Position Control by Examples

1 Device description

Consider the following device (Figure 1). It is a cable driven disc steered by a Brushless Maxon\(^1\) DC motor. It is equipped with two incremental encoders for position measurement. The first one is on the motor shaft and the second one is on the output shaft.

The device parameters are considered as follows:

- \( M_D \) and \( J_D \) are respectively the Mass and the Inertia of the disc relative to its rotation center.
- \( r_g \) is the distance of the center of mass of the disc to its rotation center.
- \( I_m \) is the inertia of the motor.
- The gear ratio of the cable based transmission is 15.

2 Device Dynamic Model

By considering \( k_{\text{vis}} \) and \( \Gamma_{\text{dry}} \), respectively the viscous friction constant and the dry friction at the output side, the Euler dynamic equation is written as follows

\[
\sum M = J_{RL} \dot{\Theta}_L = \Gamma_{\text{act}} - \Gamma_{\text{dry}} - k_{\text{vis}} \Theta_L - \Gamma_g \\
(\text{eq. 1})
\]

\[
\Gamma_g = M_D \cdot g \cdot r_g \cdot \sin(\Theta_L) \\
(\text{eq. 2})
\]

\( \Gamma_g \) is the gravity torque. \( \Gamma_{\text{act}} \) is the actuating torque reported at the output side.

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\(^1\) www.maxonmotor.ch
$J_{RL}$ is the total inertia reported at the load side

$$J_{RL} = J_D + n^2 J_m$$  \hspace{1cm} (eq. 3)

Combining (eq.2) and (eq.3) leads to:

$$J_{RL} \ddot{\theta}_L = \Gamma_{act} - \Gamma_{dry} - k_{vis} \dot{\theta}_L - M_D g \ r_g \ sin(\theta_L)$$  \hspace{1cm} (eq. 4)

3  Control implementation

3.1 Exact compensation strategy

The exact compensation control strategy consists of using the motor torque control to compensate all the resistant torques. It can be as simple as compensating the gravity, the friction or any other known effects.

Hence, the actuating control torque may be written as:

$$\Gamma_{act} = \Gamma_{act}^* + \Gamma_{compensation}$$  \hspace{1cm} (eq. 5)

Where $\Gamma_{compensation}$ is the compensating torque and $\Gamma_{act}^*$ is the additional actuating torque assuring a desired objective (position, velocity or force control). This additional torque may come from a PID regulator.

(eq. 4) may be rewritten as follows:

$$J_{RL} \ddot{\theta}_L = \Gamma_{act} - M_D g \ r_g \ sin(\theta_L) - \Gamma_{dry} - k_{vis} \dot{\theta}_L$$

Torques (effects) to be compensated

We need to compensate the gravity, viscosity and the dry friction. This must be done by rejecting these values. We pose:

$$\Gamma_{act} = \Gamma_{act}^* + M_D g \ r_g \ sin(\theta_L) + \Gamma_{dry} + k_{vis} \dot{\theta}_L$$  \hspace{1cm} (eq. 6)

That means that the compensating torque is given by:

$$\Gamma_{compensation} = M_D g \ r_g \ sin(\theta_L) + \Gamma_{dry} + k_{vis} \dot{\theta}_L$$  \hspace{1cm} (eq. 7)

![Figure 3- Compensating feedback loop](image)
The relation between $\theta_L$ and the new control variable torque is then given by the following dynamic equation:

$$I_R \ddot{\theta} = \Gamma_{act}^*$$  \hspace{1cm} (eq. 8)

**Important**

Note that the relation between $\theta_L$ and $\Gamma_{act}^*$ (eq. 4) is nonlinear. After the applied compensating feedback, the relation between $\theta_L$ and $\Gamma_{act}^*$ becomes linear this operation is also called exact nonlinear compensation or exact linearisation. Furthermore the obtained system (eq.8) is a double integrator that may be controller by any conventional controller.

![Figure 4- Position control with a an exact compensating feedback loop](image)

Finally, the control expression of the torque applied to the motor may be given by:

$$\Gamma_{act} = \Gamma_{PID} + M_D g r_g \sin(\theta_L) + \Gamma_{dry} + k_{vis} \dot{\theta}_L$$  \hspace{1cm} (eq. 9)

Where

$$\Gamma_{PID} = K_p \cdot \left( e + T_d \cdot \frac{de}{dt} + \frac{1}{T_i} \cdot \int e(\tau) d\tau \right)$$  \hspace{1cm} (eq. 10)

### 3.2 Feed forward (A priori) strategy

We can evaluate the needed torque to realize a known trajectory defined by the desired position, velocity and acceleration thanks to the dynamic inverse model. This torque is called the “a priori torque” and is defined as the motor torque for a given time-defined trajectory (eq. 11).

$$\Gamma_{ap} = \Gamma_{motor}(\theta = \theta_d(t), \dot{\theta} = \dot{\theta}_d(t), \ddot{\theta} = \ddot{\theta}_d(t))$$  \hspace{1cm} (eq. 11)

$$\Rightarrow \Gamma_{ap} = J_RL \ddot{\theta}_d + M_D g r_g \sin(\theta_d) + k_{vis} \dot{\theta}_d + \Gamma_{dry}$$  \hspace{1cm} (eq. 12)

Inertia feed forward  \hspace{1cm} “A priori”  \hspace{1cm} Gravity feed forward  \hspace{1cm} “A priori”  \hspace{1cm} Friction feed forward  \hspace{1cm} “A priori”

The final control torque is the sum of both contributions: the a priori torque computed thanks to the desired position and a closed loop torque provided by a chosen control algorithm such as a PID function (eq.13).
The actuating control torque at the load side may be expressed as follows:

\[ \Gamma_{\text{act}} = \Gamma_{\text{PID}} + J_{RL} \dot{\theta}_d + M_d g r_g \sin(\theta_d) + k_{\text{vis}} \dot{\theta}_d + \Gamma_{\text{dry}} \]  

(eq. 13)

The closed loop behavior

By considering only the a priori loop, the dynamic behavior is deduced and given by (eq. 14)

\[ J_{RL} \ddot{\theta} = \Gamma_R - J_{RL} \dot{\theta}_d + M_d g r_g \sin(\theta_d) + k_{\text{vis}} \dot{\theta}_d + \Gamma_{\text{dry}} - M_d g r_g \sin(\theta_L) - k_{\text{vis}} \dot{\theta}_L - \Gamma_{\text{dry}} \]

\[ \varepsilon = \theta - \theta_d \]

\[ \Rightarrow J_{RL} \ddot{\varepsilon} = \Gamma_R - k_{\text{vis}} \dot{\varepsilon} + \left\{ M_d g r_g \sin(\theta_d) - M_d g r_g \sin(\theta_L) \right\} \]  

(eq. 14)

Important

This behavior does not correspond to a linear system caused by the presence of the sinus function. To carry out the stability analysis of such dynamics after using a linear controller (PD, PID, ...), one must make a linearization around an operational point. Another solution is to use nonlinear stability analysis tools such as Lyapounov stability purposes.

This means that in the presence of nonlinearities, the proof of the stability when using a priori controller must be taken with a particular care. Even though, this scheme remains commonly used in robotic industrial control. This is due to its easiness of implementation while having the possibility to take into account coupling effects when multi-DOF robots are considered.