Thermodynamic variables of interest

1. Temperature (T)

2. Dew-point temperature (T_d)

3. Mixing ratio (r)
   \[ r = \frac{\rho_v}{\rho} \]

4. Specific humidity (q)
   \[ q = \frac{\rho_v}{\rho} \]

5. Potential temperature (\(\theta\)) - Poisson equation:
   It is conserved for adiabatic processes.

\[ \theta = T + \frac{g}{C_p} z \]

\[ \frac{\partial \theta}{\partial z} = \left( \frac{\partial T}{\partial z} + \frac{g}{C_p} \right) \]

\(C_p=\) specific heat at constant pressure

Lapse Rate:
\[ \Gamma = \frac{\partial T}{\partial z} \]

\(T_d:\) Temperature at which the air becomes saturated for a given water vapor pressure e

Temperature that an air parcel with absolute temperature T and pressure P would have if brought adiabatically to the pressure of 1000-mb (100 KPa)

\[ \theta = T \left( \frac{P_0}{P} \right)^{0.286} \]

Dry adiabatic lapse rate \(= 9.8 \degree C/km\)

ADIABATIC case:
\[ \frac{\partial \theta}{\partial z} = 0 \]

\[ \frac{\partial T}{\partial z} = -\frac{g}{C_p} \]
5. **Virtual Temperature** \( (T_v) \) - for unsaturated air, \( T_v \) is given by

\[
T_v = T (1 + 0.61q)
\]

*Temperature at which dry air has the same density as moist air at the same pressure*

\( q \) is the specific humidity

6. **Virtual Potential Temperature** \( (\theta_v) \) given by

\[
\theta_v = \theta (1 + 0.61q)
\]

*Definition of ATMOSPHERIC STABILITY based on \( \theta_v \)*

**Atmospheric stability:**

- **NEUTRAL:** Dry Adiabatic lapse rate \( \left( \frac{\partial \theta_v}{\partial z} = 0 \right) \) + NO convection

- **UNSTABLE:** Superadiabatic lapse rate \( \left( \frac{\partial \theta_v}{\partial z} < 0 \right) \)

- **STABLE:** Subadiabatic lapse rate \( \left( \frac{\partial \theta_v}{\partial z} > 0 \right) \)
Virtual Temperature ($T_v$): Derivation

**Dalton’s Law:** The total pressure in a mixture of perfect gases equals the sum of the partial pressures

\[ p = \sum p_i \]

**Ideal (perfect) gas law**

\[ \rho_i = \frac{p_i}{R_i T} \]

\[ R_v = \frac{R^*}{m_{H_2O}} \]

\[ \frac{R_v}{R_d} = \frac{m_d}{m_{H_2O}} \]
Virtual Temperature ($T_v$): Derivation (cont.)

Recall:
- Mixing ratio: $r = \frac{\rho_v}{\rho_d}$
- Specific humidity: $q = \frac{\rho_v}{\rho}$

\[
\rho_d = \frac{p - e}{R_d T} \quad \quad \rho_v = \frac{e}{R_v T} = \frac{0.622 e}{R_d T}
\]

\[
\rho = \rho_v + \rho_d = \frac{p}{R_d T} \left( 1 - \frac{0.378 e}{p} \right)
\]

\[
p = \rho R_d T \left( \frac{p}{p - 0.378 e} \right) = \rho R_d T \left( 1 + \frac{0.378 e}{p - 0.378 e} \right)
\]

\[
q = \frac{\rho_v}{\rho} = \frac{0.622 e}{p - 0.378 e}
\]

\[
0.622 = 18.02 / 28.97 = m_{H_2O} / m_d
\]

\[
p = \rho R_d T (1 + 0.61q)
\]

\[
T_v = T \left( 1 + 0.61q \right)
\]
Fig. 1.5
Example of the difference between mean potential temperature, $\bar{\theta}$, and mean virtual potential temperature, $\bar{\theta}_v$, given observations of mixing ratio, $\bar{r}$, and absolute temperature, $\bar{T}$. Dew point, $T_d$, is also shown.
Over the Oceans:

• Boundary layer **depth varies relatively slowly** in space and time. **Why?**

• Most changes in boundary layer depth over the oceans are **caused by synoptic or mesoscale processes**, creating vertical motion and/or advection of different air masses over the sea surface.
Near a region of **high pressure**:

- The boundary layer tends to be shallower near the center of High pressure.
  - This is due to the associated subsidence and divergence.
  - Few clouds form in this region; **Why?**
- Boundary layer depth increases on the periphery of the ‘High’ where the subsidence is weaker. Clouds are more likely to form in this region.
Near a region of **low pressure**:

- The rising motion associated with the ‘Low’ transports boundary layer air up into the free atm.
- Hence, it is often difficult to find the top of the boundary layer in this region.
- The boundary layer is not evident in a sounding. The cloud base is often used at the top of the boundary layer.
Over land in the vicinity of a high pressure area, the boundary layer is often very well defined and evolves with the diurnal cycle:

There are different sublayers within the boundary layer. The main layers are:
- surface layer
- convective mixed layer
- residual layer
- stable (nocturnal) boundary layer

It is also common to find:
- cloud layers
- entrainment zone
- capping inversion

(from Stull 1988)
Diurnal evolution of the ABL

Day time Unstable (convective) boundary layer
1:00 PM

Nocturnal Stable (stratified) boundary layer
5:00 AM

Interactive animations at:
http://eddycation.safl.umn.edu
1. **Viscous sublayer** The *lowest few millimeters* of air in the surface layer.
   - Within this layer, molecular transport of heat, moisture, and momentum is much more effective than turbulent transport.

2. **Roughness sublayer (Canopy sublayer)** Lowest portion of the surface layer, in which the influence of individual roughness elements (soil roughness, plants, buildings) can readily be discerned. In city centers, it may comprise a significant portion of the urban boundary layer, especially at night.
Mean velocity over smooth surfaces

\[ z^+ = \frac{z u_*}{\nu} \]

Logarithmic law in the surface layer (lowest 10-20%) and above viscous sublayer:

\[ \frac{U(z)}{u_*} = \frac{1}{K} \ln \left( \frac{u_* z}{\nu} \right) + 5 \]

Linear velocity profile in the Viscous sublayer

\[ k: \text{ von Karman constant (}=0.4) \]

Figure 13.18 Law of the wall. A typical data cloud is shaded.
Typical velocity profiles in NEUTRAL turbulent boundary layers over ‘smooth’ and ‘rough’ surfaces

Figure 13.19 Logarithmic velocity distributions near smooth and rough surfaces: (a) smooth wall; and (b) rough wall.

Figure 13.18 Law of the wall. A typical data cloud is shaded.

Figure 10.9 Comparison of observed velocity profiles over crops with the log law [Equation (10.14)]. [From Plate (1971).]
The Logarithmic Wind Profile
In the Surface Layer (approximately Lowest 10-20% of the boundary layer)

\[ U(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right) \]
3. **Surface layer.** Above the interfacial layer, in the heart of the surface layer, turbulent transport dominates. Typical depth: ~10-20% of boundary layer depth.

- **Gradients** of temperature, moisture and winds can be very large in the surface layer.

- The lapse rate within the surface layer is super-adiabatic under convective conditions.
4. **Mixed layer** located above the surface layer and below the entrainment zone.

- Heat, moisture and momentum are uniformly mixed within the mixed layer.
- **Turbulence** within the mixed layer is mainly convectively driven from two main sources:
  - **heat transfer** from the warm ground to the interfacial layer via conduction and then convective transport of this heat by thermals up into the mixed layer.
  - **radiative cooling** from the top of the cloud layer creating "upside down" thermals of cool, sinking air.
- **wind shear** can also generate mechanical turbulence within the mixed layer.
- The mixed layer begins to grow vertically approximately 1/2 hour after sunrise. It grows rapidly during the morning hours and reaches a maximum depth in the afternoon.

![Diagram of Mixed Layer](image_url)
Q: How does the mixed layer grow?

- The turbulence (largely the convectively driven thermals) entrains down potentially warmer, usually drier, less turbulent air down into the mixed layer.
- These variables are then well mixed within the mixed layer.

5. Entrainment Layer.

- As shown in the above figures, the entrainment layer (zone) is a stable layer above the mixed layer.
- It acts as a lid to rising thermals.
- It is often an inversion layer, but not always.
- Waves can often propagate on top of the mixed layer within and above the entrainment zone.

[Inversion: $dTdz>0$]
Example: free convection
Example: free convection
The residual layer and the nocturnal boundary layer

The Residual Layer

- Approximately 1/2 hour before sunset, the thermals in the convectively mixed boundary layer have shut off as the surface is cooling.
- Above the stable boundary layer, the residual layer is found, and can be thought of as a left-over convective mixed layer.
- The residual layer, thus, has all the properties of the recently decaying convective mixed layer.
- The static stability of this layer of air is then: ........ ???
- The residual layer is not in direct contact with the ground, and therefore, is strictly speaking, not a boundary layer.

The Stable (Nocturnal) Boundary Layer

- As evening progresses and the surface cools via radiative cooling, a shallow stable layer of air forms that is in direct contact with the ground (0-200 m or so deep).
  - strong static stability
  - weak/sporadic turbulence - often occurs in short bursts [high intermittency]
  - weak/calm winds at the surface, but increasing to supergeostrophic speeds aloft ->
  - This wind speed profile is often referred to as a low-level, or nocturnal jet. The low-level flow is often decoupled from the flow aloft within the low-level jet. It is possible for the surface winds to be calm, while, a few 10's of meters aloft, the winds can be 10-30 m/s.
  - Waves are often observed within the nocturnal boundary layer, more specifically, gravity (buoyancy) waves. Gravity waves are generated in statically stable layers of air.
Fig. 1.11  Mean virtual potential temperature, $\bar{\theta}_v$, and wind speed, $\bar{M}$, profiles for an idealized stable boundary layer in a high-pressure region.
Fig. 1.12
Profiles of mean virtual potential temperature, $\theta_v$, showing the boundary-layer evolution during a diurnal cycle starting at about 1600 local time. S1-S6 identify each sounding with an associated launch time indicated in Fig. 1.7.
Fig. 1.13  Lofting of a smoke plume occurs when the top of the plume grows upward into a neutral layer of air while the bottom is stopped by a stable layer.

Fig. 1.14  Sketch of the fumigation process, where a growing mixed layer mixes elevated smoke plumes down to the ground. Smoke plume 1 is fumigated at time F1, while plume 2 is fumigated at time F2.
Looping Plume Type

Unstable

Loopying

Fanning Plume Type

Stable

Fanning

Source: UCAR
Reynolds decomposition

\[ u_i = \bar{u}_i + u'_i \]
\[ \theta_v = \bar{\theta}_v + \theta'_v \]
\[ q = q + q' \]

( ) \_s space average
( ) \_t time average
( ) \_e ensemble average

For **homogeneous and stationary** (statistically not changing over time) turbulence:

**ERGODIC condition**

\[ ( )_e = ( )_t = ( )_s = ( ) \]

What is ‘**isotropic**’ turbulence?
- *Turbulence statistics invariant under rotation of coordinate system*

What is ‘**homogeneous**’ turbulence?
- *Turbulence statistics invariant under translation of coordinate system*
Turbulence Kinetic Energy

Mean Kinetic Energy

\[
\frac{MKE}{m} = \frac{1}{2}(\overline{u_i^2}) = \frac{1}{2}(u_1^2 + u_2^2 + u_3^2)
\]

Turbulence Kinetic Energy

\[
e = \frac{TKE}{m} = \frac{1}{2}(\overline{u_i^2}) = \frac{1}{2}(\overline{u_1'^2} + \overline{u_2'^2} + \overline{u_3'^2})
\]

Fluxes

Fig. 2.11
Momentum can be split into the three cartesian directions, based on the u, v, and w components of wind. Momentum flux can consist of the transfer of any of these three components in any of three directions: x, y, and z, yielding a total of nine momentum flux components.

From Stull (1988)
Energy Spectrum and *The energy gap*

![Graph showing energy spectrum and eddy frequency & time period]

**Fig. 2.2** Schematic spectrum of wind speed near the ground estimated from a study of Van der Hoven (1957).

*From Stull (1988)*
**FLUX: Transfer rate of a quantity per unit area per unit time**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Flux</th>
<th>Kinematic Flux</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat</td>
<td>$Q^*$</td>
<td>$Q = \frac{Q^*}{\rho_{\text{fluid}}C_p}$</td>
</tr>
<tr>
<td></td>
<td>$\left[ \frac{J}{m^2s} \right]$</td>
<td>$\left[ \frac{Km}{s} \right]$</td>
</tr>
<tr>
<td>Pollutant</td>
<td>$q_{\text{pollut.}}^*$</td>
<td>$q_{\text{pollut}} = \frac{q_{\text{pollut}}^*}{\rho_{\text{air}}}$</td>
</tr>
<tr>
<td></td>
<td>$\left[ \frac{kg_{\text{pollut.}}}{m^2s} \right]$</td>
<td>$\left[ \frac{kg_{\text{pollut}}m}{kg_{\text{fluid}}s} \right]$</td>
</tr>
<tr>
<td>Momentum</td>
<td>$F^*$</td>
<td>$F = \frac{F^*}{\rho_{\text{fluid}}}$</td>
</tr>
<tr>
<td></td>
<td>$\left[ \frac{kg \cdot m \cdot s^{-1}}{m^2s} \right]$</td>
<td>$\left[ \frac{mm}{s \cdot s} \right]$</td>
</tr>
</tbody>
</table>

**Mean Fluxes**

- $\overline{W} \cdot \overline{\theta}$
- $\overline{W} \cdot \overline{q}$
- $\overline{W} \cdot \overline{U}$

**Turbulent (Reynolds) Fluxes**

- $\overline{w'\theta'}$
- $\overline{w'q'}$
- $\overline{w'u'}$
Unstable (convective) boundary layer
Stable boundary layer (negative buoyancy)
Time series of velocity and temperature collected with a sonic anemometer at 20 Hz.
Quadrant analysis

$u' - w'$

$w' - \theta'$

Note: color scale indicates number of points per pixel
In turbulent boundary layers:

\[ u'w' < 0 \]

Turbulent (Reynolds) stresses
Turbulent stresses:

\[ u'w' < 0 \]

Turbulent fluxes:

Stable boundary layers: \[ w'\theta' < 0 \]

Untable boundary layers: \[ w'\theta' > 0 \]
Physical interpretation of turbulent flux and its sign

Unstable (convective) boundary layer

(a) Eddy mixes some air down, and some up.

Net upward heat flux

w' = neg.

θ' = neg.

θ = pos.

0 1

Stable boundary layer

(b) Eddy mixes some air up, and some down.

Net downward heat flux

w' = pos.

θ' = neg.

θ = pos.

0 1

Fig. 2.12 Idealization of the small eddy mixing process, showing (a) net upward turbulent heat flux in a statically unstable environment, and (b) net downward turbulent heat flux in a stable environment. From Stull (1988)

EXERCISE: Show that <u'w'> should always be negative.
Governing Equations for Turbulent Flows

1. Equation of State (Ideal Gas Law)

\[ p = \rho R_d T_v \]

2. Conservation of Mass (Continuity)

\[ \frac{\partial \rho}{\partial t} + \sum_{j} \left( \rho u_j \right) \frac{\partial}{\partial x_j} = 0 \]

Incompressible

\[ \frac{d \rho}{dt} = 0 \]

\[ \frac{\partial u_j}{\partial x_j} = 0 \]

3. Conservation of Momentum

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\delta_{i3} g - 2 \varepsilon_{ijk} \Omega_j u_k - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} \]

**I**  **II**  **III**  **IV**  **V**  **VI**

**I** Storage of momentum (inertia)

**II** Advection

**III** Gravity (vertical direction)

**IV** Coriolis effects (due to earth’s rotation)

**V** Pressure gradient forces

**VI** Influence of viscous stresses
4. Conservation of Moisture

\[ \frac{\partial q}{\partial t} + u_j \frac{\partial q}{\partial x_j} = \nu_q \frac{\partial^2 q}{\partial x_j^2} + \frac{S_q}{\rho} \]

5. Conservation of Heat

\[ \frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = \nu_\theta \frac{\partial^2 \theta}{\partial x_j^2} + \frac{S_\theta}{\rho C_p} \]