazard vanishes).

(a) Calculate the mean firing rate \( \nu \) for \( h = 0.5 \) and \( h = 2 \).

\( h = 0.5 \rightarrow \nu = \frac{1}{2} \) 

\( h = 2.0 \rightarrow \nu = \frac{1}{4} \)

number of points: /2

(b) What is the interval distribution \( P(s) \) for the case \( h = 2.0 \).

\( h = 2.0 \rightarrow P(s) \frac{1}{4} \)

number of points: /2

(c) we now have \( N = 200 \) neurons. The input to all neurons switches every 500ms between a value of \( h_1 \) and a value of \( h_2 \), starting at \( t = 0 \) with \( h = h_1 \).

Suppose you have a measurement device that allows you to record the spikes of each of the 200 neurons.

What is the expected number of spikes \( \bar{n}_{\text{spikes}} \) after 5 seconds of measurements? (keep the values \( h_1 \) and \( h_2 \) as well as \( \Delta \) arbitrary)

\( \bar{n}_{\text{spikes}} = \frac{1}{4} \)

number of points: /2
(d) Suppose that, during the above experiment with $N = 200$ neurons, the stimulus is switched on at $t = 0$ and is constant thereafter with a value $h_1 = h_2 = 0.5$. Suppose $\nu_0 = 200\text{Hz}$ and $\Delta = 10\text{ms}$. What is the expected population activity $A(t)$ at $t = 4\text{s}$?

$$A(t) = \ldots \ldots$$

number of points: \(/2\)

(e) Suppose that neurons have been silent for $t < 0$ (e.g., $h = -1$ for $t < 0$) and the above input $h = 0.5$ is switched on at $t = 0$.

e1 What is the expected population activity $A$ at $t = 1\text{ms}$? (justify your answer).

$$A(t) \approx \ldots \ldots$$

because \ldots \ldots

\ldots \ldots

number of points: \(/3\)

e2 Is it different from the value calculated in (d)? (justify your answer)

Different/not different \ldots

because \ldots \ldots

\ldots \ldots

number of points: \(/3\)

(e3) How many neurons do you expect to fire between $t = 0$ and $t = 10\text{ms}$? A rough estimate is sufficient: nearly all neurons, about half but rather more, about half but rather less, significantly less than half etc. [Hint: how many neurons survive without firing?]

Out of the $N = 200$ neurons, I expect that \ldots \ldots fire because

\ldots \ldots

\ldots \ldots

number of points: \(/3\)